

TSFS02 — Vehicle Dynamics and Control

Computer Exercise 2: Lateral Dynamics

*Division of Vehicular Systems
Department of Electrical Engineering
Linköping University
SE-581 33 Linköping, Sweden*



Contents

- 1 Introduction** **3**
- 1.1 Examination 3
- 1.2 Prior Knowledge and Skills 3

- 2 Preparation Tasks** **4**

- 3 Computer Exercise Tasks** **4**

- A MATLAB** **8**
- A.1 Parameter Estimation 8

- B The Magic Formula Tire Model** **8**

- C Nomenclature** **9**

1 Introduction

In this exercise you will work with simple vehicle models that describe the lateral dynamics. You will use MATLAB to parametrize and validate these models by using measurement data gathered with the test vehicle in Figure 1. This vehicle, a Volkswagen Golf -08, is equipped with various sensors to measure longitudinal and lateral velocity (optical sensor), pitch and roll angles (optical sensors), accelerations and angular rates (IMU), and position (GPS). In addition, the internal sensors of the vehicle is sampled through the CAN bus (e.g., steering wheel angle and wheel velocities).

The main purpose of the exercise is to get an understanding of the single-track model, the linear tire model and the *Magic Formula* tire model, how well these can describe the vehicle dynamics and how they can be parametrized based on real measurements.

1.1 Examination

To pass this exercise you should have fulfilled the following:

- Solved the preparation tasks (in Section 2).
- Solved all the computer exercise tasks (in Section 3).
- Answered all questions, with motivated and thoughtful answers.

The examination is done by presenting your results and answers to a course assistant at the scheduled exercise session. To speed up the examination process, it is recommended to present the tasks as you complete them, instead of saving them all to the end.

1.2 Prior Knowledge and Skills

To complete all tasks you need to:

- Be able to work with MATLAB.
- Understand and be able to sketch the single-track model.
- Understand and be able to sketch a handling diagram.
- Understand the linear tire model.
- Know about the *Magic Formula* tire model (see Appendix B), and have a grasp idea of the purpose of the different parameters.

Thus, lecture 4–7, lesson 5, and sections 1.4 and 5.2–5.6 in the course book should have been worked through before starting with the exercises.



Figure 1 Test vehicle for vehicle dynamics studies.

2 Preparation Tasks

The tasks in this section (i.e., Task 1–5) are preparation tasks that should be solved before starting with the exercise tasks in Section 3. Please verify with an assistant that you have solved these tasks correctly before starting with Task 6–9 (since your code implementations will be based on the equations you derive in the preparation tasks).

Task 1

Sketch up the single-track model with the relevant forces, angles, velocity vectors, etc., that you will be using in Task 2.

Task 2

For the single track model, derive the equations for the slip angles (α_f and α_r) and the lateral tire forces ($F_{y,f}$ and $F_{y,r}$). The slip angles should be expressed as functions of v_x , v_y , and Ω_z , and the lateral force as functions of a_y and $\dot{\Omega}_z$.

Task 3

Derive the relation between the steer angle δ and the slip angles (α_f , α_r) during steady state cornering, if the wheel base L and the cornering radius R are given.

Task 4

For Task 8, you need equations describing the dynamics of the lateral velocity v_y and the yaw rate Ω_z . Derive expressions for \dot{v}_y and $\dot{\Omega}_z$.

Task 5

Sketch handling diagrams for two different vehicles:

- A vehicle with understeer properties at low lateral accelerations, and oversteer properties at high lateral accelerations.
- A vehicle which is understeering for low lateral accelerations, and becomes even more understeered at higher lateral accelerations.

Also specify how you can calculate the quantities on the x and y axes (so that you can plot a handling diagram) using the parameters and variables available in `vehicleParameters.mat` and `ssMeasData.mat` (see Appendix C). Note that we consider steady state cornering here.

3 Computer Exercise Tasks

Start by downloading `TSFS02_Lab2_Lateral.zip` from the course homepage, unzip in an appropriate folder, and point `MATLAB` to this.

Task 6 – Identify tire parameters

In this task you should determine the parameters for the linear tire model and the Magic Formula tire model, using the provided measurement data. This data is sampled during several double lane-change tests, with different initial speeds (resulting in different levels for lateral acceleration, slip angles, etc.) to provide data that covers as much as possible of the force-vs-slip diagrams.

- Open `tireParaIdent.m` and fill in the expressions for slip angles and lateral forces from Task 2. Note that the variables are in the form of vectors, meaning you have to use `.*` and `./` for multiply and divide. Execute Cell 1–3, followed by Cell 4 to plot the forces as functions of the slip angles.

- b) Estimate the cornering stiffnesses (C_{af} , C_{ar}), e.g., with the linear least square method (see Appendix A.1 for guidelines). Plot the linear tire model in the figure (from from the previous task) with Cell 6. Is it a good fit? If not, tune the cornering stiffnesses manually until you are satisfied. Remember in which area the linear tire model is considered a good approximation.

Tip: You probably need to slightly tune the cornering stiffness manually (unless you come up with something clever for your estimation procedure).

- c) Estimate the parameters B , C , and D in the Magic Formula tire model, e.g., by using `fit` in MATLAB (see Appendix A.1 for guidelines). Assume $C \approx 1$ and $\mu_y \approx 1$, what is a good initial guess for the parameters? Plot the Magic Formula model in the figure with Cell 8, and adjust the parameters if needed.

What properties do the different parameters in the Magic Formula tire model control?

Tip: Tweak the parameter values manually to see what happens with the $F_y - \alpha$ plot.

Parameter B : _____

Parameter C : _____

Parameter D : _____

- d) When you are satisfied with your estimations, execute Cell 9 (at the bottom) to save the tire parameters (i.e., C_{af} , C_{ar} , B_f , C_f , D_f , B_r , C_r , and D_r) to the file `tireParameters.mat`.

Task 7 – Steady state cornering

You will now study how the single-track model, together with the tire models above, handles in steady state cornering compared to measurement data. The data in this task is gather during constant radius tests, for various speeds (thus, different lateral accelerations).

Note that during steady state cornering we can assume the yaw rate and lateral velocity to be constant, i.e., $\dot{\Omega}_z = 0$ and $\dot{v}_y = 0$.

- Open the file `steadyStateAnalysis.m` and fill in expressions for the lateral tire forces ($F_{y,f}$ and $F_{y,r}$) given the lateral acceleration (a_y). (For simplicity, assume small steer angles, i.e., $\cos \delta = 1$.)
- Write expressions for the slip angles, as functions of the lateral tire forces, for the linear tire model and Magic Formula.
- Fill in expressions for the steering angle δ , given the slip angles, wheelbase, and turning radius. (Do this separately for the two tire models.)
- You will now plot handling diagrams for the two tire models, and compare with measurement data, using the prepared plot scripts in Cell 5. Fill in the variables/expressions needed for a handling diagram and run the script.

How well do the models describe the vehicle motions compared to the measurement data?

Tip: When doing the measurement tests it is difficult to make sure that the turning radius is completely fixed. Therefore, use other variables than R when plotting the handling diagram for the measurement data.

- Based on the handling diagrams, how should the steering angle be changed if the velocity increase? How does this differ when we consider the different models and the measurements?

Task 8 – Transient dynamics (double lane-change maneuver)

In this task you should validate the transient dynamics of your model. This is done with a slightly modified version of the double lane-change test ISO 3888-2, which often is used for evaluating the lateral stability properties of vehicles. Figure 2 shows a sketch of the track layout for the double lane-change maneuver.

You have access to three different sets of measurement, each consisting of a single run through the double lane-change test. These three runs are performed at different entry speeds and denoted Test 1, 2, and 3, where Test 1 is the least aggressive (lowest entry speed) and Test 3 the most aggressive. Table 1 shows entry speeds and maximum values for a few interesting variables for these tests. The Youtube link below shows a measurement run recorded at the same time as Test 1–3 was sampled, and is comparable to Test 3 in terms of “aggressiveness”.

<http://www.youtube.com/watch?v=o9hefU71DL0>

In the tasks below you should implement vehicle models as ODE functions and simulate these. The simulation inputs to the models are steering angle δ and longitudinal velocity v_x , which is taken from the measurement data (i.e. you will use the same inputs as was used when performing the tests). Finally, you should compare the resulting dynamics from your simulation models with measurement data for Test 1–3.

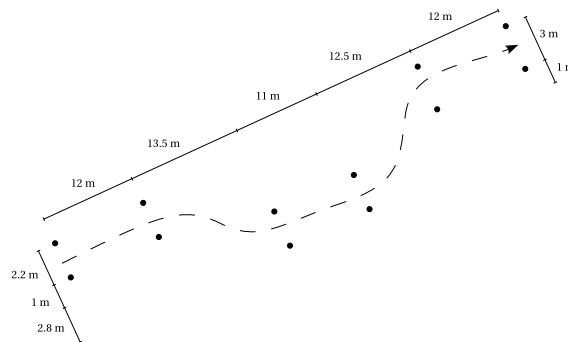


Figure 2 Track layout for the double lane-change test.

- Start by completing the simulation models for the single-track model with the linear tire model, `STL.m`, and with the Magic Formula tire model, `STMF.m`.
- In `transientAnalysis.m` you can choose which test to run by uncommenting row 21, 22, or 23. After choosing which test to run, execute the whole m-file to run the simulations and generate plots. Run all three tests and verify that the output is reasonable.
- How far, in terms of lateral acceleration, can the different tire models be considered accurate? How does these conclusions correlate to the handling diagram from Task 7?

Task 9 – Ramp steer

The last test to be investigated is a *ramp steer* test, in which the steering angle is increased slowly, meaning the vehicle should react similar to the steady state cornering on steering input.

Measurement data for this test is sampled at a different occasion, where the number of passengers in the vehicle was different. This will in particular affect two of the vehicle parameters in your models (i.e., the parameters stored in `vehicleParameters.mat`).

- Which two of the parameters do you think is most probable to have been affected (by changing the number of passengers)?

Table 1 Initial velocity and maximum values of a few interesting variables for the three measurement runs (Test 1–3) in the double lane-change test. Note that δ_{sw} is the steering angle at the hand wheel.

Variable	Test 1	Test 2	Test 3	Unit
v_{init}	38.3	51.4	62.4	km/h
$\delta_{sw,max}$	154	147	157	deg
$\dot{\delta}_{sw,max}$	615	742	1013	deg/s
ψ_{max}	0.535	0.586	0.710	rad/s
$a_{y,max}$	5.78	7.96	9.23	m/s ²
$\alpha_{f,max}$	0.062	0.097	0.124	rad
$\alpha_{r,max}$	0.034	0.060	0.102	rad
$\dot{\alpha}_{f,max}$	0.386	0.551	0.814	rad/s
$\dot{\alpha}_{r,max}$	0.239	0.400	0.690	rad/s

- b) Open rampSteer.m and run the whole script. Which variables seem to be affected, and not correlating well with the measurement data?
- c) Make a rough sensitivity analysis for the two parameters, by changing them $\pm 10\%$, and notice how the dynamics is affected.
- d) Adjust the parameters (both of them or only one), within reasonable bounds, until you have a decent fit to the measurements. Compare the results with the handling diagram from Task 7.

A MATLAB

For the exercises, there exist prepared MATLAB scripts. These need to be modified and/or completed in some of the tasks. In the rows where this is necessary, a # has been inserted to clarify where the modifications are needed (don't forget to remove the # before you run the scripts!).

Some of the m-files are divided into cells, separated by `%%`. Each cell can be evaluated individually, with right-click and *Evaluate Current Section* or `Ctrl+Enter`, which can make it easier to perform sub-tasks.

A.1 Parameter Estimation

Two different estimation functions that in these exercises could be useful for parameter estimation are *the linear least square* and the `fit` function.

Least-Square Fit

Estimating the parameter k in the linear equation $y = kx$, for a data set (measurements of y and x), can in MATLAB be done with:

```
k = x\y;
```

Note that y and x should be $n \times 1$ vectors.

The fit Function

Consider the model $y = k_1 \sin(k_2 x)$, where we have measurements for y and x . Estimating k_1 and k_2 can be done with the function `fit` in MATLAB with the following script:

```
k1_init = 1;
k2_init = 10;
fit_equation = fittype(@(k1,k2,x) k1*sin(k2*x));
fit_data = fit(x, y, fit_equation, 'StartPoint', [k1_init, k2_init]);
k1 = fit_data.k1;
k2 = fit_data.k2;
```

Here `k1_init` and `k2_init` are initial guesses for the parameters k_1 and k_2 . Note that y and x should be vectors on the format $n \times 1$.

B The Magic Formula Tire Model

The Magic Formula is an empirical model that can be used to characterize the longitudinal and lateral tire-forces in vehicle dynamics modeling and simulation. The basic form of the model is, for the lateral forces, described by

$$F_y = D \sin(C \arctan(B\alpha - E(B\alpha - \arctan(B\alpha))))$$

where D is known as the peak factor (equal to $\mu_y F_z$), C the shape factor, B the stiffness factor, and E the curvature factor. Note that the cornering stiffness can be described by $C_\alpha = BCD$.

In the exercises it is sufficient to use the slightly simplified version of the model:

$$F_y = D \sin(C \arctan(B\alpha))$$

This is equivalent to setting $E = 0$ in the longer version.

C Nomenclature

Table 2 lists all the parameters and variables that are available in the .m and .mat files, with descriptions and units. Table 3 specifies in which .mat files these variables and parameters are included.

Table 2 Nomenclature for MATLAB scripts.

Parameter	Description	Unit
t	Time	s
delta	Steer angle (at the front wheels)	rad
v	Velocity at the CoG	m/s
vx	Longitudinal velocity at the CoG	m/s
vy	Lateral velocity at the CoG	m/s
alphaf	Front slip angle	rad
alphar	Rear slip angle	rad
Fyf	Lateral tire forces, front	N
Fyr	Lateral tire forces, rear	N
ay	Lateral acceleration	m/s ²
Omegaz	Yaw rate	rad/s
dOmegaz	Yaw acceleration	rad/s ²
Caf	Front cornering stiffness	N/rad
Car	Rear cornering stiffness	N/rad
Bf, Cf, Df	Magic Formula parameters, front	
Br, Cr, Dr	Magic Formula parameters, rear	
m	Vehicle mass	kg
L	Wheelbase	m
l1	CoG to front axle	m
Iz	Vehicle inertia (about the z-axis)	kgm ²
g	Gravity constant	m/s ²

Table 3 Variables and parameters included in the .mat files.

vehicleParameters.mat	tireParaMeas.mat
L	delta
l1	vx
m	vy
Iz	ay
g	Omegaz
	dOmegaz

ssMeasData.mat	dlcMeasData.mat
rsMeasData.mat	
delta	t
v	delta
ay	vx
Omegaz	vy
	ay
	Omegaz
	alphaf
	alphar