

1. Consider a passenger car in steady state cornering which is driving at 60 km/h and experiences a lateral acceleration of  $4 \text{ m/s}^2$ . The vehicle mass is 1800 kg and the cornering stiffness, for both front and rear wheels, is  $5 \cdot 10^4 \text{ N/rad}$ . The distance from the center of gravity to the front axle is 1.1 m and 1.4 m to the rear axle.
  - a) Compute the understeer coefficient and determine if the vehicle is understeered, neutral steered, or oversteered.
  - b) Calculate the front and rear slip angles.
  - c) What is the necessary steering angle?
2. A car weighs 1600 kg and has a wheelbase of 2.8 m. The center of gravity is 1.3 m behind the front axle and the cornering stiffnesses are  $C_{\alpha f} = 4.5 \cdot 10^4 \text{ N/rad}$  and  $C_{\alpha r} = 4.7 \cdot 10^4 \text{ N/rad}$ . How much load can be put in the trunk, above the rear axle, without the car becoming oversteer?
3. A neutral steer vehicle is traveling straight ahead at a longitudinal velocity of 50 km/h. The cornering stiffnesses are  $C_{\alpha f} = 4.7 \cdot 10^4 \text{ N/rad}$  and  $C_{\alpha r} = 4.5 \cdot 10^4 \text{ N/rad}$ . What is the resulting lateral velocity if a side force  $F_y = 200 \text{ N}$  is applied at the vehicle's center of gravity?
4. A car has a wheelbase of 2.7 m. The center of gravity is located 1.2 m behind the front axle and 0.5 m above ground. 55% of the total braking force is placed on the front axle and 45% on the rear axle. The coefficient of road adhesion is  $\mu_f = 0.7$  in the front and  $\mu_r = 0.8$  rear. The coefficient of rolling resistance is  $f_r = 0.015$  and we neglect air drag. Which tires will lock up first during heavy braking on flat road?
5. Consider the brush model for a tire under the action of a driving torque. Assume that the normal pressure is uniformly distributed and that there are different friction coefficients in the adhesion region and sliding region respectively. Known data are: Length of the contact patch  $l_t = 14 \text{ cm}$ , normal load  $W = 4000 \text{ N}$ , tangential stiffness  $k_t = 15 \cdot 10^6 \text{ N/m}^2$ , longitudinal slip  $i = 3\%$ , friction coefficient in adhesion region  $\mu_p = 0.8$ , and friction coefficient in sliding region  $\mu_s = 0.65$ .
  - a) Determine how the tractive force per unit contact length  $dF_x/dx$  varies in the contact patch.
  - b) Determine the tractive force  $F_x$ .

**Answers**

1. (a)  $K_{us} = 0.0212$ . The vehicle is understeered.

(b)  $\alpha_f = 2.31^\circ$ ,  $\alpha_r = 1.82^\circ$

(c)  $\delta = 2.56^\circ$

2. 152 kg.

3. 1.5 cm/s.

4. The rear wheels lock up first.

5. (a)

$$\frac{dF_x}{dx} = \begin{cases} k_t i x, & \text{f\"or } 0 \leq x < l_c \\ \mu_s W / l_t, & \text{f\"or } l_c < x \leq l_t \end{cases}$$

where  $l_c = 5.1$  cm.

(b) 2.24 kN.