Vehicle Dynamics and Control

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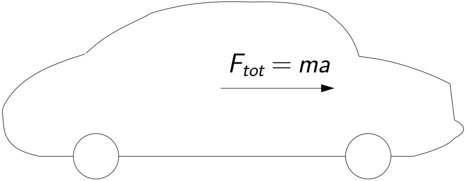
Lecture 2

Longitudinal dynamics

Model:



Forces acting on the vehicle in the longitudinal direction:

- Tractive/braking force from the wheels: F
- Rolling resistance: R_r
- Horizontal component of the gravitational force: R_g
- Aerodynamic resistance: R_a

Equation of motion in x-direction:

$$m\frac{dV}{dt} = F - R_r - R_g - R_a$$

Longitudinal dynamics: Forces

Some models for Tractive/braking force from the wheels, F, and rolling resistance, R_r, were presented the previous lecture.

The horizontal component of the gravitational force is

$$R_g = W \sin \theta_s$$

where $W = mg$ och θ_s is the slope angle.

I will use the convention that θ_s is positive in uphill slopes and negative in downhill slopes. (In the course book, it is assumed that θ_s is always positive and $R_g = \pm W \sin \theta_s$.)

Longitudinal dynamics: Aerodynamic resistance

Model for the aerodynamic resistance

$$R_a = \frac{\rho}{2} C_D A_f V_r^2$$

where

ρ: Air density

C_D: Coefficient of aerodynamic resistance

A_f: Frontal area

V_r: Speed of the vehicle relative to the wind

It will be assumed that $\rho = 1.225 kg/m^3$

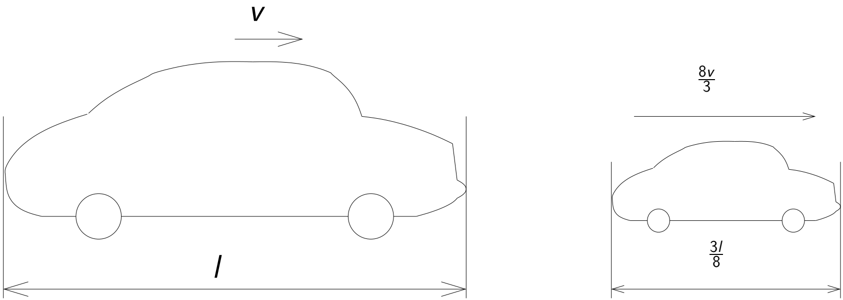
Empirical formula for frontal area

$$A_f = 1.6 + 0.00056(m - 765)$$

The frontal area A_f and the coefficient C_D for som car models can be found in Table 3.1.

Aerodynamic resistance: Wind Tunnel Experiments

To get similar air flow of air the product of the characteristic length and the velocity should be the same:

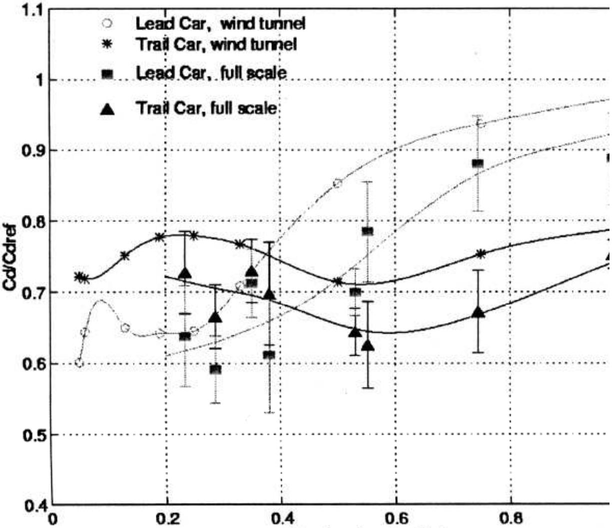


The flow of air is also influenced by

- The cross-sectional area of the wind tunnel
- The speed of the road relative the speed of the vehicle

Aerodynamic resistance

The figure shows C_D for two trucks as a function of the distance between them



Aerodynamic Lift

The flow of air also causes a lift force

$$R_L = \frac{\rho}{2} C_L A_f V_r^2$$

where C_L is the coefficient of aerodynamic lift.

Application: Mass Estimation

The mass of a truck varies depending the load carried on the trailer. To know the mass can be valuable when controlling the vehicle, e.g., when accelerating before an uphill slope.

Assume that we want to estimate the mass m and using the longitudinal equation motion

$$ma = F - R_r - R_g - R_a$$

If everything else in the equation is known except m, then the equation can be used to calculate m (using e.g. a Kalman filter).

Application: Mass estimation

Assume that rotational speed of the wheels are measured and used to estimate the speed V

Main challenges:

It may be difficult to estimate the longitudinal acceleration accurately.

The models for the propelling force F, rolling resistance R_r, and aerodynamic resistance R_a are usually not very accurate.

The slope angle θ_S is not known.

Application: Mass estimation

Assume that the signal from an accelerometer, measuring the longitudinal acceleration, is available. How can this information be used?

The longitudinal equation of motion

$$ma = F - R_r - R_g - R_a$$

can be rewritten as

$$m(a + g\sin\theta) = F - R_r - R_a$$

and the accelerometer is measuring

$$a + g\sin\theta$$

Hence, it is not necessary to know the slope angle θ.

Longitudinal model

Now, the normal forces will included in the model.

Figure 3.1 shows the forces acting on a vehicle during an acceleration.

Assume that the slope angle θ is equal to zero and the vehicle isn't moving. Then the equations of equilibrium are

$$
\begin{aligned}
W_f + W_r &= W \\
W_f l_1 - W_r l_2 &= 0
\end{aligned}
$$

and the solution is

$$
\begin{aligned}
W_f &= \frac{l_2}{L} W \\
W_r &= \frac{l_1}{L} W
\end{aligned}
$$

Figure 3.1

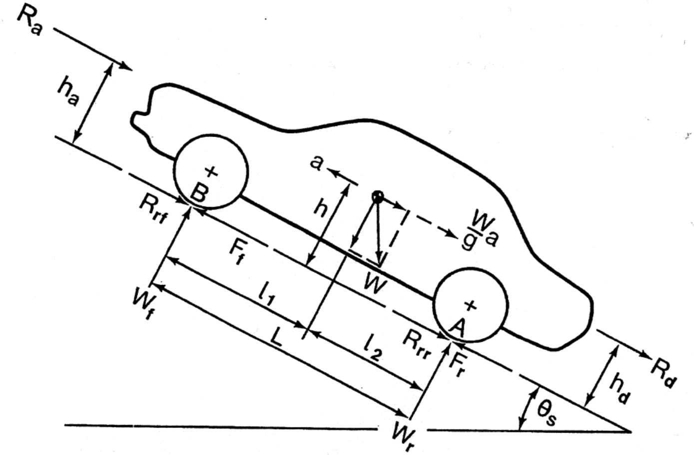


Fig. 3.1 Forces acting on a two-axle vehicle.

Longitudinal model

Now, a moving vehicle will be considered.

It will be assumed that $h_a = h_d = h$ and equilibrium of moments about two point the distance h above the points A and B gives the equations

$$
\begin{aligned}
-Wl_2 + LW_f + h(F_f - R_{rf}) + h(F_r - R_{rr}) = 0 \\
Wl_1 - LW_r + h(F_f - R_{rf}) + h(F_r - R_{rr}) = 0
\end{aligned}
$$

with the solutions

$$W_f = \frac{l_2}{L}W - \frac{h}{L}(F - R_r)$$

and

$$W_r = \frac{l_1}{L}W + \frac{h}{L}(F - R_r)$$

Maximal acceleration

Assume that the car is rear-wheel driven. What is the maximal propelling force that is possible to accomplish? The limit case is

$$F_{max} = \mu W_r + f_r W_r = (\mu + f_r)\left(\frac{l_1}{L} W + \frac{h}{L}(F_{max} - R_r)\right)$$

Solve for F_{max} and use $R_r = f_r W$

$$F_{max} = \frac{(\mu + f_r) W (l_1 - f_r h)}{L - (\mu + f_r) h}$$

The corresponding result for a front-wheel driven car is

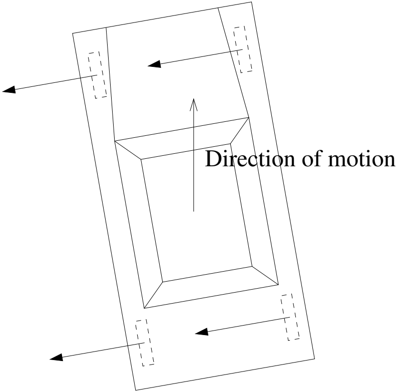
$$F_{max} = (\mu + f_r) W_f = (\mu + f_r)\left(\frac{l_2}{L} W - \frac{h}{L}(F_{max} - R_r)\right)$$

och

$$F_{max} = \frac{(\mu + f_r) W (l_2 + f_r h)}{L + (\mu + f_r) h}$$

Lateral forces and stability: Introduction

Assume that a car is moving on a straight line and the motion is perturbed:

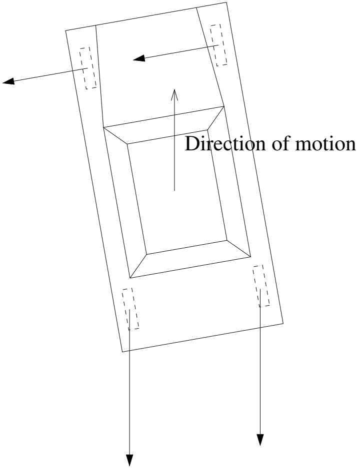


The figure shows that

- Front wheels turns the car counterclockwise (bad!?)
- Rear wheel turns the car clockwise (good!?)

Lateral forces and stability: Braking

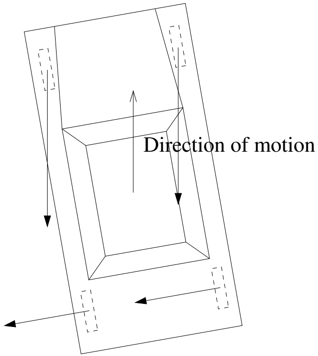
Assume that the rear wheels are used for braking causing a wheel lock-up:



The figure shows that the car will turn away from the intended direction and the car will probably become unstable.

Lateral forces and stability: Braking

Let the front wheels be used instead causing a wheel lock-up:



The figure shows that rear wheel turns the car towards the direction of the unperturbed direction. The drawback is that it becomes difficult to maneuver the car.

Brake force distribution

The objective is to distribute the forces so that the front and rear begin to slide at the same time. Given a braking force F_b, we will find coefficients K_{bf} and K_{br}, $K_{bf} + K_{br} = 1$, and distribute the braking force $F_{bf} = K_{bf} F_b$ and $F_{br} = K_{br} F_b$ to reach the objective.
Figure 3.47 shows the forces acting on the vehicle

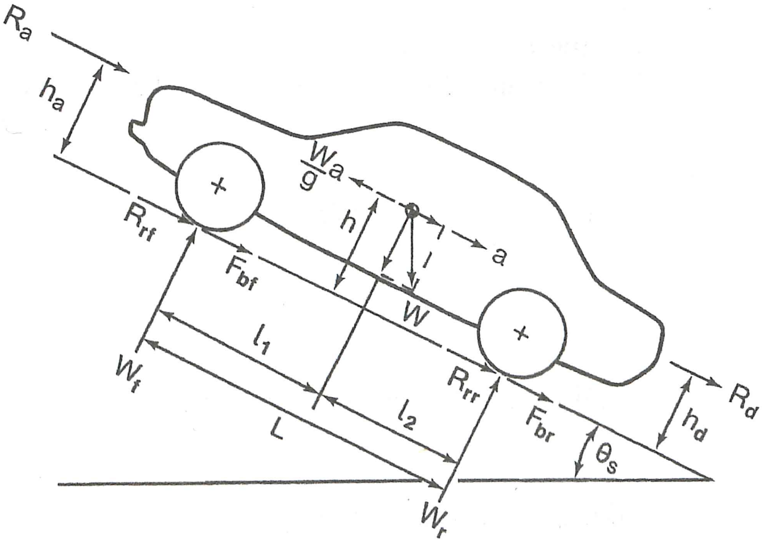
In this case we get

$$W_f = \frac{1}{L}(Wl_2 + h(F_b + f_r W))$$

och

$$W_r = \frac{1}{L}(Wl_1 - h(F_b + f_r W))$$

Figure 3.47



Brake force distribution

In the case where all wheels begin to slide we have

$$F_{bmax} = F_{bf} + F_{br} = \mu W_f - f_r W_f + \mu W_r - f_r W_r = \mu W - f_r W$$

Hence, $F_b + f_r W = \mu W$ and

$$F_{bf} = K_{bf} F_{bmax} = (\mu - f_r) W_f = \frac{(\mu - f_r) W (l_2 + h\mu)}{L}$$

and

$$F_{br} = K_{br} F_{bmax} = (\mu - f_r) W_r = \frac{(\mu - f_r) W (l_1 - h\mu)}{L}$$

The ratio of the brake forces is then

$$\frac{F_{bf}}{F_{br}} = \frac{K_{bf}}{K_{br}} = \frac{l_2 + h\mu}{l_1 - h\mu}$$

Brake force distribution: Alternative approach

Assume now that brake force distribution is given, i.e., K_{bf} and K_{br} where $K_{bf} + K_{br} = 1$. Will the front or the rear wheel lock first?

Only the brake force and rolling resistance will be considered in this case. Hence

$$ma = F_b + F_r \quad \text{and} \quad F_b + f_r W = \frac{W}{g} a,$$

First, the front wheels will be considered. The normal force is

$$W_f = \frac{W}{L}\left(l_2 + \frac{a}{g} h\right),$$

In this case the brake force is

$$F_{bf} = K_{bf} F_b = K_{bf} W \left(\frac{a}{g} - f_r\right)$$

Brake force distribution: Alternative approach

The front wheel will lock when

$$F_{bf} = \mu W_f - f_r W_f = (\mu - f_r)\frac{W}{L}\left(l_2 + \frac{a}{g}h\right)$$

It follows that

$$K_{bf} W\left(\frac{a}{g} - f_r\right) = \frac{(\mu - f_r)W}{L}\left(l_2 + \frac{a}{g}h\right)$$

and the front wheels lock when

$$\left(\frac{a}{g}\right)_f = \frac{(\mu - f_r)l_2/L + K_{bf} f_r}{K_{bf} - (\mu - f_r)h/L}$$

In the same way wed get that the rear wheels lock when

$$\left(\frac{a}{g}\right)_r = \frac{(\mu - f_r)l_1/L + K_{br} f_r}{K_{br} + (\mu - f_r)h/L}$$

Brake force distribution: Alternative approach

Given a brake force distribution.

The front wheels will lock first if

$$\left(\frac{a}{g}\right)_f < \left(\frac{a}{g}\right)_r$$

and the rear wheels will lock first if

$$\left(\frac{a}{g}\right)_r < \left(\frac{a}{g}\right)_f$$