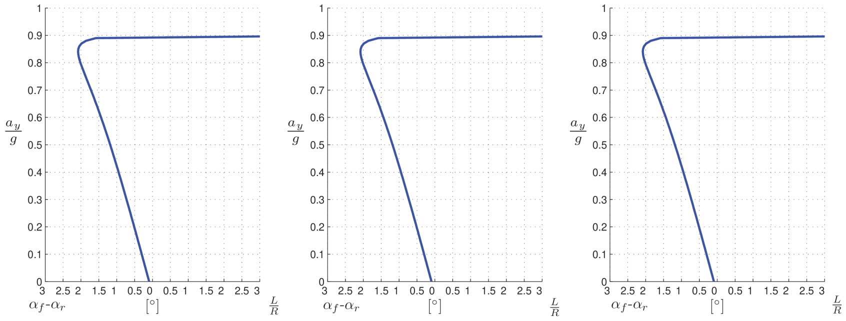
1. This is a continuation of exercise 4.1. The curves below show how $\alpha_f - \alpha_r$ depends on a_y/g. Draw curves that allows you to determine the following:

   a) The steer angle δ_f for constant curve radius $R = 100$ m.

   b) The steer angle δ_f for constant velocity 70 km/h.

   c) The quotient L/R for constant steer angle $\delta_f = 2.5°$.



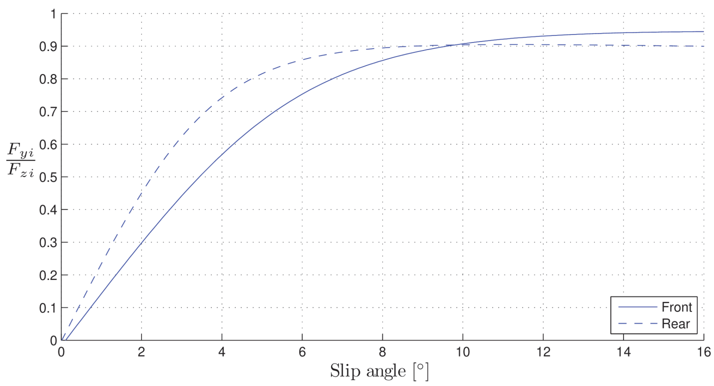
2. This is a continuation of exercise 4.1. Assume that $m = 1600$ kg, $I_z = 2800$ kgm^2, $l_1 = l_2 = 1.4$ m, and that the center of gravity is low enough so that longitudinal load transfer may be neglected. The car is traveling at 70 km/h and keeps a constant curve radius of 100 m when a braking force is applied on the rear wheels.

   a) Use the friction ellipse to determine F_{yr} if the braking force is $F_x = 0.5 \cdot F_{x,max}$.

   b) Determine the yaw acceleration $\dot{\Omega}_z$.

   c) How should δ_f change for $\dot{\Omega}_z = 0$ to hold instantaneously?

   The figure below is the tire-force characteristics from exercise 4.1.



3. In Lecture 4 the equation

$$\delta_f = \frac{L}{R} + K_{us}\frac{a_y}{g}$$

was derived for the steering angle at steady state cornering in the case of front-wheel steering. Assume that a rear-wheel steering is added to the model with the steer angle δ_r, as we did in Lecture 7. What is the corresponding relation?

4. It was shown in Lecture 7 that

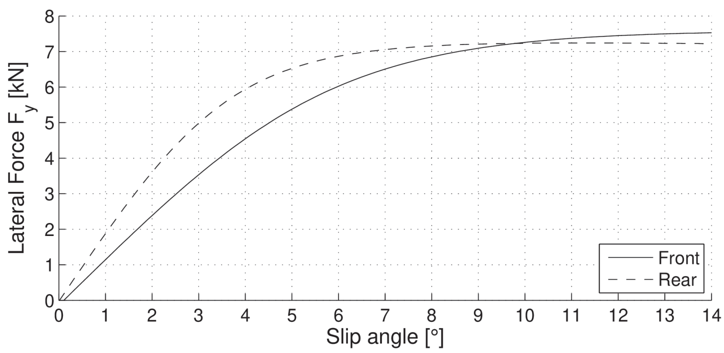
$$\begin{aligned}\begin{bmatrix} V_y(s) \\ \Omega_z(s) \end{bmatrix} &= (sM + A)^{-1}(\mathbf{u}_f\delta_f(s) + \mathbf{u}_r\delta_r(s)) \\ &= \frac{1}{\Delta}\begin{bmatrix} I_z s + a_4 & -a_2 \\ -a_3 & ms + a_1 \end{bmatrix}\left(\begin{bmatrix} 2C_{\alpha f} \\ 2l_1 C_{\alpha f} \end{bmatrix}\delta_f + \begin{bmatrix} 2C_{\alpha r} \\ -2l_2 C_{\alpha r} \end{bmatrix}\delta_r\right)\end{aligned}$$

where

$$\Delta = I_z m s^2 + (I_z a_1 + m a_4)s + (a_1 a_4 - a_2 a_3).$$

Assume that $C_{\alpha f} = C_{\alpha r} = C_\alpha$ and $l_1 = l_2 = L/2$ and consider a step in the front steer angle, $\delta_f(s) = 1/s$. Use the initial value theorem, i.e., $f(0^+) = \lim_{s\to\infty} sF(s)$, to determine the immediate response in $\dot{V}_y$ and $\dot{\Omega}_z$ in the two cases $\delta_r = \delta_f$ (rear wheels in phase) and $\delta_r = -\delta_f$ (rear wheels out-of phase), respectively. (Hint: You don't need to calculate the matrix A.)

5. The lateral forces, as a function of the slip angle, for the front and rear tires respectively is given by the following figure:



The vehicle mass is $m = 1600$ kg, the wheelbase $L = 2.6$ m, and the center of gravity is 1.2 m behind the front axle. The vehicle is driving through a long curve with radius is $R = 100$ m (assume stationary conditions). Determine the steer angle δ_f if the velocity is $v = 90$ km/h.

6. In this exercise, we study steady-state cornering and use the tire brush model. Assume that $l_1 = l_2$, $C_{\alpha f} < C_{\alpha r}$, and $\mu_r < \mu_f$.

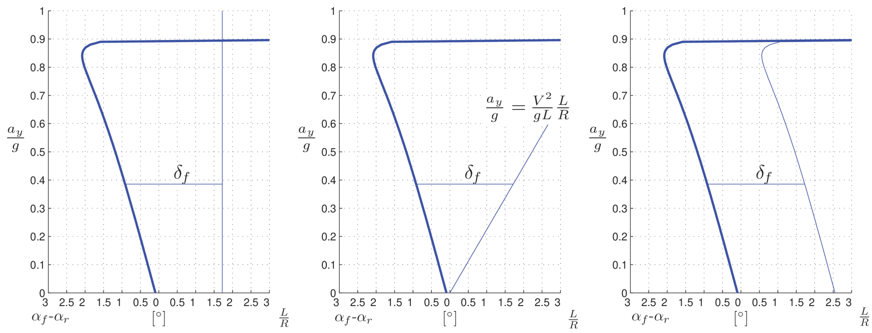
   a) Sketch the normalized curves F_{yf}/F_{zf} and F_{yr}/F_{zr} in the same figure.

   b) What value of a_y/g yields the critical value of the slip angle (according to the brush model) in the front and rear respectively.

   c) Calculate the understeer gradient as a function of a_y/g.

Answers

1. Curves:



2. a) $F_{yr} = 2.6$ kN
   b) $\dot{\Omega}_z = 0.20$ rad/s^2
   c) δ_f should be reduced by about 0.2°.
3. $\delta_f - \delta_r = \frac{L}{R} + K_{us}\frac{a_y}{g}$
4. if $\delta_f = \delta_r$:
$$\begin{bmatrix} V_y(0^+) \\ \Omega_z(0^+) \end{bmatrix} = \begin{bmatrix} \frac{4C_\alpha}{m} \\ 0 \end{bmatrix}$$
   if $\delta_f = -\delta_r$:
$$\begin{bmatrix} V_y(0^+) \\ \Omega_z(0^+) \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{2LC_\alpha}{I_z} \end{bmatrix}$$
5. $\delta_f = 3.5°$
6. a) ...
   b) $\mu_f/2$ and $\mu_r/2$ respectively.
   c) If $a_y/g < \mu_r/2$:
$$K_{us} = \frac{mg}{4}\left(\frac{1}{C_{\alpha f}} - \frac{1}{C_{\alpha_r}}\right)$$
   If $\mu_r/2 \leq a_y/g < \mu_f/2$:
$$K_{us} = \frac{mg}{4}\left(\frac{1}{C_{\alpha f}} - \frac{\mu_r^2}{4C_{\alpha_r}(\mu_r - a_y/g)^2}\right)$$
   If $\mu_f/2 \leq a_y/g$:
$$K_{us} = \frac{mg}{4}\left(\frac{\mu_f^2}{4C_{\alpha_f}(\mu_f - a_y/g)^2} - \frac{\mu_r^2}{4C_{\alpha_r}(\mu_r - a_y/g)^2}\right)$$