1. A Nissan Micra GL, that weighs 1300 kg and is equipped with radial tires, travels on a straight level road at a speed of 90 km/h.

   a) Calculate the total force stemming from rolling resistance and aerodynamic resistance.

   b) What is the required power?

   c) What would the retardation caused by rolling and aerodynamic resistance be if neutral gear was engaged?

   d) Calculate force and power again but with a headwind of 10 m/s.

2. A VW Passat, that weighs 1800 kg and is equipped with radial tires, is at a standstill in a downhill slope. No gear is engaged and the brakes are released.

   a) Determine the rolling resistance as a function of car velocity.

   b) Determine the aerodynamic resistance as a function of car velocity.

   c) Formulate the governing differential equation and determine the velocity as a function of time with a slope angle of 0.5 degrees.

   d) Determine the stationary speed as a function of the slope angle.

3. In the following four exercises we study the simplified brush model for a tire under the action of a driving torque, see page 24 and forward in the course book.

   In this task, we consider a single brush element. Assume the following variables and parameters are known; velocity V, wheel rotational velocity ω, wheel radius r, contact length l_t, normal force W, friction coefficient μ_p, and tangential stiffness of the brush elements k_t.

   a) Sketch a single brush element, with the longitudinal force $\frac{dF_x}{dx}$, the vertical force $\frac{dF_z}{dx}$, and the longitudinal elongation e depicted.

   b) Describe how the longitudinal elongation e of the brush element varies as it travels through the contact patch. That is, express e as a function of x.

   c) What is the maximum longitudinal force that can be generated by the brush element?

   d) Express the longitudinal force $\frac{dF_x}{dx}$ as a function of x for

   - a brush element in adhesive contact with the road.
   - a sliding brush element.

4. Suppose the following: The contact length of the tire is $l_t = 12$ cm, the normal force is $W = 4000$ N with a uniform distribution over contact patch, and the coefficient of road adhesion is $\mu_p = 0.8$, and the tangential stiffness is $k_t = 16 \cdot 10^6$ N/m^2.

   a) Using the brush model, sketch the curves corresponding to the ones in Figure 1.15 (i.e. graphs for $\frac{dFx}{dx}$ and $\frac{dFx}{dx}$ vs. x) for the following cases:

   - The trailing edge of the contact patch is precisely at the adhesion limit.
   - There is sliding in half of the contact length.
   - There is sliding in the entire contact length.

   b) Use your figures to determine the longitudinal force F_x for these three cases.

   c) What are the values of the longitudinal slip i in these three cases?

   d) What is the relation between the slip i and the force F_x when i is less than the critical value?

   e) Using the brush model, sketch the curve corresponding to the one in Figure 1.16. What are the characteristic differences and what weaknesses does the brush model have?

5. Consider a tire under the action of a driving torque. Suppose that $\mu_p = 0.75$, $W = 3500$ N, and $C_i = 85000$ N.

   a) Determine the longitudinal force F_x if $i = 1\%$ and $i = 5\%$ respectively.

   b) Determine the longitudinal slip i if $F_x = 1000$ N and $F_x = 2000$ N respectively.

6. Consider the brush model for a tire under the action of a driving torque and assuming that the normal pressure is uniformly distributed. Known data are: Contact length $l_t = 14$ cm, normal force $W = 5000$ N, coefficient of adhesion $\mu_p = 0.85$, and tangential stiffness $k_t = 15 \cdot 10^6$ N/m^2. The car travels at 20 km/h and the slip is $i = 3\%$.

   a) Compute the length of the region where sliding takes place.

   b) Compute the power loss in the sliding region caused by friction.

Answers

1. a) 450 N
   b) 11.2 kW
   c) 0.346 m/s^2
   d) 715 N och 17.9 kW

2. a) $F_r = 240 + 7.06 \cdot 10^{-4} V^2$ with V in km/h
   b) $F_a = 0.423 V_r^2$ with V_r in m/s
   c) $v(t) = 0$
   d) If

   $$f_r = c_1 + c_2 V^2$$

   then the stationary speed is

   $$V_{stat} = \sqrt{\frac{mg(\sin\theta_s - c_1)}{\frac{1}{2} C_D \rho A_f + mgc_2}}$$

   θ_s assumed positive in downhill.

3. a) ...
   b) $e(x) = \left(1 - \frac{V}{\omega r}\right) x$
   c) $\frac{dF_x}{dx}|_{max} = \mu_p \frac{W}{l_t}$
   d) $\frac{dF_x}{dx} = k_t x i, \qquad \frac{dF_x}{dx} = \mu_p \frac{W}{l_t}$

4. a) ...
   b) $\mu_p W/2 = 1600$ N, $3\mu_p W/4 = 2400$ N, and $\mu_p W = 3200$ N.
   c) In the first two cases, the slip is 1.4% and 2.8% respectively. The third case can never occur with this model but $i = 100\%$ is close. Follow-up question: Explain why this never can occur if the brush model is used.
   d) $F_x = C_i i$, where $C_i = 115200$ N.
   e) The brush model increases monotonically, it has no peak value such as the peak at $i \approx 20\%$ in Figure 1.16.

5. a) 850 N and 2.2 kN respectively.
   b) 1.18% and 3.24% respectively.

6. a) 7.3 cm
   b) 378 W