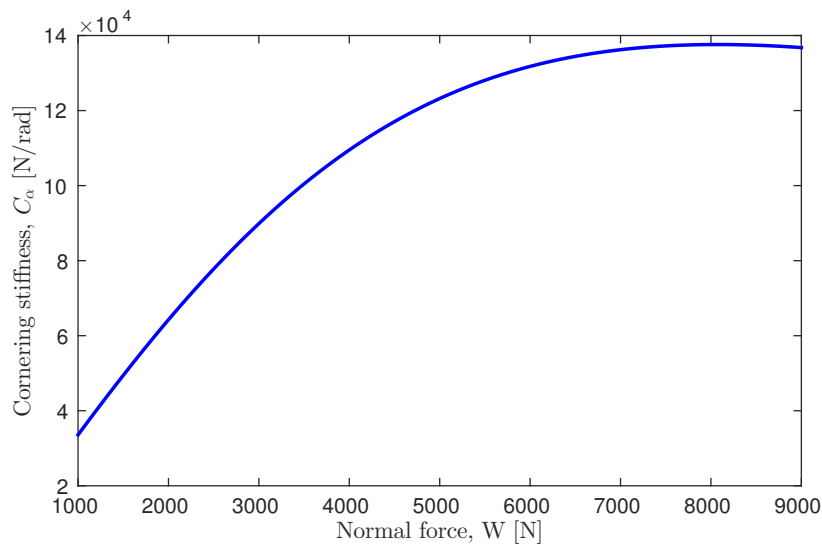


1. Utgå från borstmodellen för ett drivande hjul med normalkraften  $F_z = 49 \text{ kN}$  jämnt fördelad över kontaktytan. Friktionskoefficienten mellan väg och däck är  $\mu = 0.8$ . Antag att vilo- och glidzonen är  $8 \text{ cm}$  resp.  $6 \text{ cm}$  långa. Bestäm den longitudinella kraften  $F_x$ .
2. En bil har massan  $1800 \text{ kg}$  och kör på en plan väg. Axelavståndet är  $2.8 \text{ m}$  och tyngdpunkten ligger  $1.3 \text{ m}$  bakom framaxeln. Vi antar att  $h_a = h_d = h = 0.5 \text{ m}$  och att  $R_a + R_d = 300 \text{ N}$ . Det totala rullmotståndet är  $R_r = R_{rf} + R_{rr} = 200 \text{ N}$  och friktionskoefficienten mellan däck och underlag är  $\mu = 0.9$ .
  - a) Antag att den framåt drivande kraften  $F = F_f + F_r$  är känd. Bestäm normalkrafterna  $W_f$  och  $W_r$  som en funktion av  $F$ .
  - b) Bestäm maximal acceleration om bilen är bakhjulsdriven.
3. En bil har massan  $1900 \text{ kg}$  och kör i en uppförsbacke med lutningen  $2$  grader och håller hastigheten  $60 \text{ km/h}$ . Rullmotståndskoefficienten är  $f_r = 0.014$  och luftmotståndet ges av  $R_a = \frac{1}{2}\rho C_D A_f V^2$ , där  $\rho = 1.225 \text{ kg/m}^3$ ,  $A_f = 2.0 \text{ m}^2$  och  $C_D = 0.33$ . Hur långa sträcka tar det att retardera bilen till  $50 \text{ km/h}$  om man frikopplar?
4. Sebastian Vettel is approaching *Curva Parabolica* in his Ferrari Formula One car at Monza, reaching a speed of  $330 \text{ km/h}$  before he needs to brake. The total weight of the car is  $720 \text{ kg}$ , with a  $45/55$  weight distribution (i.e.  $45\%$  of the mass rests on the front wheels when standing still). The wheel base is  $3.4 \text{ m}$ , and the center of gravity height is  $0.3 \text{ m}$  above the ground plane. The total downforce generated at  $330 \text{ km/h}$  is  $20 \text{ kN}$ , which is divided as;  $30\%$  from the front wing,  $30\%$  from the rear wing, and  $40\%$  from the underbody/floor/diffusor. The front wing downforce acts  $0.5 \text{ m}$  in front of the front axle, the rear wing downforce  $0.2 \text{ m}$  behind the rear axle, and the remaining downforce acts  $0.45 \text{ m}$  behind the center of gravity. The friction coefficient for these kind of racing tires are considerably higher than for conventional road tires, and also varies with several variables. However, here we assume the friction coefficient is  $\mu = 1.25$ . The frontal area is  $1.4 \text{ m}^2$  and the drag coefficient is  $0.8$ . The drag force is acting at a height of  $0.4 \text{ m}$ . Rolling resistance can be neglected.
  - a) What would the deceleration due to only air drag be at this speed?
  - b) What is the maximum total deceleration that is possible when applying the brakes?

- c) Compute the optimal brake-force distribution ( $K_{b,f}$  and  $K_{b,r}$ ) he should use at this point.
5. Consider a passenger car with a total mass of 1600 kg and a wheel base of 2.8 m, where the center of gravity is located 1.2 m behind the front axle. The vehicle is loaded with 800 kg of junk, placed 0.3 m in front of the rear axle. The cornering stiffness of the car's tires varies with the normal force according to the figure below.

How is the handling characteristics affected by the additional load?

- a) Calculate  $K_{us}$  without the load.
- b) Calculate  $K_{us}$  with the load.
- c) Is it necessary to account for the change in  $C_\alpha$  due to the normal load? Compute  $K_{us}$  with the additional load (as in b), but using the  $C_\alpha$  you determined without the load (i.e.,  $C_\alpha$  from a).



6. You are given the task to develop an autonomous drifting controller for a rear-wheel driven passenger car. Consider the vehicle in steady-state (i.e.  $\dot{\beta} = 0$  and  $\dot{\Omega}_z = 0$ ), “power sliding” in a roundabout with a radius of 50 m. The body slip angle is  $\beta = 30^\circ$  and the speed is constant. The system needs to know the driving force on the rear wheels  $F_{x,r}$  and the front steer angle  $\delta$ .

The vehicle’s total mass is  $m = 1500$  kg and the wheel base is  $L = 2.4$  m. The center of gravity is in the middle of the front and rear axle ( $l_1 = l_2$ ).

In the figure below, the lateral tire force  $F_y$  is shown as a function of slip angle  $\alpha$ . This diagram describes both front and rear tire-force characteristics ( $F_{y,f}(\alpha_f)$  and  $F_{y,r}(\alpha_r)$ ).

Note that the slip angle  $\beta$  cannot be assumed small here.

- a) Compute the rear-wheel longitudinal force  $F_{x,r}$  that is necessary to achieve the desired body slip and maintain constant velocity. Assume here that we can use the friction ellipse to describe the relation between  $F_y$  and  $F_x$ , according to

$$\left(\frac{F_y}{F_{y,\alpha}}\right)^2 + \left(\frac{F_x}{F_{x,max}}\right)^2 = 1$$

where  $F_{x,max} = \mu_x W$ , and  $F_{y,\alpha}$  is found from the figure below. The longitudinal friction coefficient is  $\mu_x = 0.9$ . (In reality the friction ellipse is not suitable for slip angles of this magnitude.)

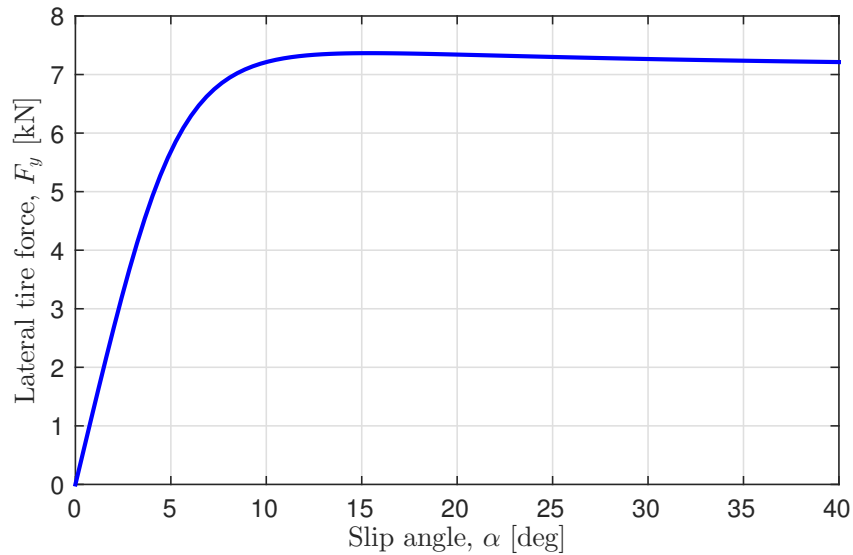
- b) Determine the steering angle  $\delta$  that is necessary to keep the vehicle in steady-state ( $\dot{\Omega}_z = 0$ ).

The force characteristics, in the figure above, is described by

$$F_y = D_y \sin(C_y \arctan\{B_y \alpha - E_y [B_y \alpha - \arctan(B_y \alpha)]\})$$

with the following parameter values

$$D_y = 7365, \quad C_y = 1.1930, \quad B_y = 8.8626, \quad E_y = -1.2076$$



## Answers

1. 2.8 kN
2. a)  $W_f = 9.5 \text{ kN} - 0.18F$ ,  $W_r = 8.2 \text{ kN} + 0.18F$   
 b)  $a_{max} \approx 4.7 \text{ m/s}^2$
3. 80 m
4. a)  $7.70 \text{ m/s}^2$   
 b)  $52.8 \text{ m/s}^2$   
 c)  $K_{b,f} = 0.552$ ,  $K_{b,r} = 0.448$
5. a)  $K_{us} = 0.0040$   
 b)  $K_{us} = -0.0104$   
 c)  $K_{us} = -0.0283$
6. a)  $F_{x,r} = 3.54 \text{ kN}$   
 b)  $\delta = \frac{L \cos \beta}{R} + \alpha_f - \alpha_r = -19.9^\circ$  (Which means we have to counter steer with  $20^\circ$ .)