Chapter 11 An introduction to Graph Thory

Section 11.1

Required knowledge

Relation: $\mathcal{R} \subseteq A \times B$ is called a relation from A to B.

Definitions

directed and undirected graph, vertex, edge, incident, adjacent, origin, terminus, loop, (closed) walk, trail, circuit, path, cycle, connected, component, multigraph, multiplicity, n-graph

Theorem

11.1 G undirected graph. Trail from a to b in $G \Rightarrow$ Path from a to b in G.

Questions

1. Try to find an application of graph theory. Explain the connection between the graph theoretical concepts and the interpretation of them in the particular chosen application.

Exercises

2, 7, 13

Section 11.2

Definitions

subgraph, spanning subgraph, induced subgraph, complete graph, complement of a graph, graph isomorphism, isomorphic graphs, self-complementary

Exercises

1, 15, 16, 20

Section 11.3

Definitions

degree of vertex, pendant vertex, k-regular graph, Euler circuit, Euler trail, incoming degree, outgoing degree, adjacency matrix, incidence matrix

Theorems

11.2 $G = (V, E)$ undirected graph or multigraph, then $\sum_{\nu \in V} \deg(\nu) = 2|E|$.

11.3(4) G undirected graph (directed graph) or multigraph. Euler circuit in G $\Leftrightarrow$ G connected and $\forall \nu \in V \exists k \in \mathbb{Z} : \deg(\nu) = 2k$ (or $id(\nu) = od(\nu)$).

Exercises

6, 8 , 19, 24, 31

Section 11.4

Definitions

planar graph, embedding, bipartite graph, complete bipartite graph, elementary subdivision, homeomorphic, platonic solids, dual, cut-set, infinite region, degree of region

Theorems

11.5	G nonplaner $\Leftrightarrow$ G contains a subgraph homeomorphic to either K_5 or $K_{3,3}$
11.6	G connected, planar, then $r = 2 + \|E\| - \|V\|$ is the number of regions.
C11.3	G connected, planar, loop-free, not multigraph, $\|E\| > 2$, then $3r \leq 2\|E\|$ and $\|E\| \leq 3\|V\| - 6$.

Exercises

12, 19, 26, 29

Section 11.5

Definitions

Hamilton cycle/path, tournament, independent, maximal independent set, independence number $\beta(G)$

Theorems

11.7	A tournament always contains a directed Hamilton path.
11.8	G loop-free, $\|V\| \geq 2$. $\forall x, y \in V : (x \neq y$ and $\deg(x)+\deg(y) \geq \|V\| - 1)$ $\Rightarrow G$ has Hamilton path
11.9	G loop-free, undirected, $V \geq 3$. $\forall$ nonadjacent $x, y \in V : (x \neq y$ and $\deg(x)+\deg(y) \geq \|V\|) \Rightarrow G$ has Hamilton path

Questions

1. Explain the connection between Th 11.8 and Th 11.9.
2. Give an example of a loop-free undirected graph that does not satisfy the condition $\forall$ nonadjacent $x, y \in V : (x \neq y$ and $\deg(x)+\deg(y) \geq |V|)$ in theorem 11.9, but has even though a Hamilton cycle.

Exercises

3, 11, 24, 25, 26

Section 11.6

Definitions

proper coloring, chromatic number $\chi(G)$, chromatic polynomial $P(G, \lambda)$, clique number $\omega(G)$

Theorems

11.10	G connected, $e \in E$, then $P(G_e, \lambda) = P(G, \lambda) + P(G'_e, \lambda)$
11.11	$P(G, \lambda)$ constant $\Leftrightarrow P(G, \lambda) = 0$
11.12	$\lvert E \rvert > 0 \Rightarrow P(G, 1) = 0$
11.14	G undirected, $G = G_1 \cup G_2$, $G_1 \cap G_2 = K_n$, then $P(G, \lambda) = [P(G_1, \lambda) P(G_2, \lambda)]/\lambda^n$

Exercises

10, 12, Supplementary exercise 21