MODELING FOR OPTIMAL CONTROL: A VALIDATED DIESEL-ELECTRIC POWERTRAIN MODEL

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ABSTRACT

An optimal control ready model of a diesel-electric powertrain is developed, validated and provided to the research community. The aim of the model is to facilitate studies of the transient control of diesel-electric powertrains and also to provide a model for developers of optimization tools. The resulting model is a four state three control mean value engine model that captures the significant nonlinearity of the diesel engine, while still being continuously differentiable.
Keywords: Modeling, Optimal Control, Diesel engine, Diesel-Electric

NOMENCLATURE

Symbol	Description	Unit
p	Pressure	Pa
T	Temperature	K
ω	Rotational speed	rad/s
N	Rotational speed	rpm
$\dot{m}$	Massflow	kg/s
P	Power	W
M	Torque	Nm
Π	Pressure ratio	-
V	Volume	m^3
η	Efficiency	-
A	Area	m^2
Ψ	Head parameter	-
Φ	Flow parameter	-
γ	Specific heat capacity ratio	-
c_p	Specific heat capacity constant pressure	J/(kg · K)
c_v	Specific heat capacity constant volume	J/(kg · K)
R	Gas Constant	J/(kg · K)
r_c	Compression ratio	-
n_{cyl}	Number if cylinders	-
$(A/F)_s$	Stoichiometric air/fuel-ratio	-
q_{HV}	Lower heating value of fuel	J/kg
u_f, u_{wg}, P_{gen}	Control signals	mg/cycle, -, W
J	Inertia	kg · m^2
BSR	Blade speed ratio	-
R	Radius	m
λ	Air/fuel equivalence ratio	-
ϕ	Fuel/air equivalence ratio	-

Table 1: Symbols used

Index	Description	Index	Description
amb	Ambient	*c*	Compressor
im	Intake manifold	*em*	Exhaust manifold
01	Compressor inlet	02	Compressor outlet
eo	Engine out	*a*	Air
e	Exhaust	*ac*	After Compressor
f	Fuel	*ice*	Engine
GenSet	Engine-Generator	*t*	Turbine
wg	Wastegate	*es*	Exhaust System
vol	Volumetric	*d*	Displaced
fric	Friction	*pump*	Pumping
ig	Indicated gross	*mech*	Mechanical
tc	Turbocharger	*ref*	Reference

Table 2: Subscripts used

INTRODUCTION

Optimal control can be an important tool to gain insight into how to control complex nonlinear multiple-input multiple-output systems. However for the model to be analyzable and also for the results to be relevant, higher demands are set on model quality. This relates both to differentiability of the model, for efficient solution processes of the optimal control problem, and also its extrapolation properties since the obtained solutions are often on the border to or outside the nominal operating region. This paper presents the modelling and final model of a diesel-electric powertrain to be used in the study of transient operation. This optimal control ready model will also be made available to the research community to further encourage optimal control studies.

The resulting model is a four state, three control, mean value engine model (MVEM) that consists of 10 submodels that are all continuously differentiable, and suitable for automatic differentiation, in the region of interest in order to enable the nonlinear program solvers to use higher order search methods.

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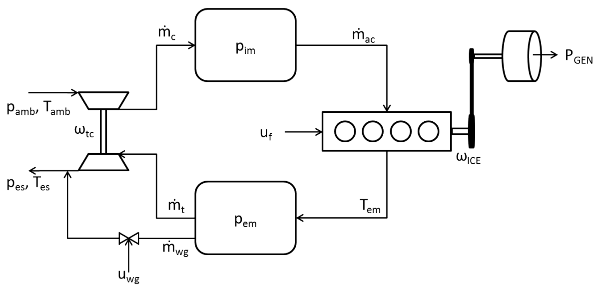

Figure 1: Structure of the model

In engine simulation the component efficiencies are often implemented as maps. In an optimal control framework such strategies are undesirable, instead the developed model includes analytically differentiable efficiency models for the compressor, turbine, cylinder massflow, engine torque and generator power. The efficiency map of the measured production engine is highly nonlinear, see Fig. 3-left, something that is well captured by the developed model, as seen in Fig. 2-left. The resulting mean relative model errors are less than 2.9% for the states and less than 5.4% for the component models.

A typical internal combustion engine normally has an efficiency ”‘island”’ located near the maximum torque line where its peak efficiency is obtained, see [1, 2, 3]. Due to the special nature of the efficiency map of the measured engine the model is also provided with a second torque model, yielding a more typical efficiency map, see Fig. 2-right.

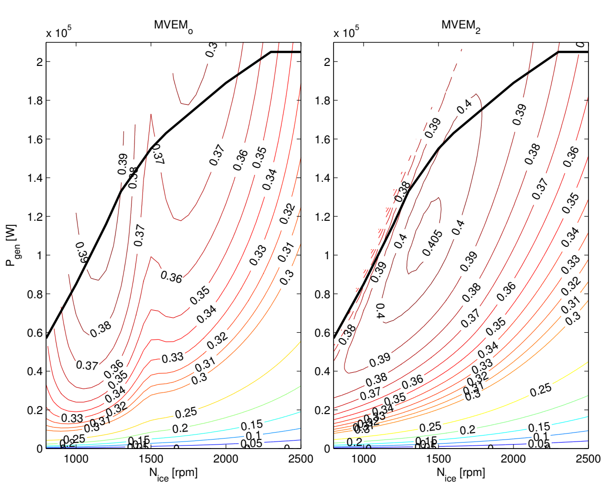

Figure 2: Efficiency of the two models, $MVEM_o$: a model trying to capture the characteristics of the modeled engine (left) and $MVEM_2$: a model representing a typical engine (right).

CONTRIBUTIONS

The contributions of the paper are three-fold: 1) A methodology how to model and parametrize a model of a diesel-electric powertrain is presented. The measurements are conducted without a dynamometer, the only requirements are a diesel-electric powertrain and sensors. 2) A model structure and modeling approach with provided equations, enabling researchers to adjust the parameters of the model to represent their own powertrain. 3) It also provides researchers without engine models or data a relevant and validated open source model on which control design or optimization can be performed.

MODEL STRUCTURE

The aim of the model is control systems design and optimization. This imposes the requirement that the model has to be detailed, but at the same time computationally fast. This leads to a 0-D or MVEM approach. Within MVEM there are two different approaches, one is black box modelling or standard system identification techniques, another is physical modelling where the engine is described using standard physical relations. Due to that one of the model aims is optimization and the solution of optimization problems often are on the border to or outside the nominal operating region the physical modeling approach is selected for its extrapolation properties. For more information about engine modelling as well as the state of the art of engine models the reader is referred to [1, 2].

MODELING

The measured and modeled engine-generator combination (GenSet) consists of a generator mounted on the output shaft of a medium-duty tier 3 diesel-engine. The engine is equipped with a charge air cooled wastegated turbocharger. The states of the developed MVEM are engine speed, ω_{ice}, inlet manifold pressure, p_{im}, exhaust manifold pressure, p_{em}, turbocharger speed, ω_{tc}. The controls are injected fuel mass, u_f, wastegate position, u_{wg}, and generator power, P_{gen}.

The submodels are models for compressor massflow and power, engine out and exhaust manifold temperatures, cylinder massflow, turbine massflow and power, wastegate massflow, engine torque and generator power, with connections between the compo-

Measured	Implemented	Measured	Implemented
ω_{ice}	State	T_{amb}	Constant
p_{im}	State	T_{01}	T_{amb}
p_{em}	State	T_{02}	not used
ω_{tc}	State	T_{im}	Constant
$\dot{m}_f$	Control (u_f)	T_{em}	Static model
u_{wg}	Control	p_{amb}	Constant
P_{gen}	Control	p_{01}	p_{amb}
$\dot{m}_c$	Static model	p_{02}	p_{im}
p_{es}	Constant	λ	Static model

Table 3: Measured variables and their implementation in the model.

nents as in Fig 1. The signals measured and also how they are implemented in the model are listed in Table 3. The data sets used are described in Appendix and listed in Table 5-7.

The tuning process is that first the component models are tuned to stationary measurements. Then the dynamic models are tuned using the results from the component tuning, and finally the whole model is tuned to both dynamic and stationary measurements. In the dynamic and full model tuning all measured signals except the states and $\dot{m}_f$ are used.

Error measure

In the modeling the following relative error is used:

$$e_{\text{rel}}(k) = \frac{y_{mod}(k) - y_{meas}(k)}{\frac{1}{M}\sum_{l=1}^{M}|y_{meas,stat}(l)|} \tag{1}$$

i.e. regardless of whether it is dynamic or stationary measurements that are considered the error is normalized by the mean absolute value from the stationary measurements. In the tuning it is the euclidean norm of this relative error that is minimized.

Dynamic Models

There are four dynamic models, two rotational states and two manifolds. The rotational states, ω_{ice} and ω_{tc} are modelled using Newton's second law

$$J_{GenSet}\frac{d\omega_{ice}}{dt} = \frac{P_{ice} - P_{mech}}{\omega_{ice}} \tag{2}$$

$$J_{tc}\frac{d\omega_{tc}}{dt} = \frac{P_t\eta_{tm} - P_c}{\omega_{tc}} \tag{3}$$

and the manifolds are modelled using the standard isothermal model [4]

$$\frac{dp_{im}}{dt} = \frac{R_a T_{im}}{V_{im}}\left(\dot{m}_c - \dot{m}_{ac}\right) \tag{4}$$

$$\frac{dp_{em}}{dt} = \frac{R_e T_{em}}{V_{em}}\left(\dot{m}_{ac} + \dot{m}_f - \dot{m}_t - \dot{m}_{wg}\right) \tag{5}$$

where in the tuning the measured intake manifold temperature, T_{im} is used but in the final model the intercooler is assumed to be ideal, i.e. no pressure loss and T_{im} constant. The dynamic models have four tuning parameters, J_{GenSet}, J_{tc}, V_{im} and V_{em}.

Compressor

The compressor model consists of two sub-models, one for the massflow and one for efficiency. In order to avoid problems for low turbocharger speeds and transients with pressure ratios $\Pi_c < 1$ a variation of the physically motivated $\Psi\ \Phi$ model in [1] is used. The idea is that Ψ approaches a maximum at zero flow and that the maximum flow in the region of interest is quadratic in ω_{tc}.

Massflow model

The pressure quotient over the compressor:

$$\Pi_c = \frac{p_{02}}{p_{01}} \tag{6}$$

Pressure ratio for zero flow:

$$\Pi_{c,max} = \left(\frac{\omega_{tc}^2 R_c^2 \Psi_{max}}{2c_{p,a}T_{01}} + 1\right)^{\frac{\gamma_a}{\gamma_a - 1}} \tag{7}$$

Corrected and normalized turbocharger speed:

$$\omega_{tc,corr,norm} = \frac{\omega_{tc}}{15000\sqrt{T_{01}/T_{ref}}} \tag{8}$$

Maximum corrected massflow:

$$\dot{m}_{c,corr,max} = c_{\dot{m}_{c,1}}\omega_{tc,corr,norm}^2 + c_{\dot{m}_{c,2}}\omega_{tc,corr,norm} + c_{\dot{m}_{c,3}} \tag{9}$$

Corrected massflow:

$$\dot{m}_{c,corr} = \dot{m}_{c,corr,max}\sqrt{1 - \left(\frac{\Pi_c}{\Pi_{c,max}}\right)^2} \tag{10}$$

The massflow is then given by:

$$\dot{m}_c = \frac{\dot{m}_{c,corr}\,p_{01}/p_{ref}}{\sqrt{T_{01}/T_{ref}}} \tag{11}$$

The surge-line is modeled using the lowest massflows for each speedline from the compressor map and is well described by the linear relationship:

$$\Pi_{c,surge} = c_{\dot{m}_{c,surge,1}}\dot{m}_{c,corr} + c_{\dot{m}_{c,surge,2}} \tag{12}$$

In an optimization context surge is undesirable why this is implemented as a constraint according to:

$$\Pi_c \leq \Pi_{c,surge} \quad (13)$$

Efficiency model

The efficiency of the compressor is modeled using a quadratic form in the flow parameter Φ and speed ω_{tc} following [1]. The dimensionless flow parameter is defined as:

$$\Phi = \frac{\dot{m}_c R_a T_{01}}{\omega_{tc} 8 R_c^3 p_{01}} \quad (14)$$

Deviation from optimal flow and speed:

$$d\Phi = \Phi - \Phi_{opt} \quad (15)$$

$$d\omega = \omega_{tc,corr,norm} - \omega_{opt} \quad (16)$$

The compressor efficiency is given by:

$$\eta_c = \eta_{c,max} - \begin{bmatrix} d\Phi \\ d\omega \end{bmatrix}^T \begin{bmatrix} Q_1 & Q_3 \\ Q_3 & Q_2 \end{bmatrix} \begin{bmatrix} d\Phi \\ d\omega \end{bmatrix} \quad (17)$$

The consumed power is calculated as the power from consumed in an isentropic process divided by the efficiency:

$$P_c = \frac{\dot{m}_c c_{p,a} T_{01} \left(\Pi_c^{\frac{\gamma_a - 1}{\gamma_a}} - 1 \right)}{\eta_c} \quad (18)$$

Initialization

The compressor has 10 tuning parameters, Ψ_{max}, $c_{\dot{m}_{c,1-3}}$, Φ_{opt}, $\eta_{c,max}$ and ω_{opt}, Q_{1-3}. The model is first fitted to the compressor map then to the stationary measurements, using data set A, but then $\dot{m}_c$ is measured and η_c and P_c are calculated according to:

$$\eta_c = \frac{T_{01}(\Pi_c^{1-1/\gamma_a} - 1)}{T_{02} - T_{01}} \quad (19)$$

$$P_c = \dot{m}_c c_{p,a}(T_{02} - T_{01}) \quad (20)$$

The results are mean/max absolute errors of [2.4/8.2, 2.3/23.2, 1.4/7.8] % for [$\dot{m}_c$, η_c, P_c] respectively.

Cylinder Gas Flow

The cylinder gas flow models are models for the air and fuel flow in to the cylinder. The airflow model is a model for the volumetric efficiency of the engine. The model used is the same as in [5] according to:

$$\eta_{vol} = c_{vol,1}\sqrt{p_{im}} + c_{vol,2}\sqrt{\omega_{ice}} + c_{vol,3} \quad (21)$$

$$\dot{m}_{ac} = \frac{\eta_{vol} p_{im} \omega_{ice} V_d}{4\pi R_a T_{im}} \quad (22)$$

The control signal u_f is injected fuel mass in mg per cycle and cylinder and the total fuel flow is thus given by:

$$\dot{m}_f = \frac{10^{-6}}{4\pi} u_f \omega_{ice} n_{cyl} \quad (23)$$

The air-fuel equivalence ratio λ is computed using:

$$\lambda = \frac{\dot{m}_{ac}}{\dot{m}_f} \frac{1}{(A/F)_s} \quad (24)$$

In diesel engines a lower limit on λ is usually used in order to reduce smoke. However in fuel cut, i.e. $u_f = 0$, $\lambda = \infty$ which is undesirable in optimization. Instead the fuel-air equivalence ratio ϕ is used and the lower limit on λ can be expressed as:

$$\phi = \frac{\dot{m}_f}{\dot{m}_{ac}}(A/F)_s \quad (25)$$

$$0 \leq \phi \leq \frac{1}{\lambda_{min}} \quad (26)$$

Initialization

The tuning parameters of the gas flow models are $c_{vol,1-3}$. The model is initialized using all stationary measurements, i.e. set A using that at stationary conditions $\dot{m}_{ac} = \dot{m}_c$. The volumetric efficiency model corresponds well to measurements with a mean/max absolute relative error of [0.9/3.7] %.

Engine torque and generator

The engine torque is not measured so the tuning of the torque models have to rely on the DC-power out from the power electronics. Then there are actually three efficiencies that should be modeled, the power electronics, the generator, and the engine efficiencies. In Fig. 3-left the total efficiency of the powertrain is shown, with the maximum power line.

First the engine torque model is tuned. In the tuning the engine torque is calculated using the stationary efficiency map of the generator, provided by the manufacturer. The efficiency of the power electronics is lumped with the generator efficiency and is here assumed to be 0.98. Then the generator model

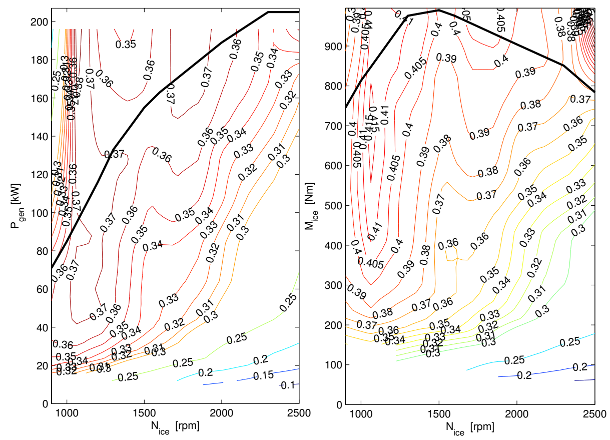

Figure 3: Efficiency of the powertrain (left) and efficiency of the engine (right)

is tuned, first using the stationary map and then measurements but with the torque calculated using the efficiency map.

Engine torque model

In Fig. 3-right the efficiency of the engine is shown, with M_{ice} calculated using the generators efficiency map and 2% losses in the power electronics assumed. The engine torque is modeled using three components, see [4], i.e. friction torque, M_{fric}, pumping torque M_{pump} and gross indicated torque, M_{ig}. The torque consumption of the high pressure pump is not modeled on it's own, but lumped in to the following models. The net torque of the engine can then be computed.

$$M_{ice} = M_{ig} - M_{fric} - M_{pump} \tag{27}$$

The pumping torque is proportional to the pressure quotient over the cylinder:

$$M_{pump} = \frac{V_d}{4\pi}\left(p_{em} - p_{im}\right) \tag{28}$$

The friction torque is modeled as a quadratic shape in engine speed:

$$M_{fric} = \frac{V_d}{4\pi} 10^5 \left(c_{fr1}\omega_{ice}^2 + c_{fr2}\omega_{ice} + c_{fr3}\right) \tag{29}$$

The indicated gross torque is proportional to the fuel energy:

$$M_{ig} = \frac{u_f 10^{-6} n_{cyl} q_{HV} \eta_{ig}}{4\pi} \tag{30}$$

Where the indicated gross efficiency is defined as:

$$\eta_{ig} = \eta_{ig,t}\left(1 - \frac{1}{r_c^{\gamma_{cyl}-1}}\right) \tag{31}$$

The torque model in (27)-(31) is fairly common, and if $\eta_{ig,t}$ is implemented as a constant maximum brake torque (MBT)-timing is assumed. A typical internal combustion engine normally has an efficiency "'island"' located near the maximum torque line where its peak efficiency is obtained, see [1, 2, 3]. However looking at Fig. 3-right this is clearly not the case. Therefore the model is provided with two different torque models, seen in Fig. 4.
Torque model 1 (TM1) is used in the model tuning and validation and is designed to capture the non-linear nature seen in Fig. 3. TM1 consists of two second order polynomials and a switching function:

$$\eta_{ig,t} = M_{f,1} + g_f\left(M_{f,2} - M_{f,1}\right) \tag{32}$$

$$g_f = \frac{1 + tanh(0.1(\omega_{ice} - 1500\pi/30))}{2} \tag{33}$$

$$M_{f,1} = c_{M_{f,1},1}\omega_{ice}^2 + c_{M_{f,1},2}\omega_{ice} \tag{34}$$

$$M_{f,2} = c_{M_{f,2},1}\omega_{ice}^2 + c_{M_{f,2},2}\omega_{ice} + c_{M_{f,2},3} \tag{35}$$

Torque model 2 (TM2) is designed and provided to represent a "'typical"' engine with an efficiency island, to be used for optimal control studies, and is thus not used in the tuning or validation. TM2 is quadratic in $\frac{u_f}{\omega_{ice}}$ and expressed as

$$\eta_{ig,t} = \eta_{ig,ch} + c_{u_f,1}\left(\frac{u_f}{\omega_{ice}}\right)^2 + c_{u_f,2}\frac{u_f}{\omega_{ice}} \tag{36}$$

The maximum power line is implemented as a limit on the net power of the engine, $P_{ice} = T_{ice}\omega_{ice}$, which is well approximated by two quadratic functions and a maximum value:

$$P_{ice} \le P_{ice,max} \tag{37}$$

$$P_{ice} \le c_{P_1}\omega_{ice}^2 + c_{P_2}\omega_{ice} + c_{P_3} \tag{38}$$

$$P_{ice} \le c_{P_4}\omega_{ice}^2 + c_{P_5}\omega_{ice} + c_{P_6} \tag{39}$$

Initialization

The two torque models have eight and six tuning parameters respectively. The tuning parameters are c_{fr1-3}, and $c_{M_{f,1},1-2}$, $c_{M_{f,2},1-3}$, or $\eta_{ig,ch}$ and $c_{u_f,1-2}$ The models are fitted using set C. For (32) it is rather straight forward. For model (36) the "island" is not

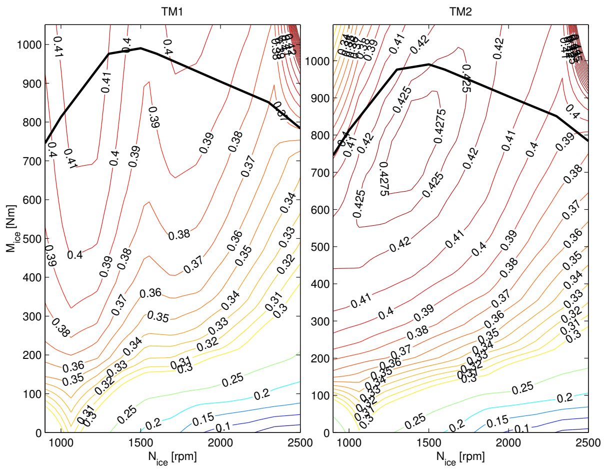

Figure 4: The two different torque models. Left: (32) certification speed . Right: (36) "‘Typical"’

visible in the measured data, therefore the parameters of $\eta_{ig,ch}$ are manually tuned and the M_{fric} model is tuned assuming MBT-timing. The mean/max absolute relative errors of TM1 are [2.2/10.9] %.

Generator model

Looking at Fig. 5 a reasonable first approximation of the relationship between mechanical and electrical power of the generator is two affine functions, something normally denoted willans line, [6], where the slope of the line depends on whether the generator is in generator or motor mode.

$$P^{+}_{mech} = e_{gen,1}P_{gen} + P_{gen,0}, \quad \text{if } P_{gen} \geq 0 \quad (40)$$

$$P^{-}_{mech} = e_{gen,2}P_{gen} + P_{gen,0}, \quad \text{if } P_{gen} < 0 \quad (41)$$

This model is not continuously differentiable so therefore to smoothen it out a switching function is used. The model is then given by:

$$P_{mech} = P^{-}_{mech} + \frac{1+\tanh(0.005P_{gen})}{2}(P^{+}_{mech} - P^{-}_{mech}) \quad (42)$$

$e_{gen,1-2}$ are seen to have a quadratic dependency on ω_{ice}, a reasonable addition to the willans line is thus to model $e_{gen,1-2}$ as:

$$e_{gen,x} = e_{gen,x-1}\omega^2_{ice} + e_{gen,x-2}\omega_{ice} + e_{gen,x-3} \quad (43)$$

which constitutes the full model.

Initialization

The generator model has seven tuning parameters, $P_{gen,0}$ and $e_{gen,1/2,1-3}$. The model is first fitted to the generator map and secondly to measurement data, using set C. The mean/max absolute relative errors of the generator model are [0.7/2.5] %.

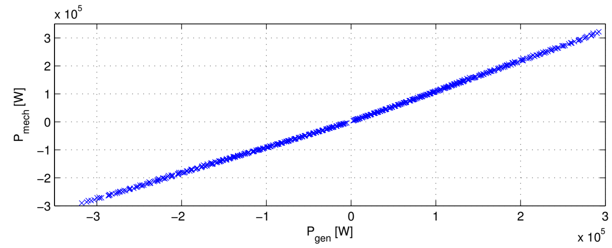

Figure 5: Mechanical generator power as a function of electrical power

Exhaust temperature

The cylinder out temperature model is based on ideal the Seiliger cycle and is a version of the model found in [5]. The model consists of the pressure quotient over the cylinder:

$$\Pi_e = \frac{p_{em}}{p_{im}} \quad (44)$$

The specific charge:

$$q_{in} = \frac{\dot{m}_f q_{HV}}{\dot{m}_f + \dot{m}_{ac}}(1 - x_r) \quad (45)$$

The combustion pressure quotient:

$$x_p = \frac{p_3}{p_2} = 1 + \frac{q_{in}x_{cv}}{c_{v,a}T_1 r_c^{\gamma_a - 1}} \quad (46)$$

The combustion volume quotient:

$$x_v = \frac{v_3}{v_2} = 1 + \frac{q_{in}(1-x_{cv})}{c_{p,a}\left(\frac{q_{in}x_{cv}}{c_{v,a}} + T_1 r_c^{\gamma_a - 1}\right)} \quad (47)$$

The residual gas fraction:

$$x_r = \frac{\Pi_e^{1/\gamma_a} x_p^{-1/\gamma_a}}{r_c x_v} \quad (48)$$

Temperature after intake stroke:

$$T_1 = x_r T_{eo} + (1 - x_r)T_{im} \quad (49)$$

The engine out temperature:

$$T_{eo} = \eta_{sc}\Pi_e^{1-1/\gamma_a} r_c^{1-\gamma_a} x_p^{1/\gamma_a - 1} \left(q_{in}\left(\frac{1-x_{cv}}{c_{p,a}} + \frac{x_{cv}}{c_{v,a}}\right) + T_1 r_c^{\gamma_a - 1}\right) \quad (50)$$

To account for the cooling in the pipes the model from [7] is used, where V_{pipe} is the total pipe volume:

$$T_{em} = T_{amb} + (T_{eo} - T_{amb}) e^{-\frac{h_{tot} V_{pipe}}{(\dot{m}_f + \dot{m}_{ac}) c_{p,e}}} \tag{51}$$

The model equations described in (45)-(50) are non-linear and depend on each other and need to be solved using fixed point iterations. In [5] it is shown that it suffices with one iteration to get good accuracy if the iterations are initialized using the solution from last time step. In an optimization context remembering the solution from last time step is difficult and also using a model that uses an unknown number of iterates is undesirable. However the loss in model precision of assuming no residual gas, i.e. $x_r = 0$, is negligible therefore this is assumed. Further, the addition of heat loss in the pipe through (51) drives x_{cv} to zero. The reduced model is then given by:

$$q_{in} = \frac{\dot{m}_f q_{HV}}{\dot{m}_f + \dot{m}_{ac}} \tag{52}$$

$$T_{eo} = \eta_{sc} \Pi_e^{1-1/\gamma_a} r_c^{1-\gamma_a} \left(\frac{q_{in}}{c_{p,a}} + T_{im} r_c^{\gamma_a - 1} \right) \tag{53}$$

$$T_{em} = T_{amb} + (T_{eo} - T_{amb}) e^{-\frac{h_{tot} V_{pipe}}{(\dot{m}_f + \dot{m}_{ac}) c_{p,e}}} \tag{54}$$

Initialization

The used temperature model has two tuning parameters, η_{sc} and h_{tot}. The first step of the initialization assumes that there is no heat loss in the manifold before the sensors. Then the complete model is fitted using the results from $T_{em} = T_{eo}$. The nominal set is used in the fitting. The mean/max absolute relative error of the temperature model is [1.9/5.4] % and the error increase from assuming $x_r = 0$ is [0.014/0.06] ‰.

Turbine and Wastegate

Since the massflow is not measured on the exhaust side, the models for wastegate and turbine have to be fitted together.

$$\Pi_t = \frac{p_{es}}{p_{em}} \tag{55}$$

Turbine

The massflow is modeled with the standard restriction model, using that half the expansion occurs in the stator and half in the rotor, see [8]:

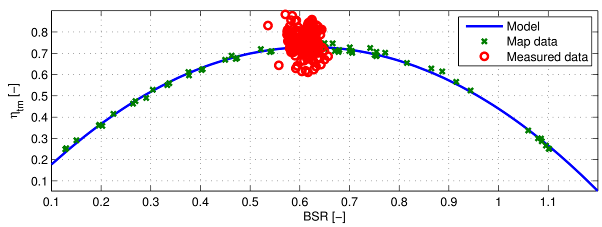

Figure 6: BSR model and its fit to map and measured data

$$\Pi_t^* = \max\left(\sqrt{\Pi_t}, \left(\frac{2}{\gamma_e + 1}\right)^{\frac{\gamma_e}{\gamma_e - 1}}\right) \tag{56}$$

$$\Psi_t(\Pi_t^*) = \sqrt{\frac{2\gamma_e}{\gamma_e - 1}\left((\Pi_t^*)^{\frac{2}{\gamma_e}} - (\Pi_t^*)^{\frac{\gamma_e + 1}{\gamma_e}}\right)} \tag{57}$$

$$\dot{m}_t = \frac{p_{em}}{\sqrt{R_e T_{em}}} \Psi_t A_{t,eff} \tag{58}$$

The turbine efficiency is modeled as a quadratic shape in blade-speed ratio (BSR), as used in [9, 8] .

$$BSR = \frac{R_t \omega_{tc}}{\sqrt{2 c_{p,e} T_{em} (1 - \Pi_t^{\frac{\gamma_e - 1}{\gamma_e}})}} \tag{59}$$

$$\eta_{tm} = \eta_{tm,max} - c_m (BSR - BSR_{opt})^2 \tag{60}$$

The power to the turbocharger is then:

$$P_t \eta_m = \dot{m}_t c_{p,e} T_{em} \eta_{tm} \left(1 - \Pi_t^{\frac{\gamma_e - 1}{\gamma_e}}\right) \tag{61}$$

Due to uncertainty of the behaviour outside the mapped region, and to avoid problems with negative turbine efficiency, a reasonable constraint is to restrict BSR to the maximum and minimum values provided in the map, i.e. $BSR_{min} \leq BSR \leq BSR_{max}$.

Wastegate

The wastegate massflow is modeled with the standard restriction model and an effective area that changes linearly in u_{wg}.

$$\Pi_{wg}^* = \max\left(\Pi_t, \left(\frac{2}{\gamma_e + 1}\right)^{\frac{\gamma_e}{\gamma_e - 1}}\right) \tag{62}$$

$$\Psi_{wg} = \sqrt{\frac{2\gamma_e}{\gamma_e - 1}\left((\Pi_{wg}^*)^{\frac{2}{\gamma_e}} - (\Pi_{wg}^*)^{\frac{\gamma_e + 1}{\gamma_e}}\right)} \tag{63}$$

$$\dot{m}_{wg} = \frac{p_{em}}{\sqrt{R_e T_{em}}} \Psi_{wg} u_{wg} A_{wg,eff} \tag{64}$$

Initialization

The initialization uses data set C. The massflow models need to be fitted together and the turbine efficiency cannot be calculated from measurements since none of the massflows are measured. Looking at the nominal data set the quadratic shape in BSR is not observed since the measurements are rather constant in BSR, see Fig. 6. Since this shape is nonexistent in the measurements the efficiency model of the turbine is locked to the map fit since otherwise it would converge to an arbitrary shape trying to capture as much as the cloud nature of the measured data as possible. One could consider adding pulse compensation factors for the massflow and efficiency but the resulting improvements are small.

The massflow models are fitted together using $\dot{m}_{ac} + \dot{m}_f = \dot{m}_t + \dot{m}_{wg} = \dot{m}_{exh}$. Friction losses according to $P_c = P_t\eta_m - w_{fric}\omega_{tc}^2$ can be added, however the parameter w_{fric} becomes small in the optimization.

The final turbine and wastegate models have three tuning parameters, $A_{t,eff}$, $\eta_{tm,max}$ and $A_{wg,eff}$. The results are mean/max relative errors of [2.3/5.4, 4.7/17.0] % for [$\dot{m}_{exh}$, $P_t\eta_{tm}$] respectively.

Exhaust flow models

Using the standard restriction model a max-expression is necessary under the square root to keep the flow real, representing choking which occurs at $\Pi_t^{-1} \approx [3.3, 1.8]$ for the turbine and wastegate. However such expressions are undesirable when using optimization tools. Instead the following expressions are used:

$$\Psi_t = c_{t,1}\sqrt{1 - \Pi_t^{c_{t,2}}} \quad (65)$$

$$\Psi_{wg} = c_{wg,1}\sqrt{1 - \Pi_t^{c_{wg,2}}} \quad (66)$$

The flow models are fitted to produce the same flow profile as the standard restriction models in (57), (63), where $c_{t,1-2}$ and $c_{wg,1-2}$ are tuning parameters.

Dynamic models

So far the models are tuned using stationary measurements. The next step is to tune the parameters of the dynamic models in (2)-(5). Since torque is not measured J_{GenSet} is fixed to it's real value and only V_{im}, V_{em} and J_{tc} are tuned. Since torque and eventual torque errors might lead to engine stalling the torque model is inverted to track the real engine speed trajectory. This will lead to that there will be almost no errors in engine speed. To fit the dynamic models data set D-I are used but only the transients in the measurements, plus a couple of seconds before and after. As in [5] the transient is also normalized to 0-1 so that the stationary point has no effect on the dynamics.

Full models

The full models are tuned using both dynamic and stationary measurements, using a similar cost function as in [5]. If the same cost function is used the model will not be able to reach the same maximum torque as the real engine for low engine speeds without λ being excessively low. Therefore to ensure that the model is able to span the entire operating range of the engine an addition is made. The model is simulated with $\lambda = \lambda_{min}$ for $N_{ice} = 800$ rpm and the models maximum torque is added to the cost function according to:

$$V_{M_{max}} = w_{M_{max}}\left(\frac{M_{ice,max,mod}(800rpm)}{M_{ice,max,meas}(800rpm)} - 1\right) \quad (67)$$

(67) assumes that the engine is smoke-limited at 800 rpm and maximum torque and thus tries to force the max torque of the model to coincide with that of the real engine, where $w_{M_{max}}$ is a weighting parameter.

To ensure reasonable behaviour also when the generator is in motoring mode this side is fitted using the efficiency map from the manufacturer with an assumed power electronics efficiency of 98%. For the stationary tuning set C is used and for the dynamics sets D-I are used. The full cost function is given by:

$$\begin{aligned} V_{tot}(\theta) = & \frac{1}{y_{dyn}M_{dyn}}\sum_{k=1}^{M_{dyn}}\sum_{y_n=1}^{y_{dyn}}\sum_{l=1}^{N_{dyn}}\frac{(e_{rel,dyn}^{y_n}(l))^2}{N_{dyn}} \\ & + \frac{1}{y_{stat}}\sum_{y_s=1}^{y_{stat}}\sum_{m=1}^{N_{stat}}\frac{(e_{rel,stat}^{y_s}(m))^2}{N_{stat}} \\ & + V_{M_{max}}^2 \end{aligned} \quad (68)$$

where y is the number of outputs, M the number of datasets and N the number of operating points in each dataset.

The models are also, as in [5], validated using only dynamic measurements and in particular all load transients, i.e. set $J_{0.1,\ 1,\ 2}$-$N_{0.1,\ 1,\ 2}$.

Table 4: Mean relative errors of the complete model. Bold marks variables used in the tuning and T, V, are the errors relative tuning and validation sets respectively.

	ω_{ice}		$\boldsymbol{p}_{im}$		$\boldsymbol{p}_{em}$		$\boldsymbol{\omega}_{tc}$	
	T	V	T	V	T	V	T	V
Dyn.	0.0	0.0	2.8	2.2	2.8	2.9	2.9	2.9
	$\boldsymbol{\dot{m}}_c$	$\boldsymbol{P}_c$	$\boldsymbol{\dot{m}}_{ac}$	$\boldsymbol{T}_{em}$	$\boldsymbol{\dot{m}}_{exh}$	$\boldsymbol{P}_t$	$\boldsymbol{P}^{+}_{mech}$	$\boldsymbol{P}^{-}_{mech}$
Stat.	2.5	1.8	2.5	2.4	3.3	5.4	3.4	1.4

RESULTS

The resulting fit to both tuning data and validation data is shown in Table 4. The variables used in the tuning are written in bold in the resulting tables. Table 4 shows that the model is a good mathematical repesentation of the measured system with state errors less than 3% and stationary errors in the same range. In Fig. 7 the state trajectories of the model are compared to measurements. There it is also seen that the agreement is good.

The pressure dynamics, and in particular the exhaust pressure, are faster than the speed dynamics therefore the resulting model is moderately stiff. This is also seen when selecting ode-solvers. In matlab ode23t and ode15s are twice as fast as the standard ode45 when simulating the model. When the states are normalized with their maximum values the relative and absolute tolerances [1e-4, 1e-7] are found to be good trade-offs between accuracy and performance.

CONCLUSION

In this paper a validated optimization ready model of a diesel-electric powertrain is presented. The resulting model is four state-three control mean value engine model, available for download in the **LiU-D-El**-package from [10]. The model is able to capture the highly nonlinear nature of the turbocharger diesel engine, and is at the same time continuously differentiable in the region of interest, to comply with optimal control software. The model is provided with two torque models to be used for optimal control studies. The first model, called $MVEM_o$ with a torque model representing the actual engine, as well as a model with a more general torque model aimed to represent a typical engine, called $MVEM_2$. Both $MVEM_o$ and $MVEM_2$ are included in the **LiU-D-El**-package together with a small example that can be downloaded fully parametrized from [10] implemented in matlab.

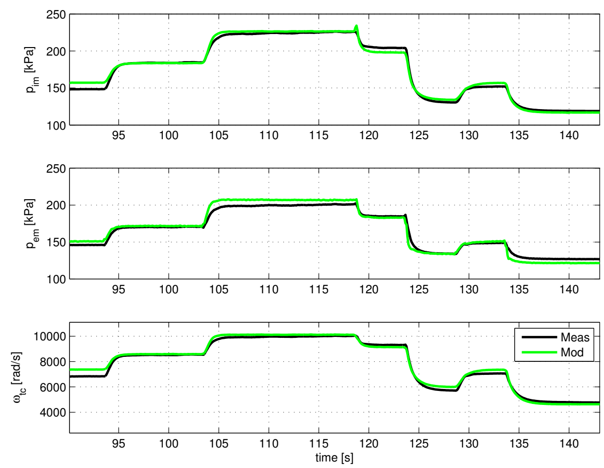

Figure 7: Model vs. measurements

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APPENDIX

DATA USED

There are a total of 192 stationary points measured. Of those 192, 53 are with the wastegate locked in a fixed position. Since injection timing is not measured those points are only used when fitting the gas flow models since there are some questions about what the engine control unit does when the wastegate control is altered. Nominal refers to unaltered wastegate, see Table 5

The dynamic data set consists of 21 measurements. The first six, D-I, are engine speed transients with constant(as close as the generator control can track) generator power and a sequence of steps in reference speed that the engine speed controller tries to track, see Table 6.

The last 15 sets are with constant reference speed, and different load steps, see Table 7. As with the speed transients the ECU controls the engine speed and the generator acts as a disturbance. The load transients are conducted at different engine speeds and then a programmed sequence of 23 power steps is performed with varying rise time, or rate at which the power changes. The first five, $J_{0.1} - N_{0.1}$ are with a ramp duration of 0.1s and the other are with 1s and 2s respectively. The total length of each set is approximately 300s.

Table 5: Stationary Data

Data Set	A	B	C
Delimiter	all	nominal	nominal & $P_{gen} > 0$
Nr. of points	192	139	127

Table 6: Speed transients

Data Set	D	E	F	G	H	I
P_{gen} [kW]	30	60	90	130	160	180
Nr. of steps	22	22	22	22	21	21

Table 7: Load transients

Data Set	$J_{0.1,\,1,\,2}$	$K_{0.1,\,1,\,2}$	$L_{0.1,\,1,\,2}$	$M_{0.1,\,1,\,2}$	$N_{0.1,\,1,\,2}$
Speed [rpm]	1100	1500	1800	2000	2200
Nr. of steps	23	23	23	23	23