# Vehicle Propulsion Systems
# Lecture 2
## Fuel Consumption Estimation & ICE

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# Outline

Repetition

Energy Consumption of a Driving Mission
- The Vehicle Motion Equation
- Losses in the vehicle motion
- Energy Demand of Driving Missions

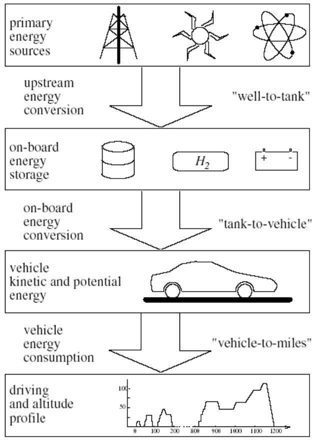
Energy demand
- Energy demand and recuperation
- Sensitivity Analysis

Forward and Inverse (QSS) Models

IC Engine Models
- Normalized Engine Variables
- Engine Efficiency



# Energy System Overview




Primary sources

Different options for on-board energy storage

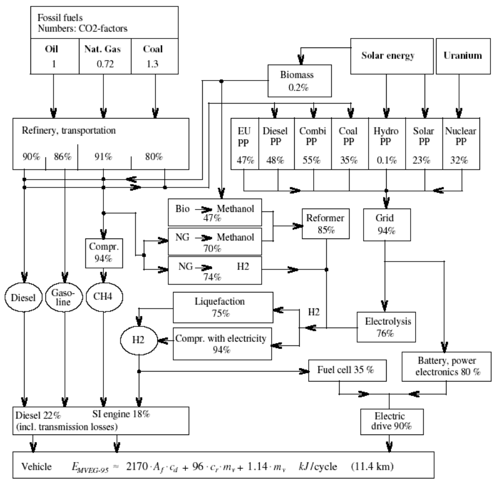
Powertrain energy conversion during driving

Cut at the wheel!

Driving mission has a minimum energy requirement.



# W2M – Energy Paths



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## Energy Consumption of a Driving Mission

- Remember the partitioning
  –Cut at the wheels.
- How large **force** is required at the wheels for driving the vehicle on a mission?



## Repetition – Work, power and Newton's law

Translational system – Force, work and power:

$$W = \int F\,dx, \qquad P = \frac{d}{dt}W = F\,v$$

Rotating system – Torque ($T = F\,r$), work and power:

$$W = \int T\,d\theta, \qquad P = T\,\omega$$

Newton's second law:

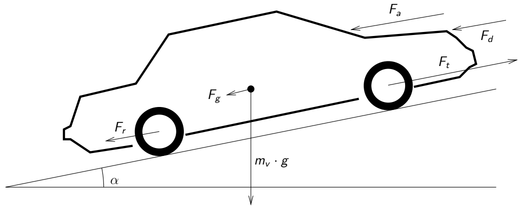
| Translational | Rotational |
|---|---|
| $m\,\frac{dv}{dt} = F_{driv} - F_{load}$ | $J\,\frac{d\omega}{dt} = T_{driv} - T_{load}$ |



## The Vehicle Motion Equation

Newton's second law for a vehicle

$$m_v \frac{d}{dt} v(t) = F_t(t) - (F_a(t) + F_r(t) + F_g(t) + F_d(t))$$




- $F_t$ – tractive force
- $F_a$ – aerodynamic drag force
- $F_r$ – rolling resistance force
- $F_g$ – gravitational force
- $F_d$ – disturbance force

## Aerodynamic Drag Force – Loss

Aerodynamic drag force depends on:
Frontal area $A_f$, drag coefficient $c_d$, air density $\rho_a$ and vehicle velocity $v(t)$

$$F_a(t) = \frac{1}{2} \cdot \rho_a \cdot A_f \cdot c_d \cdot v(t)^2$$

Approximate contributions to $F_a$

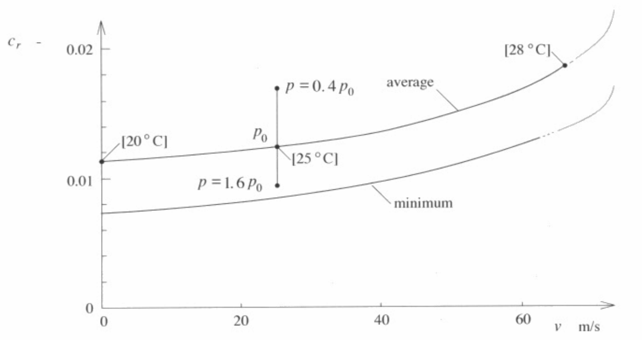
- 65% car body.
- 20% wheel housings.
- 10% exterior mirrors, eave gutters, window housings, antennas, etc.
- 5% engine ventilation.



## Rolling Resistance Losses

Rolling resistance depends on: – load and tire/road conditions

$$F_r(v, p_t, \text{surface}, \ldots) = c_r(v, p_t, \ldots) \cdot m_v \cdot g \cdot \cos(\alpha), \qquad v > 0$$




The velocity has small influence at low speeds.

Increases sharply for high speeds where resonance phenomena occur.

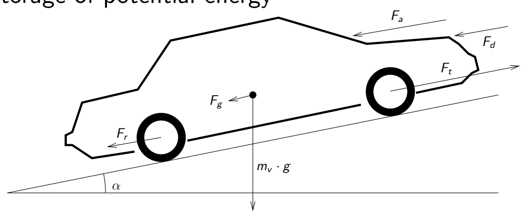
Assumption in book:
$c_r$ – constant

$$F_r = c_r \cdot m_v \cdot g$$



## Gravitational Force

- Gravitational load force
  –Not a loss, storage of potential energy



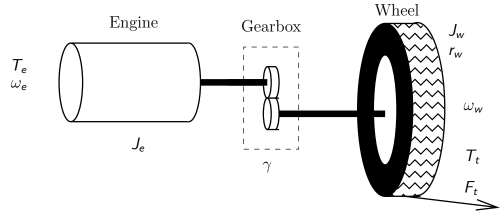

- Up- and down-hill driving produces forces.

$$F_g = m_v \, g \, \sin(\alpha)$$

- Flat road assumed $\alpha = 0$ if nothing else is stated (In the book).



## Inertial forces – Reducing the Tractive Force




$$T_e - J_e \tfrac{d}{dt}\omega_e = T_{gb} \qquad\qquad T_{gb} \cdot \gamma - J_w \tfrac{d}{dt}\omega_w = T_t$$

Variable substitution: $T_w = \gamma \, T_e$, $\qquad \omega_w \, \gamma = \omega_e$, $\qquad v = \omega_w \, r_w$

Tractive force:

$$F_t = \frac{1}{r_w}\left[\left(T_e - J_e \frac{d}{dt}\frac{v(t)}{r_w}\gamma\right) \cdot \gamma - J_w \frac{d}{dt}\frac{v(t)}{r_w}\right] = \frac{\gamma}{r_w} T_e - \left(\frac{\gamma^2}{r_w^2} J_e + \frac{1}{r_w^2} J_w\right) \frac{d}{dt} v(t)$$

The Vehicle Motion Equation:

$$\left[m_v + \frac{\gamma^2}{r_w^2} J_e + \frac{1}{r_w^2} J_w\right] \frac{d}{dt} v(t) = \frac{\gamma}{r_w} T_e - (F_a(t) + F_r(t) + F_g(t) + F_d(t))$$

# Vehicle Operating Modes

The Vehicle Motion Equation:

$$m_v \frac{d}{dt} v(t) = F_t(t) - (F_a(t) + F_r(t) + F_g(t) + F_d(t))$$

- $F_t > 0$ traction
- $F_t < 0$ braking
- $F_t = 0$ coasting

$$\frac{d}{dt} v(t) = -\frac{1}{2\, m_v} \rho_a\, A_f\, c_d\, v^2(t) - g\, c_r = -\alpha^2\, v^2(t) - \beta^2$$

Coasting solution for $v > 0$

$$v(t) = \frac{\beta}{\alpha} \tan\left(\arctan\left(\frac{\alpha}{\beta} v(0)\right) - \alpha\,\beta\, t\right)$$



# How to check a profile for traction?

The Vehicle Motion Equation:

$$m_v \frac{d}{dt} v(t) = F_t(t) - (F_a(t) + F_r(t) + F_g(t) + F_d(t)) \quad (1)$$

- Traction conditions:
  $F_t > 0$ traction, $F_t < 0$ braking, $F_t = 0$ coasting
- Method 1: Compare the profile with the coasting solution over a time step

$$v_{coast}(t_{i+1}) = \frac{\beta}{\alpha} \tan\left(\arctan\left(\frac{\alpha}{\beta} v(t_i)\right) - \alpha\,\beta\,(t_{i+1} - t_i)\right)$$

- Method 2: Solve (1) for $F_t$

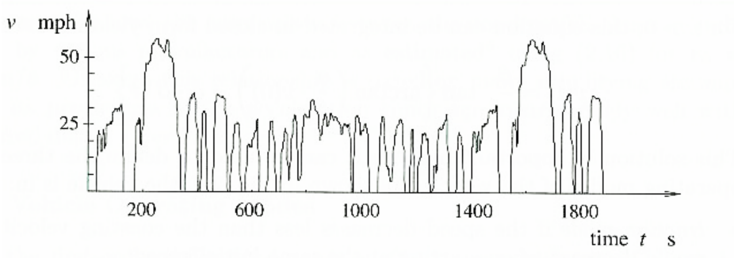
$$F_t(t) = m_v \frac{d}{dt} v(t) + (F_a(t) + F_r(t) + F_g(t) + F_d(t))$$

  Numerically differentiate the profile $v(t)$ to get $\frac{d}{dt} v(t)$.
  Compare with Traction condition ($F_t > 0$).



# Driving profiles

Velocity profile, American FTP-75 (1.5*FUDS).



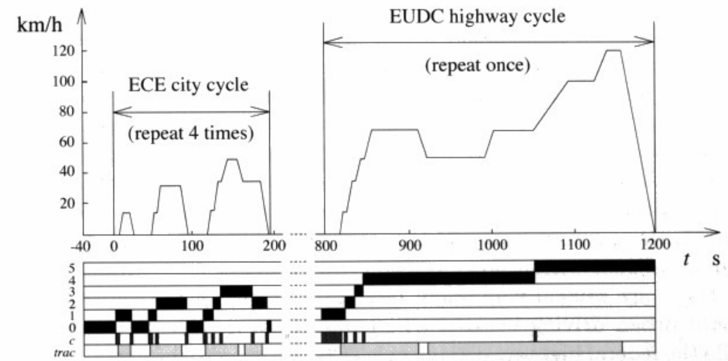

## Driving profiles in general

- First used for pollutant control now also for fuel consumption.
- Important that all use the same cycle when comparing.
- Different cycles have different energy demands.



# Driving profiles – Another example

Velocity profile, European MVEG-95

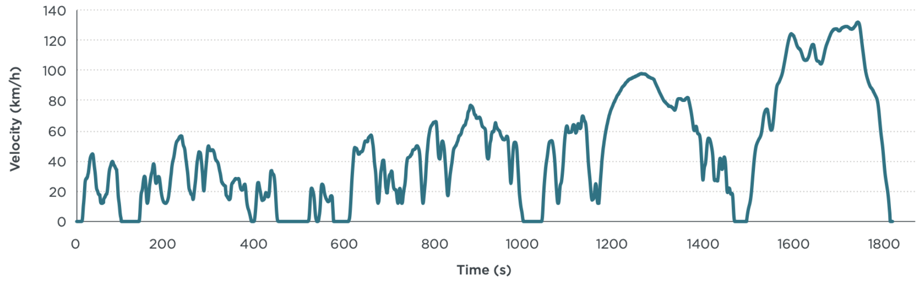



NEDC (ECE*4, EUDC)

No coasting in this driving profile.

## Driving profiles – A third example

Velocity profile, WLTC (Worldwide Harmonized Light Vehicles Test Cycle)




Adopted for new vehicles in 2017 in EU.
More demanding than NEDC.



## Mechanical Energy Demand of a Cycle

**Only** the demand from the cycle

- The mean tractive force during a cycle

$$\bar{F}_{trac} = \frac{1}{x_{tot}} \int_0^{x_{tot}} \max(F(x), 0)\, dx = \frac{1}{x_{tot}} \int_{t \in trac} F(t) v(t) dt$$

  where $x_{tot} = \int_0^{t_{max}} v(t) dt$.
- Note $t \in trac$ in definition.
- Only traction.
- Idling not a demand from the cycle.



## Evaluating the integral

Discretized velocity profile used to evaluate

$$\bar{F}_{trac} = \frac{1}{x_{tot}} \int_{t \in trac} F(t) v(t) dt$$

here $v_i = v(t_i), \quad t_i = i \cdot h, \quad i = 1, \ldots, n.$
Approximating the quantites

$$\bar{v}_i(t) \approx \frac{v_i + v_{i-1}}{2}, \qquad t \in [t_{i-1}, t_i)$$

$$\bar{a}_i(t) \approx \frac{v_i - v_{i-1}}{h}, \qquad t \in [t_{i-1}, t_i)$$

Traction approximation

$$\bar{F}_{trac} \approx \frac{1}{x_{tot}} \sum_{i \in trac} \bar{F}_{trac,i}\, \bar{v}_i\, h$$



## Evaluating the integral

Tractive force from *The Vehicle Motion Equation*

$$F_{trac} = \frac{1}{2} \rho_a A_f c_d v^2(t) + m_v g c_r + m_v a(t)$$

$$\bar{F}_{trac} = \bar{F}_{trac,a} + \bar{F}_{trac,r} + \bar{F}_{trac,m}$$

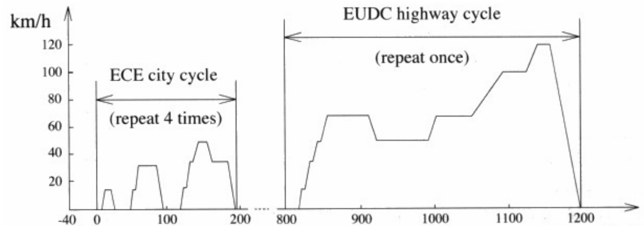
Resulting in these sums

$$\bar{F}_{trac,a} = \frac{1}{x_{tot}} \frac{1}{2} \rho_a A_f c_d \sum_{i \in trac} \bar{v}_i^3 h$$

$$\bar{F}_{trac,r} = \frac{1}{x_{tot}} m_v g c_r \sum_{i \in trac} \bar{v}_i h$$

$$\bar{F}_{trac,m} = \frac{1}{x_{tot}} m_v \sum_{i \in trac} \bar{a}_i \bar{v}_i h$$

## Values for cycles




Numerical values for the cycles: {MVEG-95, ECE, EUDC}

$$\bar{X}_{trac,a} = \frac{1}{x_{tot}} \sum_{i \in trac} \bar{v}_i^3\, h = \qquad \{319, 82.9, 455\}$$

$$\bar{X}_{trac,r} = \frac{1}{x_{tot}} \sum_{i \in trac} \bar{v}_i\, h = \qquad \{0.856, 0.81, 0.88\}$$

$$\bar{X}_{trac,m} = \frac{1}{x_{tot}} \sum_{i \in trac} \bar{a}_i\, \bar{v}_i\, h = \qquad \{0.101, 0.126, 0.086\}$$



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Adopting appropriate units and packaging the results as an Equation

$$\bar{E}_{\text{MVEG-95}} \approx A_f\, c_d\, 1.9 \cdot 10^4 + m_v\, c_r\, 8.4 \cdot 10^2 + m_v\, 10 \qquad kJ/100km$$

Tasks in Hand-in assignment



## Approximate car data

The energy need for the MVEG-95 cycle per 100 km.

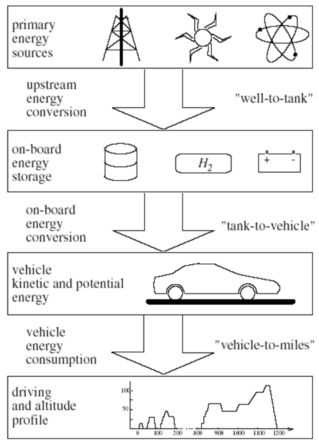
$$\bar{E}_{\text{MVEG-95}} \approx A_f\, c_d\, 1.9 \cdot 10^4 + m_v\, c_r\, 8.4 \cdot 10^2 + m_v\, 10 \qquad kJ/100km$$

| | SUV | full-size | compact | light-weight | PAC-Car II |
|---|---|---|---|---|---|
| $A_f \cdot c_d$ | 1.2 m$^2$ | 0.7 m$^2$ | 0.6 m$^2$ | 0.4 m$^2$ | .25 · .07 m$^2$ |
| $c_r$ | 0.017 | 0.017 | 0.017 | 0.017 | 0.0008 |
| $m_v$ | 2000 kg | 1500 kg | 1000 kg | 750 kg | 39 kg |
| $\bar{P}_{\text{MVEG-95}}$ | 11.3 kW | 7.1 kW | 5.0 kW | 3.2 kW | |
| $\bar{P}_{\text{max}}$ | 155 kW | 115 kW | 77 kW | 57 kW | |

Average and maximum power requirement for the cycle.



## Energy System Overview




Primary sources

Different options for on-board energy storage

Powertrain energy conversion during driving

Cut at the wheel!

Driving mission has a minimum energy requirement.

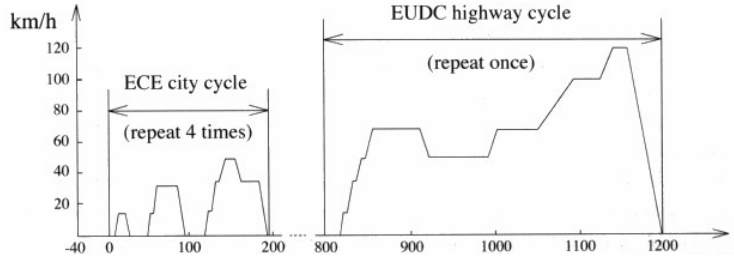
## Outline





## Energy demand again – Recuperation

- Previously: Considered energy demand from the cycle.
- Now: The cycle can give energy to the vehicle.




Recover the vehicle's kinetic energy during driving.



## Perfect recuperation

- Mean required force

$$\bar{F} = \bar{F}_a + \bar{F}_r$$

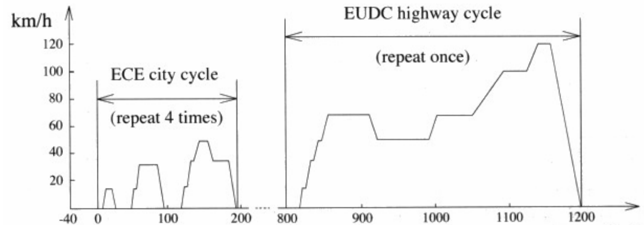
- Sum over all points

$$\bar{F}_a = \frac{1}{x_{tot}} \frac{1}{2} \rho_a A_f c_d \sum_{i=1}^{N} \bar{v}_i^3 h$$

$$\bar{F}_r = \frac{1}{x_{tot}} m_v g c_r \sum_{i=1}^{N} \bar{v}_i h$$



## Perfect recuperation – Numerical values for cycles




Numerical values for MVEG-95, ECE, EUDC

$$\bar{X}_a = \frac{1}{x_{tot}} \sum_i \bar{v}_i^3 h = \qquad \{363, 100, 515\}$$

$$\bar{X}_r = \frac{1}{x_{tot}} \sum_i \bar{v}_i h = \qquad \{1, 1, 1\}$$

$$\bar{E}_{\text{MVEG-95}} \approx A_f c_d 2.2 \cdot 10^4 + m_v c_r 9.81 \cdot 10^2 \qquad kJ/100km$$

## Comparison of numerical values for cycles

- Without recuperation.

$$\bar{X}_{trac,a} = \frac{1}{x_{tot}} \sum_{i \in trac} \bar{v}_i^3 \, h = \qquad \{319, 82.9, 455\}$$

$$\bar{X}_{trac,r} = \frac{1}{x_{tot}} \sum_{i \in trac} \bar{v}_i \, h = \qquad \{0.856, 0.81, 0.88\}$$

$$\bar{X}_{trac,m} = \frac{1}{x_{tot}} \sum_{i \in trac} \bar{a}_i \, \bar{v}_i \, h = \qquad \{0.101, 0.126, 0.086\}$$

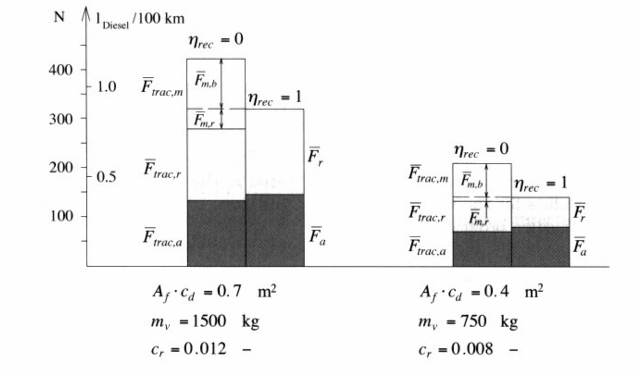
- With perfect recuperation

$$\bar{X}_a = \frac{1}{x_{tot}} \sum_i \bar{v}_i^3 \, h = \qquad \{363, 100, 515\}$$

$$\bar{X}_r = \frac{1}{x_{tot}} \sum_i \bar{v}_i \, h = \qquad \{1, 1, 1\}$$



## Perfect and no recuperation




Mean force represented as liter Diesel / 100 km.



## Sensitivity Analysis – Design changes

- Cycle energy reqirement (no recuperation)

$$\bar{E}_{\text{MVEG-95}} \approx A_f \, c_d \, 1.9 \cdot 10^4 + m_v \, c_r \, 8.4 \cdot 10^2 + m_v \, 10 \qquad kJ/100km$$

- Sensitivity analysis

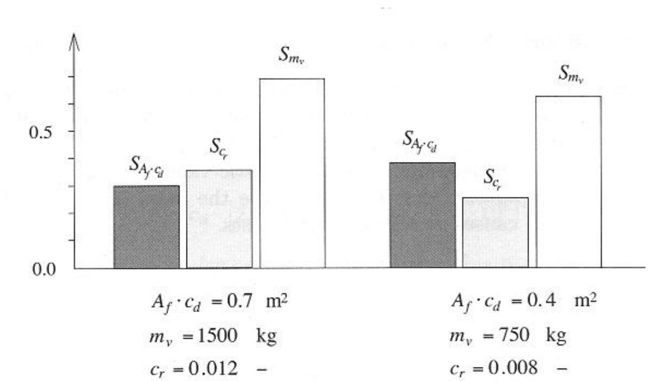
$$S_p = \lim_{\delta p \to 0} \frac{\left[\bar{E}_{\text{MVEG-95}}(p + \delta p) - \bar{E}_{\text{MVEG-95}}(p)\right] / \bar{E}_{\text{MVEG-95}}(p)}{\delta p / p}$$

$$S_p = \lim_{\delta p \to 0} \frac{\left[\bar{E}_{\text{MVEG-95}}(p + \delta p) - \bar{E}_{\text{MVEG-95}}(p)\right]}{\delta p} \frac{p}{\bar{E}_{\text{MVEG-95}}(p)}$$

- Consider the vehicle design parameters:
  - $A_f \, c_d$
  - $c_r$
  - $m_v$

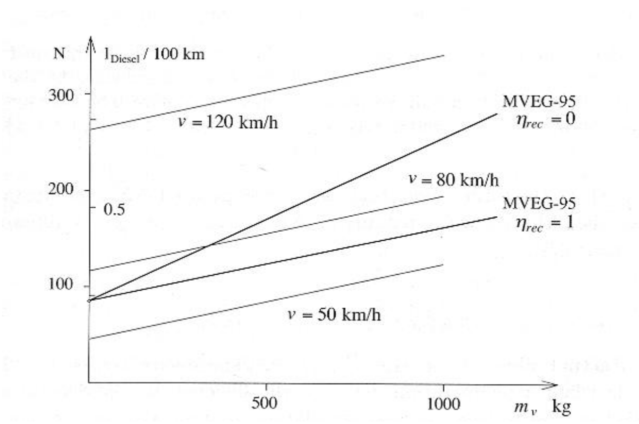


## Sensitivity Analysis



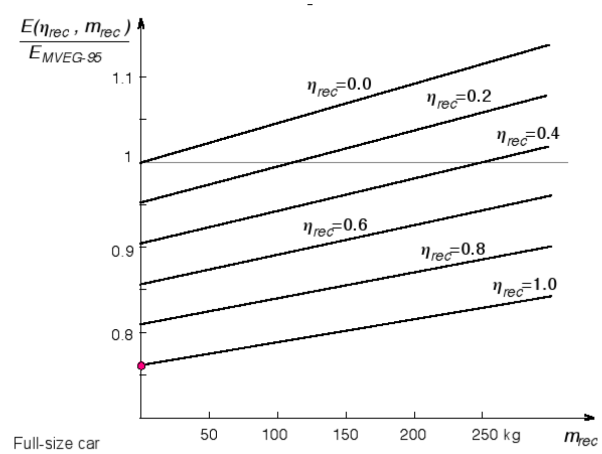

Vehicle mass is the most important parameter.

## Vehicle mass and fuel consumption






## Realistic Recuperation Devices



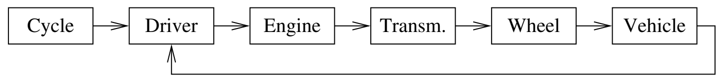



## Outline





## Dynamic approach




- Forward simulation.
- Drivers input $u$ propagates to the vehicle and the cycle
- Drivers input ⇒ . . . ⇒ Driving force ⇒ Losses ⇒ Vehicle velocity ⇒ Feedback to driver model
- Available tools (= Standard simulation) can deal with arbitrary powertrain complexity.

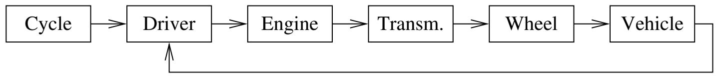
## Quasistatic approach


- Backward simulation
- Driving cycle ⇒ Losses ⇒ Driving force ⇒ Wheel torque ⇒ Engine (powertrain) torque ⇒ . . . ⇒ Fuel consumtion.
- Available tools are limited with respect to the powertrain components that they can handle.
  – The models need to be prepared for inverse simulation.
- Considering new acausal tools such as Modelica opens up possibilities.
- See also: *Efficient Drive Cycle Simulation*, Anders Fröberg and Lars Nielsen (2008) . . .



## Two Approaches for Powertrain Simulation

- Dynamic simulation (forward simulation)




–"Normal" system modeling direction
–Requires driver model

- Quasistatic simulation (inverse simulation)

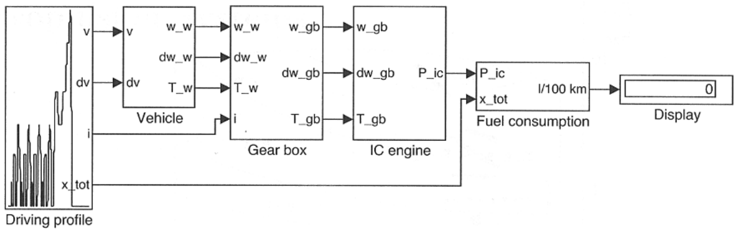

–"Reverse" system modeling direction
–Follows driving cycle exactly

- Model (or calculation) causality



## QSS Toolbox – Quasistatic Approach

- IC Engine Based Powertrain




- The Vehicle Motion Equation – With inertial forces:
  $$\left[m_v + \frac{1}{r_w^2}J_w + \frac{\gamma^2}{r_w^2}J_e\right]\frac{d}{dt}v(t) = \frac{\gamma}{r_w}T_e - (F_a(t) + F_r(t) + F_g(t) + F_d(t))$$
- Gives efficient simulation of vehicles in driving cycles



## Outline

## Causality and Basic Equations

High level modeling – Inputs and outputs

- Causalities for Engine Models


- Engine efficiency

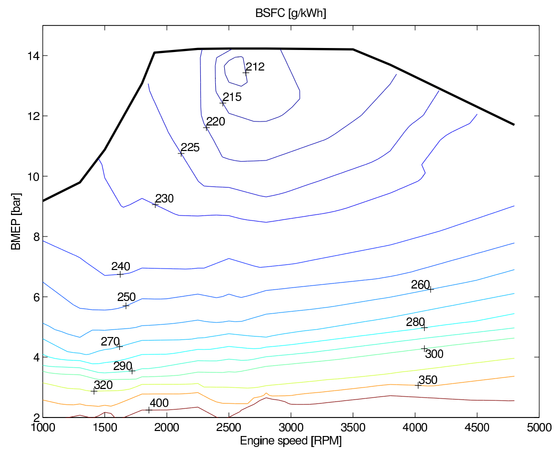
$$\eta_e = \frac{\omega_e T_e}{P_c}$$

- Enthalpy flow of fuel (Power $\dot{H}_{fuel} = P_c$)

$$P_c = \dot{m}_f \, q_{LHV}$$



## Engine Efficiency Maps




Measured engine efficiency map.

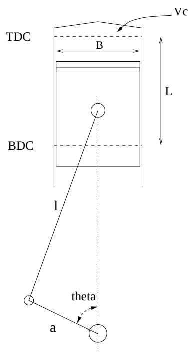
Used very often for fuel consumption assessment.

The engineering perspective, design/evaluation.

–What to do when map-data isn't available?



## Engine Geometry Definitions




Cylinder, Piston, Connecting rod, Crank shaft

- Bore, $B$
- Stroke, $S = 2\,a$
- Number of cylinders $z$
- Cylinder swept volume, $V_d = \frac{\pi B^2 S}{4}$
- Engine swept volume, $V_D = z \frac{\pi B^2 S}{4}$
- Compression ratio $r_c = \frac{V_{max}}{V_{min}} = \frac{V_d + V_c}{V_c}$



## Definition of MEP

- Mean Effective Pressure (MEP) $= \frac{\text{Work}}{\text{Displacement Volume}} = \frac{4\,\pi\,T_e}{V_D}$
- MEP – normalizes the work output with the size of the engine.
- The engineering perspective.
  If we can build a good model in the MEP domain,
  then we can scale it with $V_D$ and get a generic engine model,
  with which we can evaluate the design impact of different engine sizes.
  Opens up possibilities for selecting engines for optimal fuel economy for a vehicle, etc.

## Normalized Engine Variables

- Mean Piston Speed ($S_p = mps = c_m$):

$$c_m = \frac{\omega_e \, S}{\pi}$$

- Mean Effective Pressure (MEP=$p_{me}$ ($N = n_r \cdot 2$)):

$$p_{me} = \frac{N \, \pi \, T_e}{V_d}$$

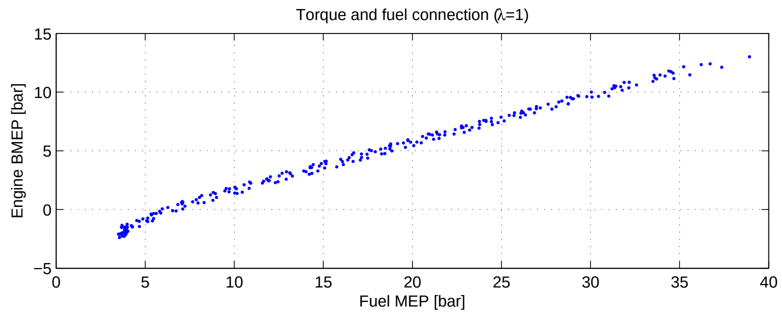
- Used to:
  - Compare performance for engines of different size
  - Design rules for engine sizing.
    At max engine power: $c_m \approx 17$ m/s, $p_{me} \approx$ 1e6 Pa (no turbo)
    $\Rightarrow$ engine size
  - Connection:

$$P_e = z \, \frac{\pi}{16} \, B^2 \, p_{me} \, c_m$$



## Torque modeling through – Willans Line

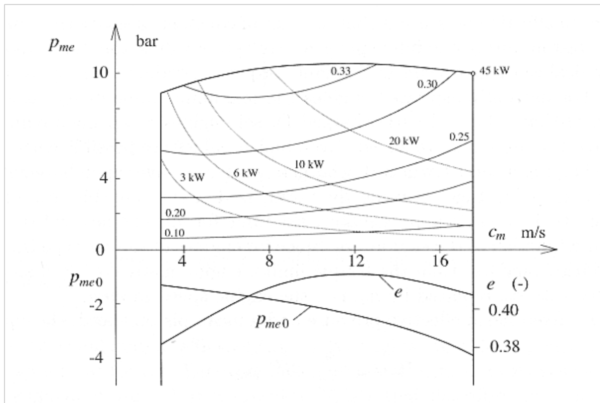
- Measurement data: x: $p_{mf}$ y: $p_{me} = BMEP$




- Linear (affine) relationship – Willans line $p_{me} = e(\omega_e) \cdot p_{mf} - p_{me,0}(\omega_e)$
- Engine efficiency: $\eta_e = \frac{p_{me}}{p_{mf}}$



## Engine Efficiency – Map Representation




### Willans line parameters

Engine speed dependent

- $e(\omega_e)$
- $p_{me,0}(\omega_e)$