

Cylinder state estimation from measured cylinder pressure traces - A Survey^{*}

Lars Eriksson^{*} Andreas Thomasson^{*}

^{} Vehicular Systems, Dept. of Electrical Engineering
Linköping University, SE-581 83 Linköping, Sweden,
larer@isy.liu.se, andreas.thomasson@liu.se*

Abstract: In the search for improved performance and control of combustion engines there is a search for the sensors that gives information about the combustion profile and the state of the gases in the combustion chamber. A particular interest has been given to the potential use of the cylinder pressure sensor and there is quite a lot of work that has been made in this area. This paper provides a comprehensive list of references and summarizes applications and methods for extracting information from the cylinder pressure sensor about the combustion and the gas state. The summary highlights the following topics related to cylinder pressure: measurement chain, cylinder torque, extraction of the burn profile, combustion placement, knocking, cylinder air mass, air to fuel ratio, residual gas estimation, and cylinder gas temperature estimation. The focus in the summary is on the latter topics about the gas state but thermodynamic analysis of the combustion process also gets a longer treatment since many methods for information extraction rely on the thermodynamic properties.

Keywords: Internal combustion engines, Engine modeling, Engine control, Cylinder pressure

1. INTRODUCTION

Cylinder pressure measurement has been used for engine research and development since John Southern, Watt's assistant, in 1796 invented an instrument to visually record the cylinder pressure throughout the entire expansion-exhaust cycle. He called this an "indicator diagram", Cummins (2000). It was first and mainly used for analysis of combustion and has later been used for control. An early control application was Cook et al. (1946) where an analog control system was designed and implemented by NACA to control the ignition timing based on the maximum pressure rise rate. In the mid 60's, with the early development of computer controlled engine management systems, several researchers have been investigating the use of cylinder pressure for estimation and control. Cylinder pressure based engine management has thus been a topic for quite some time and many applications and uses have been proposed. As a result, reviews of cylinder pressure based methods have been presented in the past. Two reviews of engine control methods using cylinder pressure sensors are found in Powell (1993); Iorio et al. (2003).

The main contribution of this paper is a literature survey with an overview of different applications where the cylinder pressure has come to use until present day. The focus in this paper is on the methods for gas state estimation, mass and composition, but other applications will be discussed as well.

2. OVERVIEW OF APPLICATIONS

The cylinder pressure has found its use in many applications and is a valuable tool in engine development and engine control and calibration. The measurement chain from sensor to estimation applications are illustrated in Figure 1. A set of applications are outlined in the list below, some of these are illustrated in Amann (1985) that highlights "cylinder-pressure measurement and its use in engine research", outlining applications such as heat release analysis, cyclic variability, and abnormal combustion like knock.

- Computation of indicated mean effective pressure (IMEP) and cylinder torque (Tq).
- Extraction of combustion process information such as mass fraction burned (MFB) profiles and heat release analysis (HR).
- Estimation of combustion phasing θ_{comb} for combustion timing control.
- Detection of knock intensity (KI) for knock control.
- Estimation of cylinder gas and wall temperature.
- Estimation of masses in the cylinder, total mass m_{cyl} , air mass m_a , residual and EGR mass m_r ,
- Estimation of air/fuel ratio, λ .

Where the last two are related as they relate air and fuel properties to each other, and they are the focus in this paper.

2.1 Measurement Chain

A prerequisite for the accuracy of the estimation methods and output is the quality of the signal and the accuracy of its acquisition. There are some well know papers for example "Measurement and analysis of pressure data"

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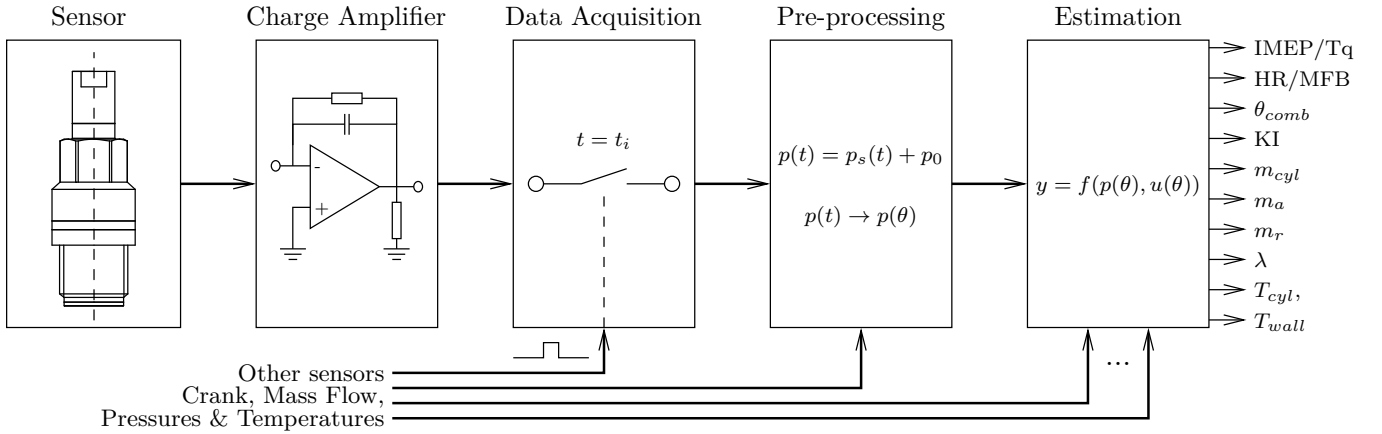


Fig. 1. Illustration of the cylinder pressure based measurement and estimation chain from sensor to calculate or estimated outputs.

is described in Lancaster et al. (1975), and “Methods of Processing Cylinder-Pressure Transducer Signals to Maximize Data Accuracy” in Randolph (1990). In the measurement chain, illustrated in Figure 1, the pressure sensor is first and it is often a piezo electric transducer that outputs a charge in response to a force on the sensor. This charge needs to be integrated to give a measured voltage output, therefore the need for the charge amplifier. The combustion chamber fluctuates in temperature during the cycle, and since the sensor can have different responses for different sensor temperatures, this influence the sensor and is called thermo-shock, see Rosseel et al. (1999) for an evaluation.

Due to the thermo shock and the integrating effect of the charge amplifier that could saturate, it is necessary to add a bleeding effect to the integrator which gives the measured cylinder pressure an unknown offset. An important step in the pressure analysis is to find the offset, so that the pressure gets the correct absolute level: this is called pressure pegging (Randolph, 1990). There are many approaches to this but the two most common ones are to set the cylinder pressure equal to the intake manifold pressure during the intake stroke. The other one is to use the polytropic relation and adjust the level of the pressure so that it agrees with the polytropic process. Different pegging methods are evaluated in Brunt and Pond (1997).

Either the sampling can be triggered by a crank angle encoder so that the sampling is event based or the cylinder pressure can be sampled time based together with crank related signal so that it can be related to the crank rotation. In this process it is necessary to perform a synchronization with the crank angle phasing and this is often called TDC determination. Different approaches for TDC determination are found in Staś (1996); Morishita and Kushiyama (1998); Staś (2000); Nilsson and Eriksson (2004); Tunestål (2011)

3. IMEP & TORQUE

With the sampled pressure synchronized with the crank rotation θ and thus the cylinder volume $V(\theta)$ known, the instantaneous torque $Tq(\theta)$ that the cylinder and crank mechanism produces on the crank shaft can be calculated. With the pressure $p(\theta)$, piston area A and lever arm $l(\theta)$ the gas pressure torque is expressed by:

$$Tq(\theta) = (p(\theta) - p_0) A l(\theta) = (p(\theta) - p_0) \frac{dV(\theta)}{d\theta} \quad (1)$$

where p_0 is the crank case pressure, i.e. the pressure at the bottom of the piston. To get crank acceleration from the driving torque accurately especially at dynamic and high speed conditions one also needs to account for the mass and moment of inertia effects of the piston and connecting rod in the linkage Schagerberg and McKelvey (2003).

The indicated work that the gas produces on the piston can be calculated from the $p dV$ work as

$$W_{ind} = \oint_{cycle} p(\theta) \frac{dV(\theta)}{d\theta} d\theta \quad (2)$$

When integrating over a cycle the crank case contribution vanishes since $\oint p_0 dV = 0$. The accuracy of these calculations in (1) and (2) will depend on the accuracy of the sensor and the correctness of the TDC determination, i.e. phasing between the pressure and volume. The implications of cylinder pressure and phasing errors are investigated in Brunt and Emtage (1996). Based on the indicated work the indicated mean effective pressures can be calculated

$$IMEP = \frac{W_{ind}}{V_d} \quad (3)$$

where V_d is the displacement of a single engine cylinder.

3.1 Cylinder Balancing

With cylinder pressures from the different cylinders the work produced by each individual cylinder can be balanced, either by adjusting the cylinder individual fuel injection or the combustion timing or both, depending on what is needed to eliminate the unbalance that has been identified. Cylinder balancing applications using cylinder pressures are mentioned in e.g. Sellnau et al. (2000); Schiefer et al. (2003); Husted et al. (2007); Willems et al. (2010)

4. HEAT RELEASE & MFB

An engine’s in-cylinder pressure reflects the piston work produced by the gas, the amount of chemical energy released in the combustion, and several other processes. The *heat release analysis* can be done taking into account the effects of volume change, heat transfer, and mass loss on

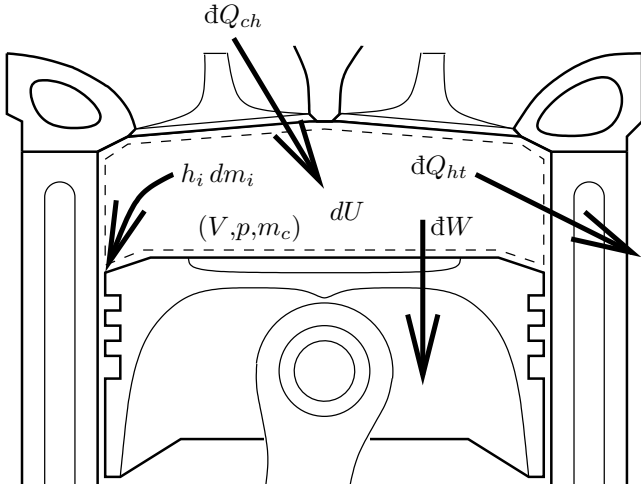


Fig. 2. Sketch of the combustion process in the cylinder, that defines the sign conventions used in the pressure and heat release models.

the cylinder pressure within the framework of the first law of thermodynamics (Gatowski et al., 1984). This analysis also allows a consistency check of the pressure data itself. Assuming that the heat release is proportional to the mass that burns then a normalized curve can be calculated and it is often referred to as the mass fraction burned trace.

Heat release analysis is often done in the framework of single-zone models. Some common assumptions made for the single zone models are as follows

- the cylinder contents and its thermodynamic state is uniform throughout the whole chamber
- the combustion is modeled as a release of heat
- the heat released from the combustion occurs uniformly in the chamber
- the gas mixture is an ideal gas

The basis for the majority of the heat release models is the first law of thermodynamics; the energy conservation equation. Associated with the analysis of combustion processes in internal combustion engines, e.g. heat release analysis, the literature follows a certain sign convention. The sign convention is illustrated in the schematic picture of an engine's cylinder in Figure 2.

The most common expression can be derived by considering the components of the heat transfer Q : the first component is the chemical energy in the fuel which releases energy as heat Q_{ch} , the other is the heat transfer to the walls which by definition cools (but sometimes also heats) the fluid Q_{ht} , where the heat transfer *from* the gases in the chamber to the cylinder wall is represented with a positive Q_{ht} . Work input is positive and the only work that is considered is the work that the fluid does on the piston W_p which is of the opposite sign to the definition $W = -W_p$. These sign conventions yield the most frequently used formulation of the first law for heat release analysis

$$\dot{Q}_{ch} = dU_s + \dot{d}W_p + \dot{d}Q_{ht} - \sum_i h_i dm_i \quad (4)$$

Before proceeding we will rewrite the basic heat release equation (4) by inserting the reversible work $\dot{d}W_p = p dV$ and utilizing the assumption of an ideal gas. The aim

is to eliminate the work and sensible energy differentials. First we express the sensible energy differential using the temperature and mass differential,

$$U_s = mu(T) \Rightarrow dU_s = m c_v dT + u(T) dm$$

In the next step the temperature differential is eliminated using the ideal gas state equation

$$T = \frac{pV}{mR} \Rightarrow dT = \frac{pV}{mR} \left(\frac{dp}{p} + \frac{dV}{V} - \frac{dm}{m} \right)$$

and using the ideal gas relations i.e. $c_p(T) - c_v(T) = R$ and $\gamma(T) = \frac{c_p(T)}{c_v(T)}$, resulting in

$$\dot{d}Q_{ch} = \frac{1}{\gamma(T) - 1} V dp + \frac{\gamma(T)}{\gamma(T) - 1} p dV + \dot{d}Q_{ht} - \sum_i (h_i(T') - u_i(T)) dm_i \quad (5)$$

where T' is the temperature of the flow dm_i and T is the temperature in the cylinder. Now the energy differential has been eliminated. It is worth to point out that c_p , c_v , γ , h_i and u_i all can depend on the temperature but this might not be written out explicitly in the sections to come.

With this equation as foundation several different models have been derived. In the following section four commonly used methods will be summarized where two are based directly on the (5) while the two other are indirectly related to it. First an approach that was presented the apparent heat release method presented by Krieger and Borman (1967) then the much used Gatowski et al. method presented 1984 and then the method presented in a landmark paper by Gerald M. Rassweiler and Lloyd Withrow presented 1938 and finally a computationally efficient method developed by Frederic Matekunas at GM 1986.

4.1 Apparent Heat-Release

The first work in heat release analysis based on the first law of thermodynamics was proposed in Krieger and Borman (1967) and was called the computation of *apparent heat release*. This single zone model is based on the first law of thermodynamics but take neither heat transfer nor crevice flows into account. The resulting equation for the chemical heat release is then

$$\dot{d}Q_{ch} = \frac{1}{\gamma(T) - 1} V dp + \frac{\gamma(T)}{\gamma(T) - 1} p dV \quad (6)$$

If the mass of mixture burned is assumed to be proportional to the amount of released heat at each step then the trace can be normalized to a mass fraction burned trace. This trace is fairly close to the heat release computed by the Rassweiler-Withrow method.

4.2 The Gatowski et al. (1984) Model

Gatowski et al. (1984) develops, tests, and applies a heat release analysis procedure that maintains simplicity, compared to the net heat release that adds the effects of heat transfer and crevice flows. The thermodynamic properties are represented by a linear approximation for $\gamma(T)$.

Using the relation between the mass flux into the crevice and into the cylinder $dm = -dm_{cr}$. The thermodynamic

properties are represented by a model for γ that is linear in temperature

$$\gamma(T) = \gamma_{300} + bT$$

The importance of the ratio of specific heats is pointed out in more detail in Klein (2007) where it is shown that it is one of the most important parameters when modeling the pressure and heat release traces.

Inserting the crevice model and the linear model for $\gamma(T)$, we can express c_v as follows

$$c_v(T) = \frac{R}{\gamma(T) - 1} = \frac{R}{\gamma_{300} + bT - 1}$$

The following differential equation can now be derived for the energy balance (with the crank angle selected as independent variable),

$$\begin{aligned} \frac{dQ_{ch}}{d\theta} &= \frac{1}{\gamma - 1} V \frac{dp}{d\theta} + \frac{\gamma}{\gamma - 1} p \frac{dV}{d\theta} + \frac{dQ_{ht}}{d\theta} \\ &+ \left(\frac{1}{\gamma - 1} T + T' + \frac{1}{b} \ln \left(\frac{\gamma' - 1}{\gamma - 1} \right) \right) \frac{V_{cr} dp}{T_w d\theta} \end{aligned} \quad (7)$$

In Gatowski et al. (1984) the heat transfer was modeled using the expression from Woschni (1967). There also exists several other correlations. The work Shayler et al. (1993) compared the correlations from Annand (1963), Woschni (1967), and Eichelberg 1939 and found that the heat transfer to the coolant best agreed with the Woschni correlation. At the same time, Hayes et al. (1993) also examined the accuracy of the Woschni correlation and reported good agreement with the measurement results for a 4 cylinder spark ignited engine.

The magnitude of the rate of energy transfer by convection modeled using Newton's law of cooling

$$\dot{Q}_{ht} = h A \Delta T = h_c A (T - T_w) \quad (8)$$

Where Woschni models the heat transfer coefficient in the following way

$$h_c = \frac{253.291 B^{-0.2} C_1 p^{0.8} \left(\frac{0.0034(p_f - p_m) T_{ivc} V_{dis} C_2}{p_{ivc} V_{ivc}} + U_p \right)^{0.8}}{T^{0.53}}$$

where the variables and units are:

B	Cylinder bore	[m]
p	Cylinder pressure	[atm]
w	Characteristic speed	[m/s]
T	Mean charge temperature	[K]
p_f	Pressure for firing cycle	[atm]
p_m	Pressure for motored cycle	[atm]
U_p	Mean piston speed	[m/s]

4.3 Rassweiler-Withrow Method

The Rassweiler-Withrow method (Rassweiler and Withrow, 1938), does not rely directly on the general framework following from the first law of thermodynamics but it is still important to cover since it is computationally efficient and many use the method for determining the mass fraction burned. The mass fraction burned $x_b = \frac{m_b}{m_{tot}}$ can be viewed as a normalized version of the heat release trace Q_{ch} such that it assumes values in the interval [0,1]. The relation between the mass fraction burned and amount of heat released can be justified by noting that the energy released from a system is almost proportional to the mass of fuel that is burned. A cornerstone for the method is the fact that

pressure and volume data can accurately be represented by the polytropic relation

$$pV^n = \text{constant} \quad (9)$$

where the exponent $n \in [1.25, 1.35]$ gives a good fit to experimental data for both compression and expansion processes in an engine (Lancaster et al., 1975). The exponent n is termed the polytropic index. It differs from γ since some of the effects of heat transfer are included implicitly in n . It is comparable to the average value of γ_u for the unburned mixture during the compression phase, prior to combustion. But due to heat transfer to the cylinder walls, index n is greater than γ_b for the burned mixture during expansion (Heywood, 1988).

In the Rassweiler-Withrow method, the actual pressure change $\Delta p = p_{j+1} - p_j$ during the interval $\Delta\theta = \theta_{j+1} - \theta_j$, is assumed to be made up of a pressure rise due to combustion Δp_c , and a pressure rise due to volume change Δp_v ,

$$\Delta p = \Delta p_c + \Delta p_v, \quad (10)$$

which is justified by (5). The pressure change due to volume change during the interval $\Delta\theta$ is approximated by the polytropic relation (9), which gives

$$\Delta p_v(j) = p_{j+1,v} - p_j = p_j \left(\left(\frac{V_j}{V_{j+1}} \right)^n - 1 \right). \quad (11)$$

Applying $\Delta\theta = \theta_{j+1} - \theta_j$, (10) and (11) yields the pressure change due to combustion as

$$\Delta p_c(j) = p_{j+1} - p_j \left(\frac{V_j}{V_{j+1}} \right)^n. \quad (12)$$

By assuming that the pressure rise due to combustion in the interval $\Delta\theta$ is proportional to the mass of mixture that burns, the mass fraction burned at the end of the j 'th interval thus becomes

$$x_{b,RW}(j) = \frac{m_b(j)}{m_b(\text{total})} = \frac{\sum_{k=0}^j \Delta p_c(k)}{\sum_{k=0}^M \Delta p_c(k)}, \quad (13)$$

where M is the total number of crank angle intervals and $\Delta p_c(k)$ is found from (12). The result from a mass fraction burned analysis is shown in figure 3, where the mass fraction burned profile is plotted together with the corresponding pressure trace. In the upper plot two cylinder pressure traces, one from a fired cycle (solid) and one from a motored cycle (dash-dotted) are displayed.

Relation to Heat Release Rate In Klein (2007) it is noted that there is a close relation between the classical Rassweiler-Withrow method and heat release analysis. If a heat-release trace is sought, the pressure change due to combustion in (5), $dp_c = \frac{n-1}{V} dQ$, can be rewritten and approximated by

$$\Delta Q_{RW}(j) = \frac{V_{j+1/2}}{n-1} \Delta p_c(j), \quad (14)$$

where the volume V during interval j is approximated with $V_{j+1/2}$ (the volume at the center of the interval), and $\Delta p_c(j)$ is found from (12). The heat-release trace is then found by summation.

4.4 Matekunas Pressure Ratio Algorithm

The concept of pressure ratio is another computationally efficient method for determining a trace similar to the heat release trace. It was developed by Matekunas at GM during

the 1980's, see Matekunas (1986) and Sellnau et al. (2000) for a description. This method does not rely directly on thermodynamic framework either but it is also interesting to cover this method since it is even more computationally efficient than the Rassweiler-Withrow method. In particular it does not require numeric integration from a start angle to the angle where we want to evaluate or analyze the mass fraction burned trace. Instead this method directly gives the approximation to the mass fraction burned at any crank angle.

The pressure ratio is defined using the ratio between a pressure from a firing cycle, $p_f(\theta)$, and the pressure from a motored cycle, $p_m(\theta)$,

$$PR(\theta) = \frac{p_f(\theta)}{p_m(\theta)} - 1 \quad (15)$$

The pressure ratio can be normalized with its maximum,

$$PR_N(\theta) = \frac{PR(\theta)}{\max PR(\theta)} \quad (16)$$

which results in a trace that is very similar to the mass fraction burned. An alternative way to normalize $PR(\theta)$ is to use the pressure ratio at a certain crank angle, when the combustion is assumed to be complete, e.g. selecting 100° as termination angle gives

$$PR_N(\theta) = \frac{PR(\theta)}{PR(100^\circ)}$$

Motored pressures can usually be exchanged for a polytropic model with reasonable accuracy, i.e.

$$p_m(\theta) = p_f(\theta_{\text{ref}}) \left(\frac{V(\theta_{\text{ref}})}{V(\theta)} \right)^n$$

where the pressure sample $p_f(\theta_{\text{ref}})$ has been determined at a reference angle before the combustion has started.

The method produces traces that are similar to the mass fraction burned profiles. The difference between them has been investigated in Eriksson (1999), and for the operating points used, the difference in position for $PR_N(\theta) = 0.5$ was in the order of 1-2 degrees. This suggests $PR_N(\theta)$ can be used as the mass fraction burned trace $x_{b,MPR}$.

The cylinder pressure in the upper plot of Figure 3 is analyzed and in the middle plots the mass fraction burned traces from Rassweiler-Withrow, Gatowski and Matekunas pressure ratio are shown. In the lower plot the heat release rate provided by the Gatowski method is given. A set of combustion descriptors for combustion phasing are also highlighted in the figure.

5. COMBUSTION PHASING FOR IGNITION CONTROL

In the search for improved fuel economy there has been an interest in placing the combustion so that the Maximum Brake Torque (MBT) is achieved. Currently most controllers use feedforward (open loop) from engine sensors but there is an interest to include feedback control so that the combustion can be placed optimally even under unmeasured disturbances and handle varying fuel properties and humidity. Several methods for determining the placement of the combustion have been proposed and those most frequently encountered are listed below. The combustion has its optimum placement if we place the combustion so that:

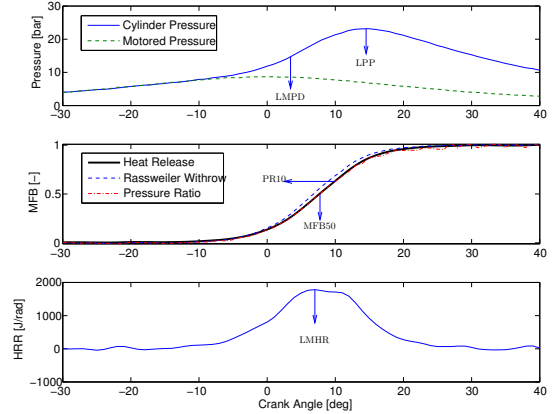


Fig. 3. Cylinder pressure, mass fraction burned and heat release rate. Some different combustion phase descriptors are also marked.

- LMPD – the maximum $dp/d\theta$ occurs at 3° ATDC, Cook et al. (1946).
- LPP – the location of the Pressure Peak (LPP) is at 15° ATDC, Hubbard et al. (1976).
- MFB50 – 50 % mass fraction burned (MFB50) occurs at 9° ATDC, Bargende (1995).
- PR10 – controlling the pressure ratio management at 10° ATDC to 0.55 Matekunas (1986).
- LMHR – Location of Maximum Heat Release Rate (LMHR) Pipitone (2008) at 9° ATDC.

These rules of thumb for combustion placement can be used directly in feedback control schemes that adjust the start of combustion so that the indicator ends up at the desired value.

6. KNOCK INTENSITY DETECTION

Knock detection and knock control from cylinder pressure sensors is a much studied topic and overviews are found in e.g. Burgdorf and Denbratt (1997); Puzinauskas (1992); Xiaofeng et al. (1993) where the latter summarizes and compares the following six different methods that later appears in many other evaluations:

- IMPO – Integral Modulus of the Pressure Oscillation, introduced by (Arrigoni et al., 1972).
- MAPO – Maximum Amplitude of the Pressure Oscillation, used by many (Leppard, 1982; Chun and Heywood, 1989).
- IMPOG – The Integral of the Modulus of the Pressure Oscillation Gradient, (Ferraro et al., 1985).
- PJMFB – Pressure Jump and Mass Fraction Burned (Klimstra, 1984; Chun and Heywood, 1989).
- KI20 – Knock Index, energy over 20 degrees, Konig and Sheppard (1990).
- D3PDT – Third Derivative of the Pressure Trace, Checkel and Dale (1986, 1989).

The first 5 all study a band-pass or high-pass filtered version of the pressure and study the amplitudes and energy contents. While the last looks at the peak negative value of the third derivative of the pressure crank angle trace as it was shown to give a measure of autoignition quantity

and hence of knock presence and severity. A classification into 8 possible classes of methods is made in Shahlari and Ghandhi (2012) the 8 classes come from the permutations among the following basic features

Frequency domain or	Time Domain
Pressure	or Heat release
Single Value	or Average Value

7. CYLINDER GAS STATE

The cylinder gas composition and temperature has always been important states in the internal combustion engine. For long time these states have therefore been subjected to different estimation and modeling approaches. A review of air charge estimation methods for spark ignited engines is presented in Wang et al. (2016), with the focus methods more commonly used in production such as MAF sensor, speed-density methods, input estimation and closed loop observers. Cylinder pressure based methods are only briefly mentioned. This section aims to give an overview of the methods proposed in the literature that to some extent uses cylinder pressure to calculate trapped mass, air charge, residual mass, air/fuel ratio and cylinder temperature.

Earlier research papers have compared a few of these methods before. An evaluation of three methods for calculating trapped air mass was presented in Worm (2005a). It compared using a standard volumetric efficiency approach to the ΔP method and the approach by Mladek and Onder (2000), both of the latter are described later in this section. A review of three methods for residual mass estimation are presented in Ortiz-Soto et al. (2012). Two methods use conditions at exhaust valve opening and closing, the third method is specific to HCCI engines with negative valve overlap and re-compression after EVC, proposed by Fitzgerald et al. (2010). These methods among others are included in the overview in this section.

For the mass estimation methods that include cylinder pressure, once the mass is known, cylinder temperature can be computed through the ideal gas law (17) since the mass, pressure and volume is known. The opposite is of course also true, if the cylinder temperature is estimated first, the total mass in the cylinder can be calculated. This is done in the majority of the methods presented in the following sections, and are therefore applicable to both cylinder mass and temperature estimation.

7.1 Ideal gas law and polytropic relation

Determining the total trapped mass in the cylinder is often a step in calculating the amount of trapped air, the residual gas mass and the air/fuel ratio. Several methods have been proposed that utilizes the cylinder pressure to varying degree. At the core of most methods are two important relations. The first is the ideal gas law

$$pV = mRT \quad (17)$$

that relates pressure, volume, mass and temperature for an ideal gas. The second important equation is the polytropic relation

$$pV^k = C \quad (18)$$

where k and C are constants that can be estimated for good fit to compression or expansion in a combustion engine.

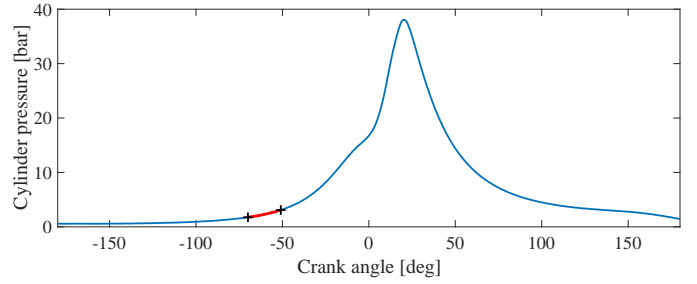


Fig. 4. Cylinder pressure trace from an SI engine. The two black crosses marks the points 70 and 50 crank angle degrees before top dead center, used for point a and b in the ΔP method in Arsie et al. (2014).

Sometimes k can be assumed known and constant, for example the value $k = 1.32$ is used for the compression in Mladek and Onder (2000). In general it depends on the gas composition, temperature and heat transfer. In Colin et al. (2007) k , and C in (18) are estimated by least squares to the measured pressure trace and the parameters then used in the cylinder mass estimate.

This relation can also be used for pegging the cylinder pressure sensor (adjusting for sensor offset). The estimate

$$\hat{p}_{cyl} = \frac{C}{V^k} \quad (19)$$

is used to calculate $\hat{p}_{cyl}(BDC)$ and the offset in the cylinder pressure signal is calculated by the difference between $\hat{p}_{cyl}(BDC)$ to the mean intake manifold pressure Colin et al. (2007). Alternatively if an accurate value for the polytropic exponent k is known from the gas composition, (18) can be used to estimate the sensor offset, since the estimated parameters C and k depend on the offset Müller and Isermann (2001); Sinnamon and Sellnau (2008).

7.2 The ΔP method

This method was pioneered by in Akimoto et al. (1989) and has since been used with various modification in for example Hart et al. (1998); Worm (2005b); Desantes et al. (2010); Arsie et al. (2014). The core of the method is to relate the pressure increase between two points during the compression stroke to the trapped mass in the cylinder. By utilizing the ideal gas law and the polytropic relation (18) the following expression can be derived

$$m_{cyl} = \frac{\Delta p V_a}{R T_a} \left\{ \left(\frac{V_a}{V_b} \right)^k - 1 \right\}^{-1} \quad (20)$$

where the subscripts a and b refer to two predetermined crank angles during the compression. The points 70 and 50 degrees before top dead center are used in Arsie et al. (2014), see Fig. 4. Additionally the polytropic exponent k and the temperature at the starting point T_a has to be known or estimated.

The temperature at the starting point T_a depends strongly on atmospheric temperature, boost pressure, compressor and intercooler efficiency Desantes et al. (2010). Additionally the residual gas fraction, the temperature of the residual gas, heat transfer from the block to the intake gas and compression until the point a will influence the starting temperature.

In the work by Desantes et al. (2010) linear and quadratic correlations are suggested for $T_a^{-1} \propto m_{cyl}/\Delta p$. The expression for the linear correlation is

$$T_a^{-1} \propto \frac{m_{cyl}}{\Delta p} = [b_1 \ b_2 \ b_3 \ b_4 \ b_5 \ b_6 \ b_7] \times \begin{bmatrix} 1 \ N \ p_{im} \ \frac{p_{em}}{p_{im}} \ T_w \ T_{im} \ m_f \end{bmatrix}^T \quad (21)$$

and the correlation with quadratic factors is given by

$$T_a^{-1} \propto \frac{m_{cyl}}{\Delta p} = [d_1 \ d_2 \ d_3 \ d_4 \ d_5 \ d_6 \ d_7 \ d_8 \ d_9 \ d_{10}] \times \begin{bmatrix} 1 \ N \ p_{im} \ \frac{p_{em}}{p_{im}} \ T_w \ T_{im} \ m_f \ N^2 \ N m_f \ m_f^2 \end{bmatrix}^T \quad (22)$$

where m_f is the injected fuel mass. The following correlation directly for T_a with fewer parameters are suggested in Arsie et al. (2014)

$$T_a = c_0 + c_1 N^2 + c_2 m_f + c_3 \frac{1}{N^3} + c_4 m_f^3 N \quad (23)$$

7.3 Fitting pressure trace during compression

Closely related to the ΔP method, all measured points in the pressure trace between two points during the compression can be fitted to the polytropic relation (9). Two observers that does this are proposed in Giansetti et al. (2007), in the first m_{cyl} is calculated by iteratively reducing the error ϵ in

$$\epsilon(i) = \sum_{\alpha=1}^{n_\alpha} \frac{\hat{p}_{cyl}(\alpha) - p_{cyl}(\alpha)}{n_\alpha} \quad (24)$$

$$\hat{p}_{cyl}(i, \alpha) = p_{cyl}(\alpha) \left(\frac{\hat{T}_{cyl}(i, \alpha)}{\hat{T}_{ref}(i)} \right)^{\frac{k(\alpha)}{k(\alpha)-1}} \quad (25)$$

$$\hat{T}_{cyl} = (i, \alpha) = \frac{p_{cyl}(\alpha) V_{cyl}(\alpha)}{\hat{m}_{cyl}(i) R} \quad (26)$$

where α is the angles between two points during the compression and i is an iteration index. This requires the reference temperature T_{ref} which is calculated by the temperatures in the intake and exhaust manifolds, cylinder pressure, heat transfer, and a residual gas fraction estimate, $\hat{\chi}_{res}$, which is determined by another model. An iteration is performed with a proportional update of the mass estimate

$$\hat{m}_{cyl}(i+1) = \hat{m}_{cyl}(i) + k_p \epsilon(i) \quad (27)$$

between each iteration until $\epsilon(i)$ is below a predefined threshold, see Giansetti et al. (2007) for details. The other observer minimizes

$$J = \sum_{\alpha=1}^{n_\alpha} (\hat{p}_{cyl}(\alpha) - p_{cyl}(\alpha))^2 \quad (28)$$

over the arguments \hat{m}_{cyl} and $\hat{\chi}_{res}$, where no iteration is required at the expense of a minimization problem.

7.4 Conditions at specific points during the cycle

The pressure, temperature and volume at different specific points during the cycle is often used as a part of the calculations. In Colin et al. (2007), cylinder mass is determined through

$$\hat{m}_{cyl} = \frac{C V^{1-k}(IVC)}{RT(IVC)} \quad (29)$$

where k and C are the coefficients in the polytropic relation (18), which are estimated to best fit the compression

trace between two specific points, see Section 7.1. In Müller and Isermann (2001) the conditions at TDC is used to estimate the residual mass, which is then used to calculate the partial pressure of air in the cylinder and hence the air mass.

A method for estimating total mass, air charge and residual gas fraction based on conditions at IVC and MFB50 is proposed in Mladek and Onder (2000). The procedure starts with an initial total mass estimate based on the ideal gas law at MFB50, with a fixed temperature and an initial calculation of the gas constant based on 10% residual gas fraction.

$$m_{Tot} = \frac{p_{50} V_{50}}{R_{50} T_{50}} \quad (30)$$

Based on this initial estimate, the temperature at IVC and the temperature of the residual gas is estimated. Additionally the fresh charge (FC) temperature is estimated based on the intake pressure and pressure at IVC. The residual gas fraction is then determined by considering the energy balance at IVC, after some manipulation the following expression is found

$$x_{RG} = \frac{c_{v,FC}(\bar{T}_{IVC} - T_{FC})}{c_{v,RG}(T_{RG} - \bar{T}_{IVC}) + c_{v,FC}(\bar{T}_{IVC} - T_{FC})} \quad (31)$$

This method is evaluated in Worm (2005a) together with the ΔP approach, with the conclusion that it has potential to be very accurate and requires almost no pre-work.

7.5 Port flow calculations

The mass in the cylinder can also be estimated by modeling the flow through the intake and exhaust valves during the valve opening periods. Usually the valve flow is described by the equation for compressible flow

$$\begin{aligned} \dot{m} &= \frac{p_{us}}{\sqrt{R T_{us}}} C_d A \Psi(\Pi_{lim}) \\ \Psi(\Pi_{lim}) &= \sqrt{\frac{2\gamma}{\gamma-1} \left(\Pi_{lim}^{\frac{2}{\gamma}} - \Pi_{lim}^{\frac{\gamma+1}{\gamma}} \right)} \\ \Pi_{lim} &= \max \left(\frac{p_{ds}}{p_{us}}, \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}} \right) \end{aligned} \quad (32)$$

where p_{us} and T_{us} are the pressure and temperature upstream of the valve, p_{ds} the pressure downstream of the valve and the product $C_d A$ is the effective flow coefficient that will vary with cam shaft angle. In both Ohyama (2002); Qu et al. (2012) this is used together with models for the intake and exhaust manifold volume, the throttle flow and the flow out of the exhaust to simulate the gas exchange process, estimating fresh charge and residual mass. Similar calculations is compared to experiments on a single cylinder engine with a gas sampling valve in Schwarz and Spicher (2003), and are found to correlate well. Port flow calculations is also used as reference in Raidt (2016), where an artificial neural network is trained to output air flow and residual gas fraction.

7.6 Frequency analysis of pressure trace

A method that is based on the resonance frequency in the cylinder pressure trace, excited by the combustion event, has been used in for example Hickling et al. (1983);

Guardiola et al. (2014); Broatch et al. (2015); Guardiola et al. (2016). The resonance frequency in a cylinder can be described by

$$f_{cyl} = \frac{aB}{\pi D} \quad (33)$$

where $a = \sqrt{\gamma RT} = \sqrt{\gamma pV/m}$ is the speed of sound, D is the cylinder diameter and B is the Bessel coefficient for the first radial mode. The Bessel coefficient depends only on geometry and therefore is only a function of the crank angle $B(\alpha)$ for a given engine. The frequency will change as the geometry change during the expansion, and some method to extract $f_{cyl}(\alpha)$ is needed, for example Short Time Fourier Transform (STFT) applied over a crank angle window at the end of combustion Guardiola et al. (2014). The total mass in the cylinder can then be estimated by

$$\hat{m}_{cyl} = \left(\frac{B(\alpha) \sqrt{\gamma p(\alpha) V(\alpha)}}{\pi D f_{cyl}(\alpha)} \right) \quad (34)$$

After the total mass has been determined the temperature at the exhaust is calculated with the ideal gas law, and the residual gas is modeled by considering the exhaust as an isentropic process

$$T_{EVO} = \frac{p_{EVO} V_{EVO}}{m_{cyl} R} \quad (35)$$

$$m_{rg} = \frac{p_{EVC} V_{EVC}}{T_{EVO} \left(\frac{p_{EVC}}{p_{EVO}} \right)^{\frac{\gamma-1}{\gamma}}} \quad (36)$$

and from there air mass is determined by subtracting the residual mass from the total mass estimate.

7.7 Other methods

A method specific to HCCI engines with negative valve overlap is presented in Fitzgerald et al. (2010). The method first relates the temperature measured at the exhaust port to the temperature at the end of the blowdown process, when the pressure reaches ambient, by the offset $\Delta T_{cyl/exh}$

$$T_{cyl,1atm} = T_{exh} + \Delta T_{cyl/exh} \quad (37)$$

In the next step the temperatures at exhaust valve opening, T_{EVO} , and exhaust valve closing, T_{EVC} , is calculated from $T_{cyl,1atm}$ by utilizing the cylinder pressure trace and considering heat losses through blow down process and the re-compression respectively. From T_{EVO} and T_{EVC} the total trapped mass and the residual mass can then be calculated with the ideal gas law.

Another approach to trapped mass estimation, where the cylinder pressure trace during part of the compression is transformed into a two-dimensional graphical signature, is presented in Youssef et al. (2011). This signature is then correlated to the trapped mass.

8. DIRECT AIR/FUEL RATIO ESTIMATION

Air fuel ratio is often estimated indirectly by estimating the air charge and residual mass by some of the methods described in the previous sections. There are however approaches that estimates this variable directly.

8.1 Statistical moments

The statistical moments of the pressure trace has been used as a basis for direct estimation of the air/fuel ratio based

on early work in Gilkey and Powell (1985); Fiorini et al. (1988). The following regression polynomial is proposed in Arsie et al. (2014)

$$\phi = a_0 + a_1 M_2 + a_2 \frac{1}{M_2} + a_3 \frac{M_3}{M_2} + a_4 N^2 \quad (38)$$

which is a slight development to the polynomials suggested in Arsie et al. (1998); Gassenfeit and Powell (1989). The variables M_i are the i :th central moments normalized by the area of the pressure cycle in motored condition:

$$M_i = \frac{\int_{\theta_{ign}}^{\theta_{ign} + \Delta\theta_{comb}} (\theta - \theta_c)^i p(\theta) d\theta}{A_u} \quad (39)$$

$$A_u = \int_{\theta_{ign}}^{\theta_{ign} + \Delta\theta_{comb}} p_{back}(\theta) d\theta \quad (40)$$

$$p_{back}(\theta) = p(\theta_{IVC}) \left(\frac{V(\theta_{IVC})}{V(\theta)} \right)^m \quad (41)$$

where p_{back} is the motored pressure and θ_c is the expected value of θ , defined as

$$\theta_c = \frac{M_1}{M_0} = \frac{\int_{-\infty}^{\infty} \theta p(\theta) d\theta}{\int_{-\infty}^{\infty} p(\theta) d\theta} \quad (42)$$

In Fiorini et al. (1988) a similar polynomial for air/fuel ratio estimation is presented, but using FFT descriptors of the cylinder pressure trace of the same interval instead of the moments. It is shown in the paper that the methods are theoretically equivalent in continuous time, but have different computational load and accuracy for the same discrete angular resolution.

8.2 G-ratio

Another approach is proposed in Patrick and Powell (1990), that suggests using G-ratio, defined as the ratio of average molecular weights before and after combustion. The air/fuel ratio is expressed in terms of the normalized fuel/air ratio ϕ , and the following model is suggested

$$\hat{\phi} = \begin{cases} b_1(\text{G-ratio}) + b_2 & \text{for } \phi \leq 1.0 \\ b_3(\text{G-ratio}) + b_4 & \text{for } \phi > 1.0 \end{cases} \quad (43)$$

The G-ratio are then expressed in terms of pressures and temperatures as

$$(\text{G-ratio}) = \frac{G_4}{G_1} = \frac{P_1 T_4 V_1 M_4}{P_4 T_1 V_4 M_1} \quad (44)$$

where the subscripts "1" and "4" refers to before and after combustion. For port injected engines $M_1 \approx M_4$, and if the points "1" and "4" are measured at the same volume, the equation can be simplified to

$$(\text{G-ratio}) = \frac{P_1 T_4}{P_4 T_1} \quad (45)$$

The P-ratio P_1/P_4 are determined by a cylinder pressure sensor, but since in-cylinder temperature is seldom available, the following approximation is proposed

$$(\text{T-ratio}) = \frac{T_4}{T_1} \approx \frac{T_{4m}}{T_{1m}} \frac{1}{(1 + c_1 N)(1 + c_2 p_{im})} \quad (46)$$

where T_{1m} and T_{4m} are measured temperatures in the intake and exhaust manifold, N is the engine speed, p_{im} is the pressure in the intake manifold and c_1 and c_2 are fitting coefficients.

9. EFFECTIVE COMPRESSION RATIO

The effective compression ratio is supposed to describe the actual compression of the gas in the cylinder and is useful for describing the effect of late intake valve closing He et al. (2008). Commonly it is defined as the ratio of cylinder volume at intake valve closing to the cylinder volume at top dead center, but this is not the best to explain experimental results, and an alternative method is proposed in He et al. (2008) based on a linear fit to the compression trace in the $\log(p)$ - $\log(V)$ diagram. The linear fit is extrapolated down to the intake manifold pressure and the volume at the intersection is defined as the effective IVC volume. The effective compression ratio is then defined as the ratio of the effective IVC volume and the volume at top dead center.

10. CYLINDER WALL TEMPERATURE

A method for estimating the cylinder wall temperature based on cylinder pressure is presented in Arsie et al. (1998, 2015). The method is based on the polytropic relation (18). If there is no heat transfer between the gas and the surroundings, the polytropic exponent would be equal to the ratio of specific heats $\gamma = c_p/c_v$. At the beginning of compression the cylinder wall would typically be hotter than the gas, and therefore heat will flow from the wall to the gas. This will make the polytropic index k higher than γ . The opposite is true later in the compression stage, as the gas temperature increases to above the wall temperature, then heat will flow from the gas to the cylinder wall, decreasing the polytropic index. By estimating k as a function of crank angle and identifying the point where $k = \gamma$, the point where the gas temperature and cylinder wall temperature is found. If the cylinder mass is known the ideal gas law can then be used to derive the gas temperature and hence the cylinder wall temperature. The method is evaluated in Arsie et al. (2015) on two Diesel engines, one naturally aspirated and one turbocharged with VGT and EGR. It is concluded that the temperature estimation quantitatively respond as expected to changes in load, intake pressure and EGR rate, but since no in-cylinder temperature measurement is available the absolute accuracy is hard to evaluate.

11. CONCLUSION

The paper has presented a survey of the use of cylinder pressure sensors in internal combustion engine estimation and control. Cylinder pressure sensors have been used to estimate large number engine variables such as torque, IMEP, heat release, knock, cylinder mass and temperature, air/fuel ratio, residual gas fraction, and cylinder wall temperature. Since the estimation is done on a cycle to cycle basis, these methods have the potential to be faster and more accurate during transients compared to for example MAF sensors for mass estimation, that is placed further away from the cylinders, or thermocouples for temperature estimation, that have slow dynamics. With increasing demands of accurate control of the combustion, required to reduce fuel consumption and emissions, in combination with the amount of information that can be extracted from the cylinder pressure, this sensor has potential to be an important source of information in the future for internal combustion engine control.

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