Vehicle Dynamics and Control

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Lecture 1

Course Literature

Course book is *Theory of Ground Vehicles*, 4th edition, by J.Y. Wong
You can borrow a copy during the course.

Chapter 1: Mechanics of Pneumatic Tires

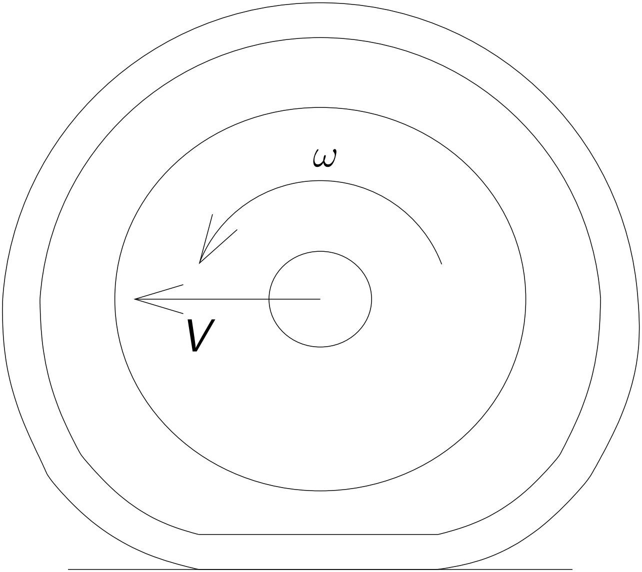
Chapter 3: Performance Characteristics of Road Vehicles

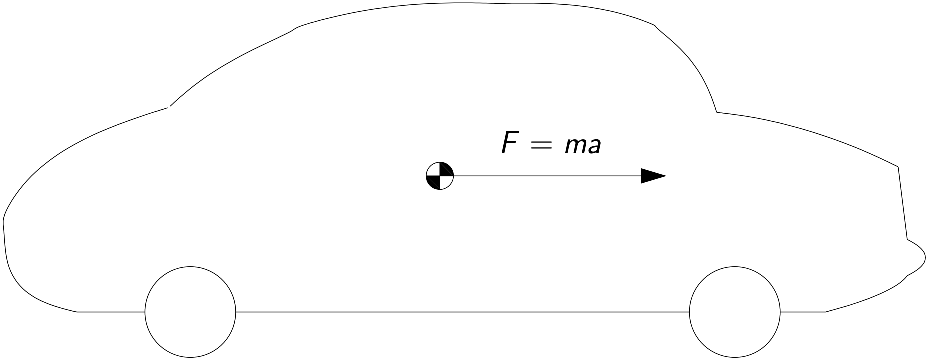
Chapter 5: Handling Characteristics of Road Vehicles

Chapter 7: Vehicle Ride Characteristics

Some additional material is taken from the books *Vehicle Dynamics, Stability and Control*, 2nd edition, D. Karnopp, and *Tire and Vehicle Dynamics*, H. Pacejka.

Chapter 1: Mechanics of Pneumatic Tires

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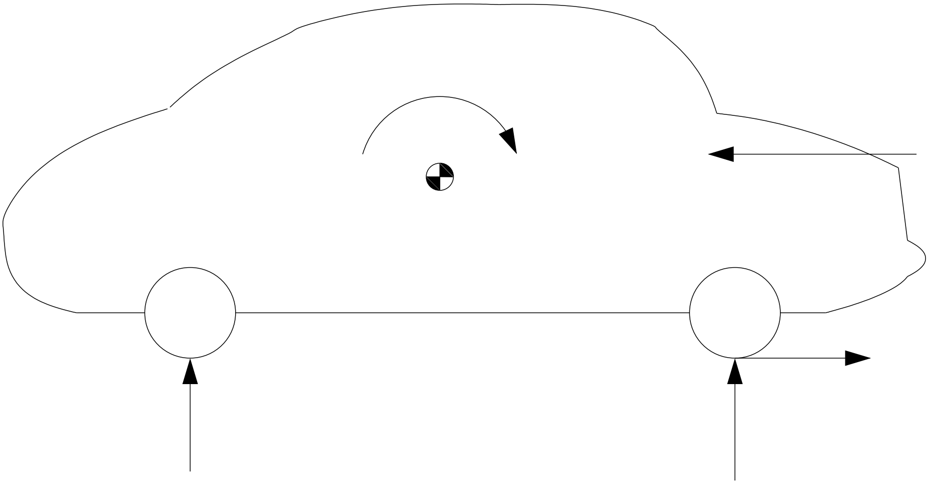
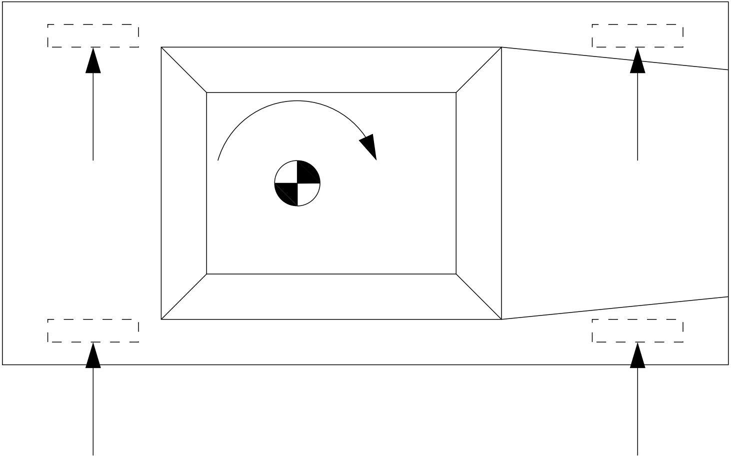
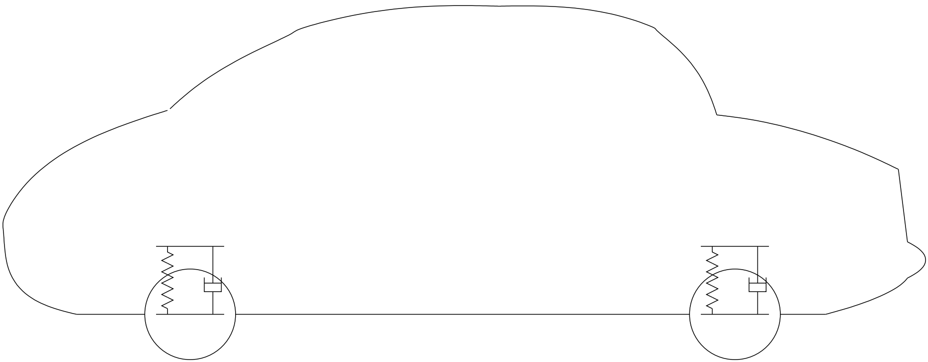
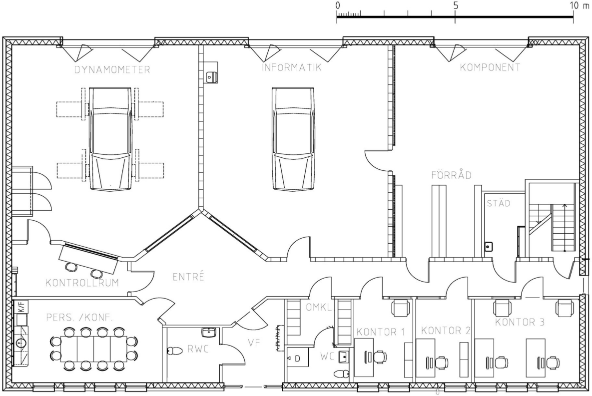






L-building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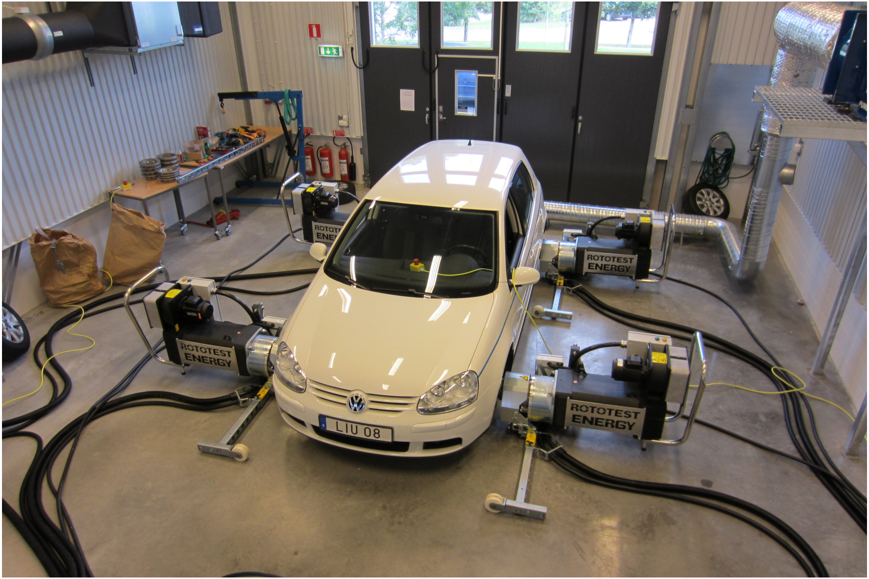
L-building

L-building

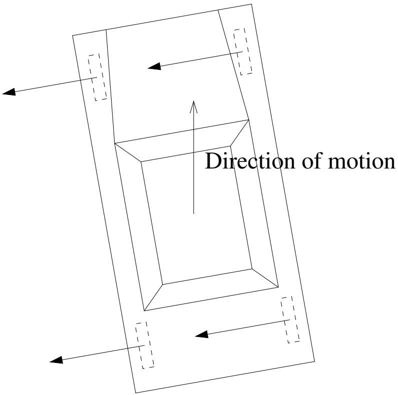


Sensor Set-up

1. Slip angle sensor
2. Pitch/roll angle sensors
3. IMU
4. GPS
5. Vehicle CAN

L-building

Stability



Stability

Stability

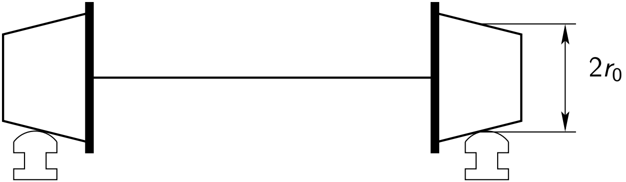
Stability

Tapered wheels

Why is the wheels on a train tapered?

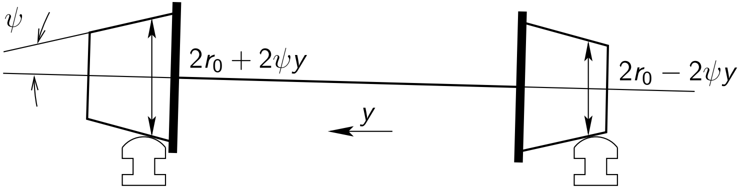
Tapered wheels: Basic motion

Consider a wheelset with tapered wheels on a rail. In the steady motion/basic motion, the wheels are moving on a straight line in the longitudinal direction:



Tapered wheels: A train taking a turn

One reason for using tapered wheels is illustrated in the following figure showing a wheelset of a train taking a right turn:



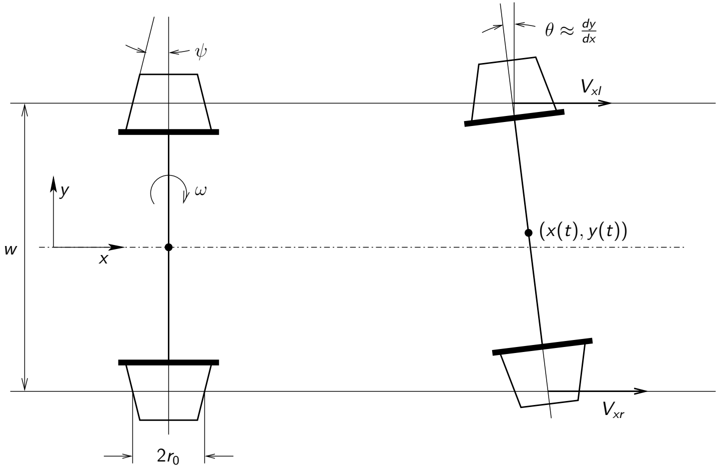
The longitudinal speed is larger for the outside wheel V_{xl} than for the inside wheel V_{xr}, but the rotational speed ω is the same. The basic motion in this case includes a constant drift y in the lateral direction, which compensates for this difference:

$$V_{xl} = (r_0 + \psi y)\omega, \quad V_{xr} = (r_0 - \psi y)\omega$$

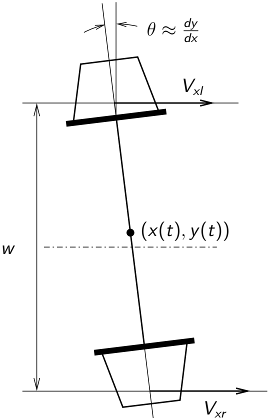
Tapered wheels: Perturbed motion

What will happen if the basic motion is perturbed?

Basic motion is shown to the left and perturbed motion to the right:



Tapered wheels



Lateral drift causes a difference in the longitudinal velocity of the wheels in the same way as before:

$$V_{xl} = (r_0 + \psi y)\omega$$
$$V_{xr} = (r_0 - \psi y)\omega$$

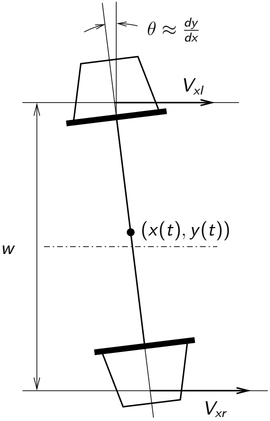
The longitudinal velocity of the center of gravity is now given by:

$$\dot{x} = \frac{V_{xl} + V_{xr}}{2} = r_0\omega$$

The approximation $\frac{dy}{dx} \approx \theta$ gives the lateral velocity:

$$\dot{y} = \frac{dy}{dx}\frac{dx}{dt} = \theta r_0\omega$$

Tapered wheels



Using $V_{xl} = (r_0 + \psi y)\omega$ and $V_{xr} = (r_0 - \psi y)\omega$ the angular velocity can be written as

$$\dot{\theta} = \frac{V_{xr} - V_{xl}}{w} = -\frac{2\psi y \omega}{w}$$

Differentiating $\dot{y} = \theta r_0 \omega$ and using the expression for the angular velocity above, the following differential equation for y is obtained:

$$\ddot{y} + \frac{2 r_0 \psi \omega^2}{w} y = 0$$

Tapered wheels: Harmonic oscillation

For a wheelset with positive taper angle (as in the figure) the solution

$$\ddot{y}(t) + \frac{2r_0\psi\omega^2}{w} y(t) = 0$$

is a harmonic oscillation

$$y(t) = \cos(\omega_n t + \phi)$$

with natural frequency

$$\omega_n = \sqrt{\frac{2r_0\psi}{w}}\omega$$

If there is friction in the system, then the wheelset will return to the basic motion asymptotically.

Tapered wheels: Unstable system

For a wheelset with negative taper angle the solutions of the differential equation

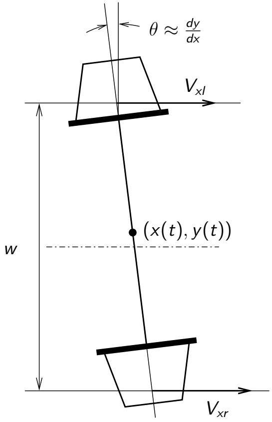
$$\ddot{y}(t) + \frac{2r_0\psi\omega^2}{w}y(t) = 0$$

are

$$y(t) = C\exp\left(\pm\sqrt{\frac{2r_0\psi}{w}}\omega\right)$$

which means that the a small perturbation would cause an exponential growth of the lateral displacement and the system is clearly unstable.

Tapered wheels: Spatial coordinates



The dynamic equation

$$\ddot{y}(t) + \frac{2r_0\psi\omega^2}{w}y(t) = 0$$

can be rewritten by using the relations

$$\ddot{y} = \frac{d^2y}{dx^2}\dot{x}^2, \quad \omega^2 = \frac{\dot{x}^2}{r_0^2}$$

and the result is the following:

$$y''(x) + \frac{2\psi}{wr_0}y(x) = 0$$

A model that doesn't depend on speed.

Tire

Figure 1.1: Tire construction

Figure 1.2: Coordinates, forces, and moments.

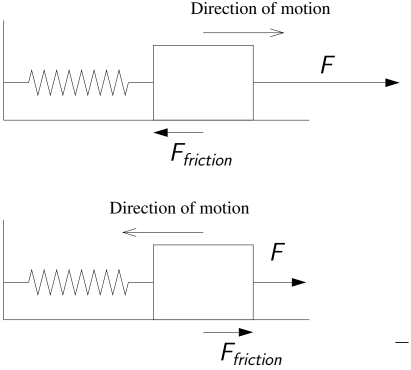
Rolling resistance

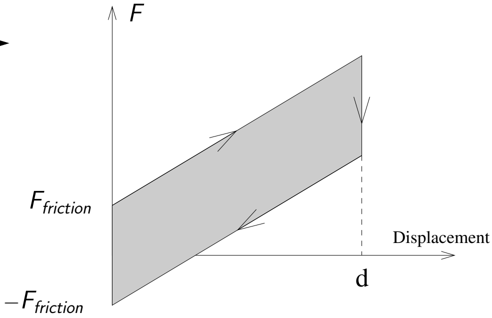
The rolling resistance of tires is primarily caused by the hysteresis in tire materials due to the deflection of the carcass while rolling.
Other less important contributors to the rolling resistance are:

- Friction between the tire and the road caused by sliding
- Air circulating inside the tire

Hysteresis

Exampel of a hysteresis loop caused by friction:



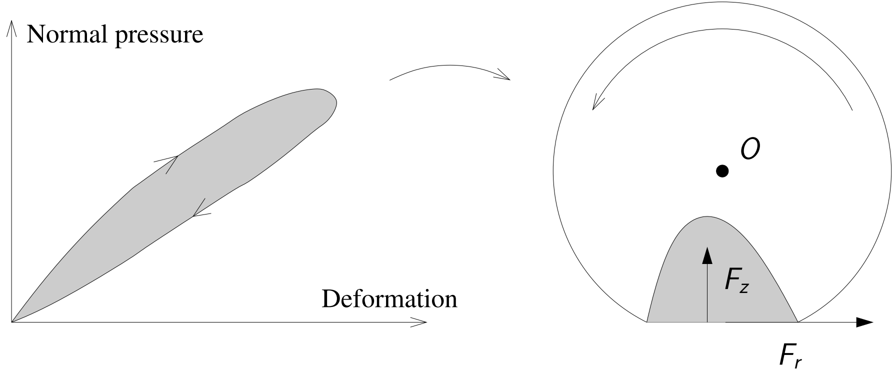


The energy loss due to hysteresis is equal to the shaded in the figure:

$$2 \cdot d \cdot F_{friction}$$

Rolling resistance: Hysteresis

The center of normal pressure is shifted in the direction of motion due to the hysteresis



The applied wheel torque on free-rolling tire is zero. Therefore, a horizontal force R_r at the contact patch must exists to maintain equilibrium. This force is called known as the rolling resistance.

Rolling resistance

The coefficient of rolling resistance f_r is defined as the ratio of the rolling resistance R_r to the normal load W, i.e., $f_r = R_r/W$.
Empirical formulas for calculating the rolling resistance coefficient as a function of speed V , based on experimental data:

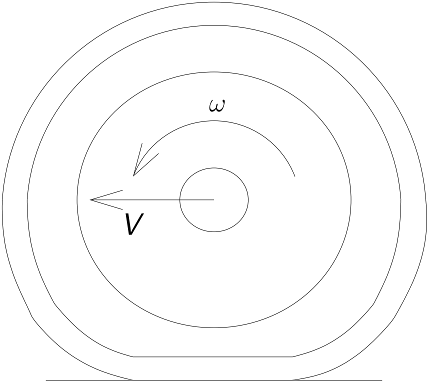
Radial-ply passenger car tire: $f_r = 0.0136 + 0.40 \times 10^{-7} V^2$

Radial-ply truck tire: $f_r = 0.006 + 0.23 \times 10^{-6} V^2$

Other factors that affect the rolling resistance:

- Surface texture, Figure 1.5.
- Inflation pressure, Figure 1.7 and 1.8.
- Internal temperature, Figur 1.11 and 1.12.

A Tire Under the Action of a Driving Torque



Definitions:

Rolling radius of a free-rolling tire: $r = V/\omega$,

Effective rolling radius under the action of a driving torque: $r_e = V/\omega$,

where V is the linear speed of the tire center, and ω is the angular speed.

A Tire Under the Action of a Driving Torque

Longitudinal slip

$$i = \left(1 - \frac{V}{\omega r}\right) \times 100\% = \left(1 - \frac{r_e}{r}\right) \times 100\%$$

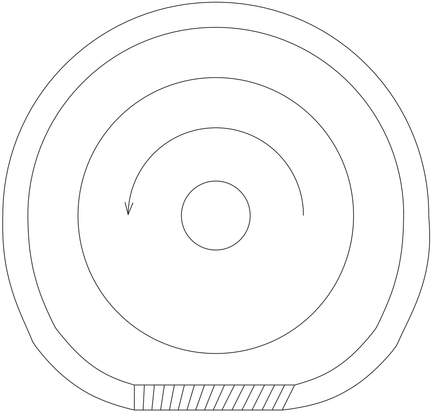
Limit cases:

Free-rolling tire: $i = 0$

The tire is not moving: $i = 100\%$ om $V = 0$,

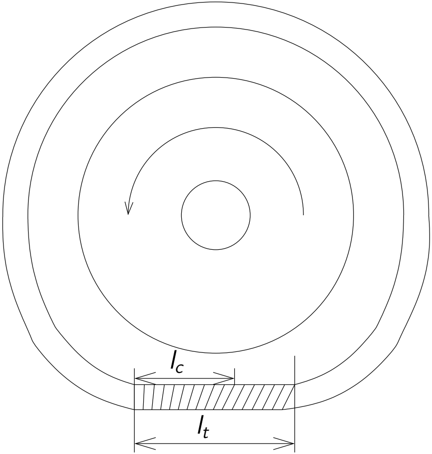
Driving Wheel: The Brush Model

The brush model is a very simple physical model of tire. The tread of the tire is modeled as elastic bristles attached to the rim, and longitudinal force is generated by the deflection of the brush elements.



Driving Wheel: The Brush Model

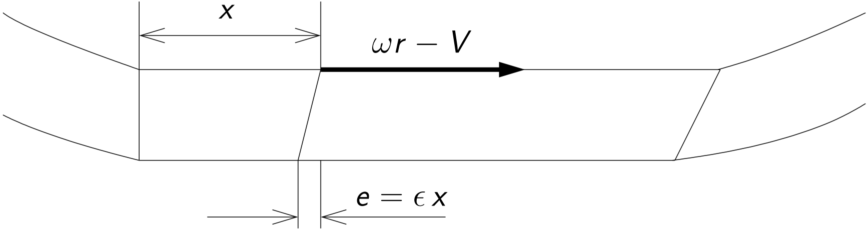
The contact patch is assumed to rectangular and can be divided into an adhesion region ($0 \leq x \leq l_c$) and a sliding region ($l_c \leq x \leq l_t$).



Driving Wheel: The Brush Model

The objective is to find the length of the adhesion region l_c. When does the longitudinal force becomes so large that the bristles begins to slide?

Consider a bristle in the adhesion region



The velocity at the rim is $\omega r - V$. The time since the bristle first touch the ground is $t = x/(\omega r)$. The deflection at the distance x is:

$$e(x) = (\omega r - V)\frac{x}{\omega r} = \left(1 - \frac{V}{\omega r}\right) x = ix$$

Driving Wheel: The Brush Model

User a linear model for the relation between deflection and longitudinal force per unit of length:

$$\frac{dF_x}{dx} = k_t e = k_t ix$$

It is assumed that normal force W is uniformly distributed in the contact region,

$$\frac{dF_z}{dx} = \frac{W}{l_t}$$

where l_t is the length of the contact region.

Assumption: The bristle will not slide if

$$\frac{dF_x}{dx} < \mu_p \frac{dF_z}{dx}$$

where μ_p is the coefficient of friction.

Driving Wheel: The Brush Model

The condition can be written

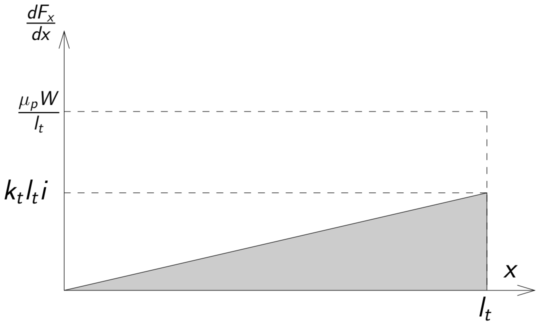
$$k_t i x < \mu_p \frac{W}{l_t}$$

First case: When is there no sliding region?
Answer: When $x = l_t$ fulfills the condition above, i.e.

$$k_t l_t i < \frac{\mu_p W}{l_t} \quad \text{or} \quad i < \frac{\mu_p W}{k_t l_t^2} \equiv i_c$$

Driving Wheel: The Brush Model

The distribution of the longitudinal force in this case ($i < i_c$)



$$F_x = \text{Area of the shaded region} = \frac{1}{2} k_t l_t^2 i \equiv C_i i$$

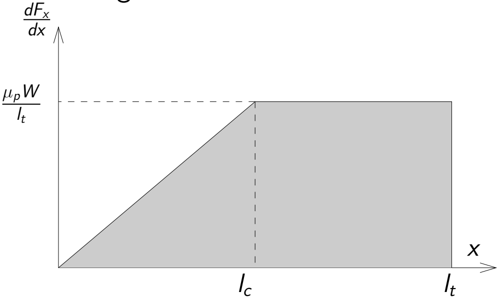
In the limit case $i = i_c = \frac{\mu_p W}{k_t l_t^2}$ is

$$F_x = \frac{1}{2} k_t l_t^2 \frac{\mu_p W}{k_t l_t^2} = \frac{\mu_p W}{2} \equiv F_{xc}$$

Driving Wheel: The Brush Model

The second case: There is a sliding region ($i > i_c$).

The distribution of the longitudinal force in this case:

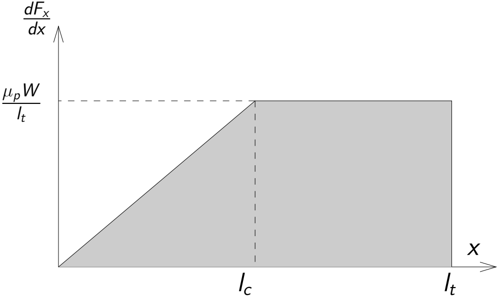


How do we calculate the length of the adhesion region l_c?

Driving Wheel: The Brush Model

Solution: Recall that the bristle will not slide if $k_t ix < \mu_p W / l_t$, i.e.,

$$x \leq \frac{\mu_p W}{k_t l_t i} \equiv l_c$$



The longitudinal force is equal to the shaded area

$$F_x = \frac{1}{2}\frac{\mu_p W}{l_t} l_c + \frac{\mu_p W}{l_t}(l_t - l_c) = \mu_p W \left(1 - \frac{1}{2}\frac{l_c}{l_t}\right)$$

The Brush Model: Summary

Critical values if longitudinal slip and force:

$$i_c = \frac{\mu_p W}{k_t l_t^2} = \frac{\mu_p W}{2C_i} \text{ och } F_{xc} = \frac{\mu_p W}{2} = C_i i_c$$

There is no sliding region when $i \leq i_c$ eller $F_x \leq F_{xc}$ and in this case

$$F_x = \frac{k_t l_t^2}{2} i = C_i i$$

If $i > i_c$ eller $F_x > F_{xc}$, then the length of the adhesion region is

$$l_c = \frac{\mu_p W}{k_t l_t i}$$

and the longitudinal force is

$$F_x = \mu_p W \left(1 - \frac{1}{2}\frac{l_c}{l_t}\right) = \mu_p W \left(1 - \frac{\mu_p W}{4 C_i i}\right)$$

Braking Wheel: The Brush Model

The skid is defined

$$i_s = \left(1 - \frac{\omega r}{V}\right) \times 100\% = \left(1 - \frac{r}{r_e}\right) \times 100\%$$

when a braking torque is applied to the wheel.

Limit cases:

Free-rolling tire: $i_s = 0$

Locked wheel: $i_s = 100\%$

Relations between i and i_s:

$$i = -\frac{i_s}{1 - i_s}$$

and

$$i_s = -\frac{i}{1 - i}$$

Braking Wheel: Summary

$$C_s = \left. \frac{\partial F_x}{\partial i_s} \right|_{i_s=0}$$

Critical values of skid and longitudinal force

$$i_{sc} = \frac{\mu_p W}{2C_s + \mu_p W}$$

$$F_{xc} = \frac{C_s i_{sc}}{1 - i_{sc}} = \frac{\mu_p W}{2}$$

No slide region ($i_s < i_{sc}$):

$$F_x = \frac{C_s i_s}{1 - i_s}$$

With slide region ($i_s \geq i_{sc}$):

$$F_x = \mu_p W \left(1 - \frac{\mu_p W (1 - i_s)}{4 C_s i_s} \right)$$