Vehicle Dynamics and Control

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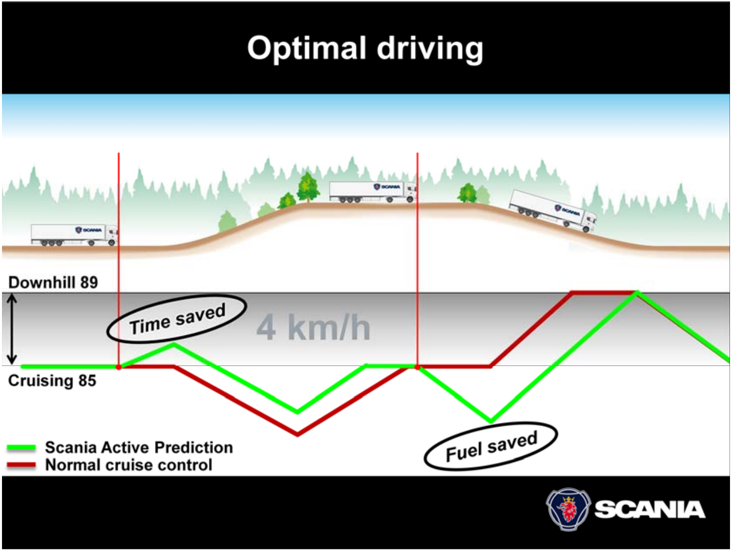
Lecture 3

Longitudinal control: Cruise control

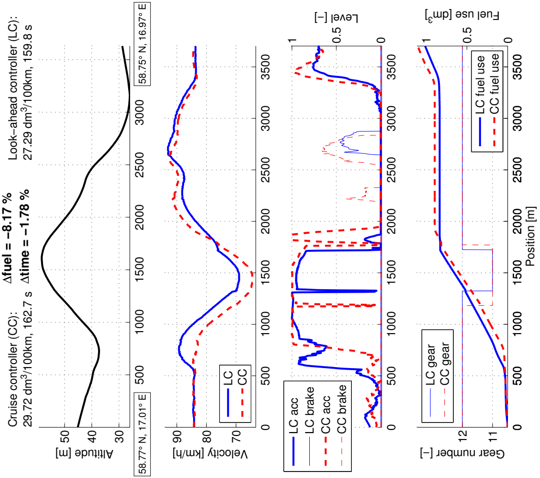
"RunSmart Predictive Cruise: How it Works
Unlike standard cruise control, where the truck tries to maintain a set speed regardless of the terrain ahead, RunSmart Predictive Cruise looks up to one mile ahead of the truck's location and anticipates road grades by using GPS and 3D digital map technology. The system adjusts the actual speed of the truck for maximum fuel efficiency based on the terrain while staying within 6 percent of the set speed."

Press release from Freightliner Trucks, March 19 2009

Longitudinal control: Cruise control



Comparison between Look-ahead Controller (LC) and Conventional controller (CC)



Longitudinal control

Some examples

- CC Cruise Control
- ACC Adaptive Cruise Control
- CA Collision avoidance
- ABS Anti-Blockier-System

ACC

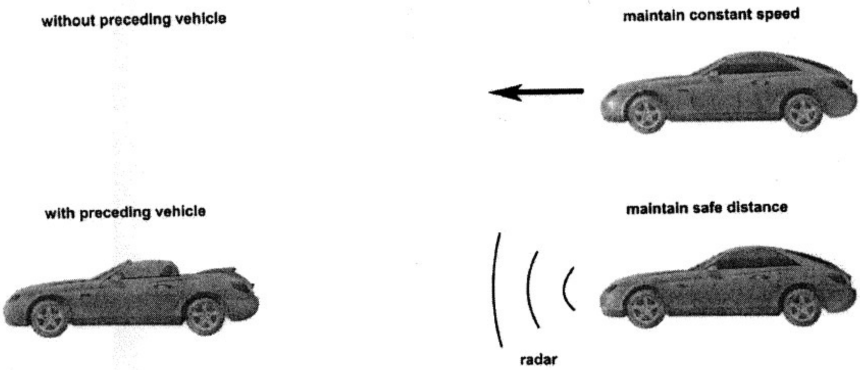
Use radar or other senors to measure the distance to other vehicles.

Control brakes and acceleration

Three different modes

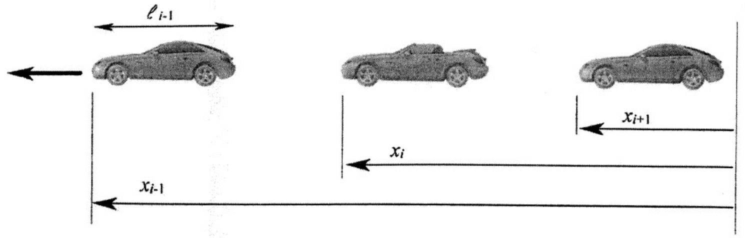
- Cruise control
- Keep distance to a vehicle in front of you
- Brake to avoid collision

ACC


without preceding vehicle
maintain constant speed
with preceding vehicle
maintain safe distance
radar

ACC: String stability

In a long caravan with ACC in all vehicles, string stability is important



ACC String stability

Consider a caravan where x_i, $i = 1, 2, \dots$ the positions of the vehicles

Define

$$\delta_i = x_i - x_{i-1} + L_{des}$$

where L_{des} is the desired distance.

A simple longitudinal model of the vehicle

$$\ddot{x}_i = u_i$$

where the acceleration u_i is the control signal

Assume that the following control strategy is used

$$u_i = -k_p \delta_i - k_v \dot{\delta}_i$$

ACC String stability

It is possible to show that the transfer function relating the spacing errors of two consecutive vehicles is

$$G(s) = \frac{\delta_i(s)}{\delta_{i-1}(s)} = \frac{k_v s + k_p}{s^2 + k_v s + k_p}$$

The gain is

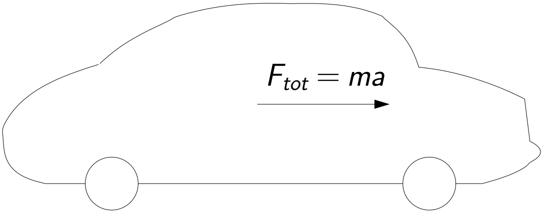
$$|G(i\omega)| = \sqrt{\frac{k_p^2 + k_v^2\omega^2}{(k_p - \omega^2)^2 + k_v^2\omega^2}}$$

and it is straightforward to show that $|G(i\omega)| > 1$ if $\omega < \sqrt{2k_p}$. This means that the amplitude of a low frequency oscillation increases when it is transferred backwards in the caravan.

Source: *Vehicle Dynamics and Control*, Rajesh Rajamani

Longitudinal dynamic

From the previous lecture:



Equation of longitudinal motion:

$$m\frac{dV}{dt} = F - R_r - R_g - R_a$$

Position as independent variable

In the cruise control application, the grade resistance R_g is a function of position. In this and many other cases it is natural to use position as independent variable.

The right-hand side of differential then becomes

$$m\frac{dv}{dt} = m\frac{dv}{dx}\frac{dx}{dt} = mv\frac{dv}{dx} = \frac{m}{2}\frac{d(v^2)}{dx} = \frac{d(mv^2/2)}{dx}$$

and we obtain

$$\frac{m}{2}\frac{d(v^2)}{dx} = F - R_r - R_g - R_a$$

Note that the previously introduced models for R_r and R_a are linear functions of v^2.

Kinetic energy

It follows that

$$d(mv^2/2) = (F - R_r - R_a)\,dx - mg\,dh$$

where

- $d(mv^2/2)$:change of kinetic energy.
- $(F - R_r - R_a)\,dx$: work
- $mg\,dh = mg\sin\theta_s\,dx$: change of potential energy

Stopping distance: First case

Given an initial speed V, the objective is to determine the stopping distance S. From the previous slides:

$$\frac{m}{2}\frac{d(v^2)}{dx} = -F_b - R_r - R_g - R_a$$

Before the general case is analyzed, some special cases will be considered First case: Neglect all forces except F_b. In this case we get:

$$\frac{m}{2}d(v^2) = -F_b\,dx$$

Calculate the integrals from start to stop

$$\int_{V^2}^{0} \frac{m}{2}\,d(v^2) = -\int_0^S F_b\,dx$$

Note how the intervals of the integrals were chosen!

Stopping distance: First case

The results is

$$\frac{mV^2}{2} = F_b S$$

i.e.

$$\text{Initial kinetic energy} = \text{Stopping distance} \times \text{Brake force}$$

and

$$S = \frac{mV^2}{2F_b}$$

Stopping distance: Second case

The second case includes the grade resistance $mg \sin \theta_s$:

$$\int_{V^2}^{0} \frac{m}{2}\, d(v^2) = -\int_{0}^{S} (F_b + mg \sin \theta_s)\, dx$$

and the result is

$$\frac{mV^2}{2} = F_b S + mg \sin \theta_s S$$

i.e.

$$\begin{aligned} \text{Initial kinetic energy } =& \text{Stopping distance} \times \text{Brake force} \\ &+ \text{Change in potential energy} \end{aligned}$$

and

$$S = \frac{mV^2/2}{F_b + mg \sin \theta_s}$$

Stopping distance: General case

$$\frac{m}{2}\frac{d(v^2)}{dx} = -F_b - mg\sin\theta_s - f_r mg\cos\theta_s - C_{ae}v^2$$

It is a separable differential equation

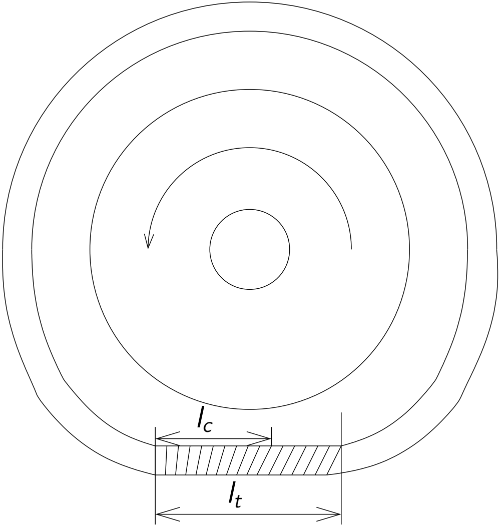
$$\frac{m}{2}\int_{V^2}^{0}\frac{d(v^2)}{F_b + mg\sin\theta_s + f_r mg\cos\theta_s + C_{ae}v^2} = -\int_0^S dx$$

and

$$\begin{aligned}
S &= \frac{m}{2C_{ae}}\log\left(\frac{F_b + mg\sin\theta_s + f_r mg\cos\theta_s + C_{ae}V^2}{F_b + mg\sin\theta_s + f_r mg\cos\theta_s}\right)\\
&= \frac{m}{2C_{ae}}\log\left(1 + \frac{C_{ae}V^2}{F_b + mg\sin\theta_s + f_r mg\cos\theta_s}\right)
\end{aligned}$$

Brush model

From Lecture 1



Brush model: Normal pressure

It was assumed that the normal pressure was constant in the contact region.

According to Figure 1.15 a parabola shaped distribution seem more reasonable, i.e.,

$$\frac{dF_z}{dx} = Cx(l_t - x)$$

In the adhesion region:

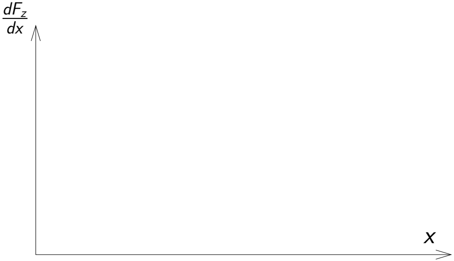
$$\frac{dF_x}{dx} = k_t \cdot i \cdot x$$

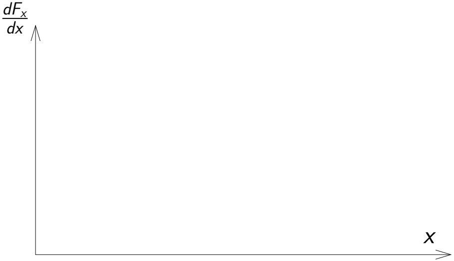
and

$$\frac{dF_x}{dx} < \mu_p \frac{dF_z}{dx}$$

Brush model: Normal pressure

Sketch the normal and longitudinal force distributions:





The longitudinal force F_x is the area under the curve to the right.

Brush model: Sliding friction

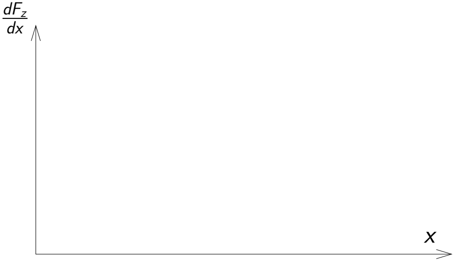
Figure 1.16 shows the longitudinal force as a function of slip.

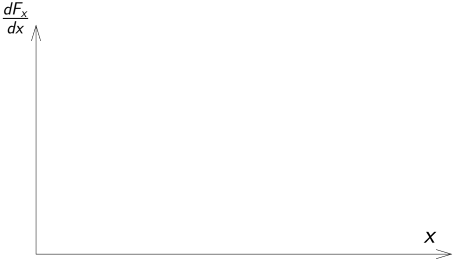
It can be seen that the force reaches a peak value and then decreases.

Assume that the sliding friction μ_s is lower than the friction μ_p in the adhesive region, and that the normal force distribution is constant in the contact region.

Brush model: Sliding friction

Sketch the normal and longitudinal force distributions:





The longitudinal force F_x is the area under the curve to the right.

Estimation of coefficient of friction μ

The coefficient of friction has been an important part when alakyzing the braking and acceleration performance. Now, one example will be presented on how to estimate the coefficient.

The approach is based on the approximation

$$\frac{F_x}{W} = K(\mu) \cdot i$$

where it is assumed that the gradient K is a function of μ. If we first find an estimate of K, then we can calculate μ.

To be able to calclulate K, we will first estimate

- Longitudinal force F_x
- Normal force W
- Longitudinal slip i

Friction: Longitudinal force F_x

Assume that longitudinal acceleration is measured.

Longitudinal model

$$ma = F_x - R_a - R_r - R_g$$

The longitudinal force is now given by

$$F_x = m(a + g \sin \theta_s) + R_a + R_r$$

where

- m is estimated mass
- $a + g \sin \theta_s$ is measured by the accelerometer
- R_a and R_r is calculated using empirical models

Friction: Normal force W

Normal force

$$W_f = \frac{l_2}{L} mg - \frac{h}{L}(R_a + m(a + g \sin \theta_s))$$

$$W_r = \frac{l_1}{L} mg + \frac{h}{L}(R_a + m(a + g \sin \theta_s))$$

Friction: Longitudinal slip i

Assume that the car is front-wheel driven.

Then there is no slip at the rear-wheel:

$$i_r = 1 - \frac{V_x}{\omega_r r_r} = 0$$

Modern cars have sensors measuring angular speed with high precision, since this information is needed by the ABS-system.

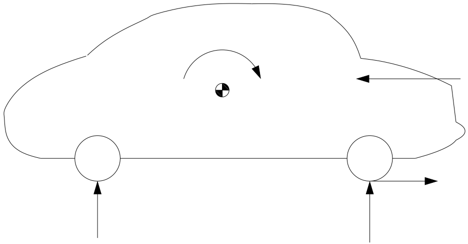
The sensors at the rear wheels can therefore be used to calculate V_x and then the slip at the front wheel can be calculated

$$i_f = 1 - \frac{V_x}{\omega_f r_f}$$

Now, the F_{xf}, W_f, and i_f at the front wheels are known and it possible to estimate K and μ

Brake force distribution

From lecture 2



$$\frac{K_{bf}}{K_{br}} = \frac{l_2 + h\mu}{l_1 - h\mu}$$

$$K_{bf} + K_{br} = 1$$

Electronic Brake-force Distribution

In this case we get

$$W_f = \frac{1}{L}(Wl_2 + h(F_b + F_r))$$

och

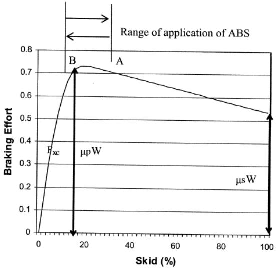
$$W_r = \frac{1}{L}(Wl_1 - h(F_b + F_r))$$

If we neglect aerodynamic resistance we get

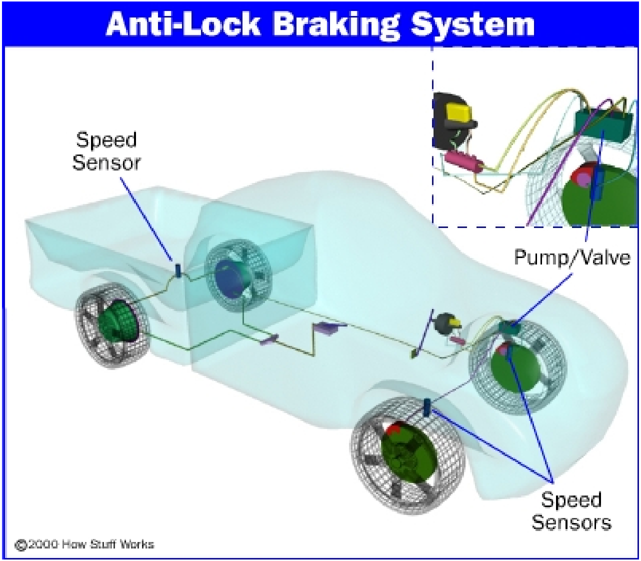
$$F_b + F_r = m(a + g\sin\theta)$$

Assume that we measure the longitudinal acceleration. How can we use this information to distribute the brake forces so that all wheels start sliding at the same time, without knowing μ?

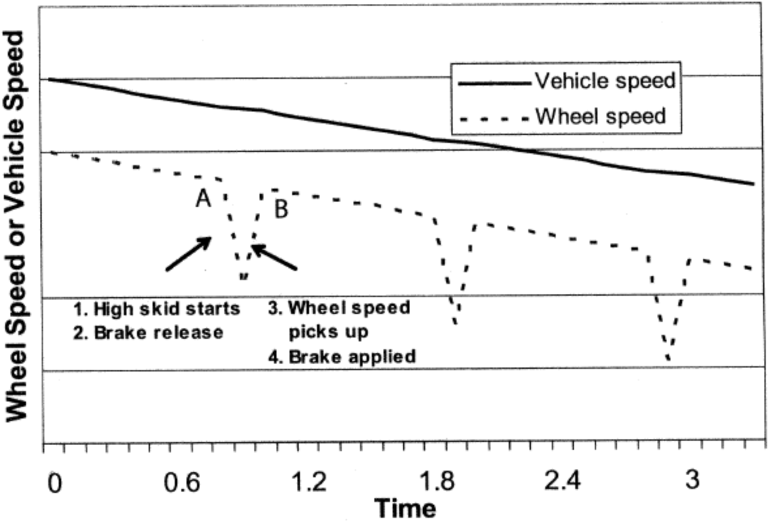
ABS: Introduction



ABS


Anti-Lock Braking System
Speed
Sensor
Pump/Valve
Speed
Sensors
©2000 How Stuff Works

ABS: A simple control strategy



ABS

When the wheels locks you loose

- Brake force
- Stability
- Ability to control the vehicle

The objective of the ABS system is to prevent wheel lock-up

ABS: Detecting wheel lock-up

If the wheels do not slide, then

$$\dot{\omega} r \approx a \leq \mu g$$

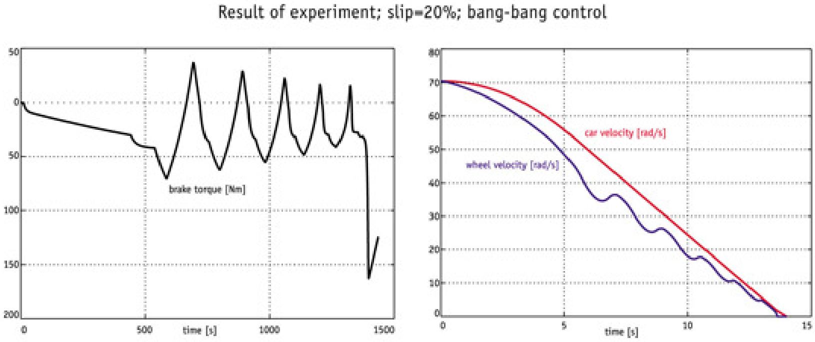
This can be used to detect when the wheels are locking, either by measuring the acceleration a or using an estimate of μ.

Another option is to use the skid

$$i_s = 1 - \frac{r\omega}{V}$$

where V is estimated.

ABS


Result of experiment; slip=20%; bang-bang control
brake torque [Nm]
time [s]
car velocity [rad/s]
wheel velocity [rad/s]
time [s]