

# Vehicle Propulsion Systems

## Lecture 11

### Case Study 6 Fuel Cell Vehicle and Optimal Control

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## Outline

### Repetition

Case study 6: Fuel Optimal Trajectories of a Racing FCEV  
Model compilation

### Model simplification

Formulating the optimal control problem

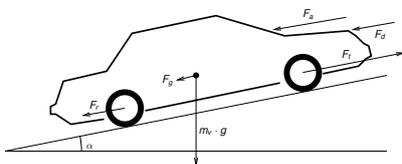
### Optimal Controllers Solutions

Some Additional Material – Fuel Consumption

## The Vehicle Motion Equation

Newtons second law for a vehicle

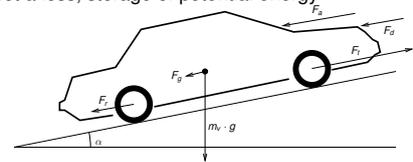
$$m_v \frac{d}{dt} v(t) = F_t(t) - (F_a(t) + F_r(t) + F_g(t) + F_d(t))$$



- ▶  $F_t$  – tractive force
- ▶  $F_a$  – aerodynamic drag force
- ▶  $F_r$  – rolling resistance force
- ▶  $F_g$  – gravitational force
- ▶  $F_d$  – disturbance force

## Gravitational Force

- ▶ Gravitational load force  
–Not a loss, storage of potential energy



- ▶ Up- and down-hill driving produces forces.

$$F_g = m_v g \sin(\alpha)$$

- ▶ Flat road assumed  $\alpha = 0$  if nothing else is stated (In the book).

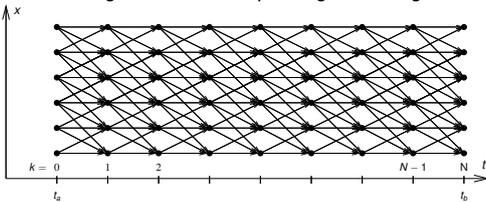
## Deterministic Dynamic Programming – Basic algorithm

$$J(x_0) = g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k)$$

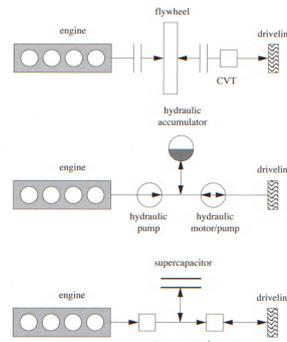
$$x_{k+1} = f_k(x_k, u_k)$$

Algorithm idea:

Start at the end and proceed backward in time to evaluate the optimal cost-to-go and the corresponding control signal

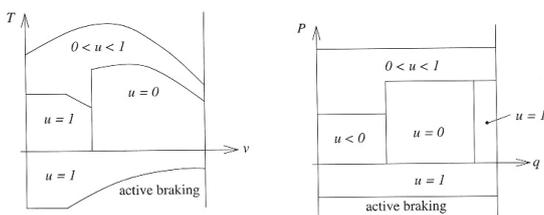


## Examples of Short Term Storage Systems



## Heuristic Control Approaches

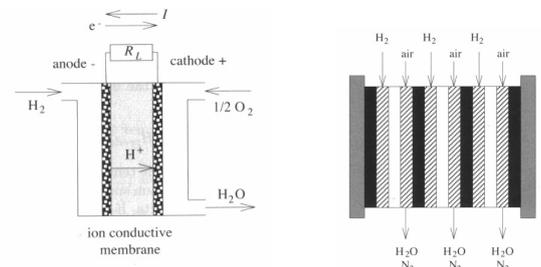
- ▶ Parallel hybrid vehicle (electric assist)



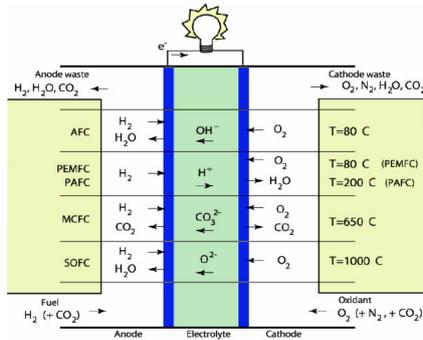
- ▶ Determine control output as function of some selected state variables:  
vehicle speed, engine speed, state of charge, power demand, motor speed, temperature, vehicle acceleration, torque demand

## Fuel Cell Basic Principles

- ▶ Convert fuel directly to electrical energy
- ▶ Let an ion pass from an anode to a cathode
- ▶ Take out electrical work from the electrons
- ▶ Fuel cells are stacked ( $U_{cell} \leq 1V$ )



## Overview of Different Fuel Cell Technologies

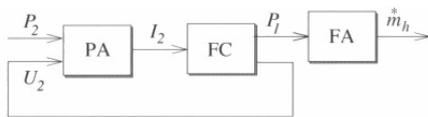


## Hydrogen Fuel Storage

- ▶ Hydrogen storage is problematic – Challenging task.
- ▶ Some examples of different options.
  - ▶ High pressure bottles
  - ▶ Liquid phase – Cryogenic storage,  $-253^\circ\text{C}$ .
  - ▶ Metal hydride
  - ▶ Sodium borohydride  $NaBH_4$

## Quasistatic Modeling of a Fuel Cell

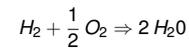
- ▶ Causality diagram



- ▶ Power amplifier (Current controller)
- ▶ Fuel amplifier (Fuel controller)
- ▶ Standard modeling approach

## Fuel Cell Thermodynamics

- ▶ Starting point reaction equation



- ▶ Open system energy – Enthalpy H

$$H = U + pV$$

- ▶ Reversible energy – Gibbs free energy G

$$G = H + TS$$

- ▶ Open circuit cell voltages

$$U_{rev} = -\frac{\Delta G}{n_e F}, \quad U_{id} = -\frac{\Delta H}{n_e F}, \quad U_{rev} = \eta_{id} U_{id}$$

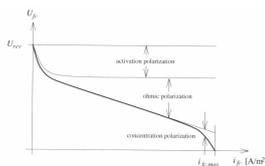
$F$  – Faradays constant ( $F = q N_0$ )

- ▶ Under load

$$P_l = I_{fc}(t) (U_{id} - U_{fc}(t))$$

## Fuel Cell Performance – Polarization curve

- ▶ Polarization curve of a fuel cell  
Relating current density  $i_{fc}(t) = I_{fc}(t)/A_{fc}$ , and cell voltage  $U_{fc}(t)$



Curve for one operating condition

- ▶ Fundamentally different compared to combustion engine/electrical motor
- ▶ Excellent part load behavior  
–When considering only the cell

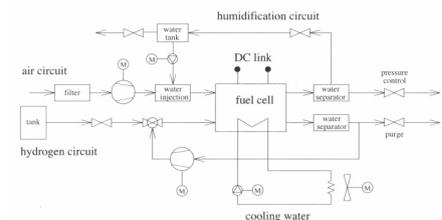
## Fuel Cell System Modeling

- ▶ Describe all subsystems with models

$$P_2(t) = P_{st}(t) - P_{aux}(t)$$

$$P_{aux} = P_0 + P_{em}(t) + P_{ahp}(t) + P_{hp}(t) + P_{cl}(t) + P_{cf}(t)$$

em–electric motor, ahp – humidifier pump, hp – hydrogen recirculation pump, cl – coolant pump, cf – cooling fan.



- ▶ Submodels for:  
Hydrogen circuit, air circuit, water circuit, and coolant circuit

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Optimal Controllers Solutions

Some Additional Material – Fuel Consumption

## Problem Setup

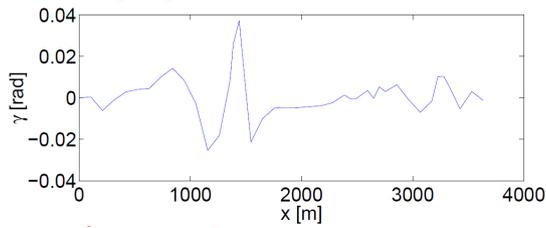
- ▶ Run a fuel cell vehicle optimally on a racetrack



- ▶ Start up lap
- ▶ Repeated runs on the track
- ▶ Path to the solution
  - ▶ Measurements – Model
  - ▶ Simplified model
  - ▶ Optimal control solutions

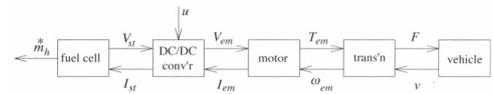
## Problem Setup – Road Slope Given

Road slope  $\gamma = \alpha(x)$

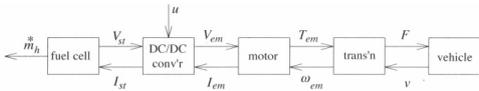


## Model Causality

Model causality – Dynamic model



## Model Component – Fuel Cell



- ▶ Current in the cell and losses

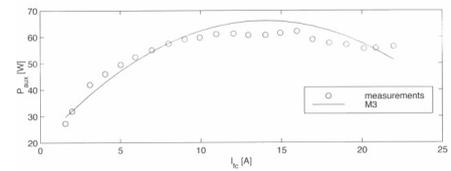
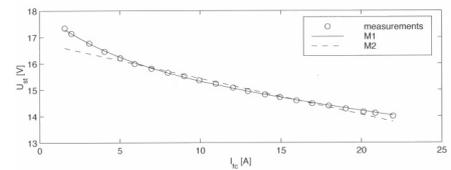
$$I_{fc}(t) = I_{fc}(t) + I_{aux}(t)$$

- ▶ Current and hydrogen flow

$$\dot{m}_H(t) = c_9 I_{fc}(t)$$

- ▶ Next step: Polarization curve and auxiliary consumption

## Fuel Cell – Polarization and Auxiliary Components



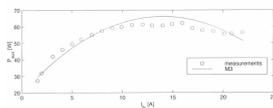
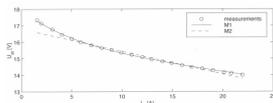
## Fuel Cell – Polarization and Auxiliary Components

- ▶ Polarization curve

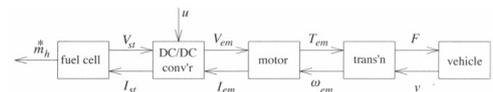
$$U_{st}(t) = c_0 + c_1 \cdot e^{-c_2 I_{fc}(t)} - c_3 \cdot I_{fc}(t)$$

- ▶ Auxiliary power

$$P_{aux}(t) = c_6 + c_7 \cdot I_{fc}(t) + c_8 \cdot I_{fc}^2(t)$$



## Model Component – DC Motor Controller



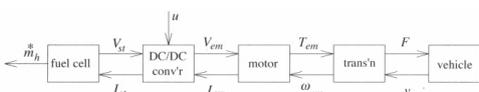
- ▶ DC motor voltage (from control signal  $u$ )

$$U_{em}(t) = \kappa \omega_{em}(t) + K R_{em} u(t)$$

- ▶ Current requirement at the stack

$$I_{st} = \frac{U_{em}(t) I_{em}(t)}{\eta_c U_{st}(t)}$$

## Model Component – DC Motor



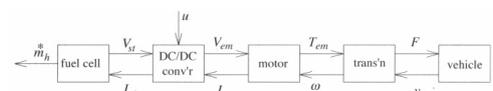
- ▶ DC motor current

$$I_{em}(t) = \frac{U_{em}(t) - \kappa \omega_{em}(t)}{R_{em}}$$

- ▶ DC motor torque

$$T_{em}(t) = \kappa_{em} I_{em}(t)$$

## Model Component – Transmission and Wheels



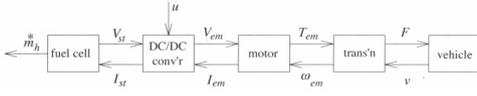
- ▶ Tractive force

$$F(t) = \eta_t^{\pm 1} \frac{\gamma T_{em}(t)}{r_w}$$

- ▶ Rotational speed

$$\omega_{em}(t) = \frac{\gamma v(t)}{r_w}$$

## Model Compilation 1 – Vehicle



- ▶ The vehicle tractive force can now be expressed as

$$F(t) = \frac{\eta_t \gamma}{r_w} \kappa_{em} K u(t)$$

- ▶ Dynamic vehicle velocity and position model

$$\frac{d}{dt} v(t) = h_1 u(t) - h_2 v^2(t) - g_0 - g_1 \alpha(x(t))$$

$$\frac{d}{dt} x(t) = v(t)$$

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## Detour

- ▶ Occam's razor: The explanation of any phenomenon should make as few assumptions as possible. Shave of those who are unnecessary.
- ▶ Law of Parsimony: Among others a factor in statistics: In general, mathematical models with the smallest number of parameters are preferred as each parameter introduced into the model adds some uncertainty to it.
- ▶ Another viewpoint. Try to simplify the problem you solve as much as possible.  
–Neglect effects and be proud when you are successful!

## Optimal Control Problems

- ▶ Start of the cycle

$$v(0) = 0, \quad x(0) = 0$$

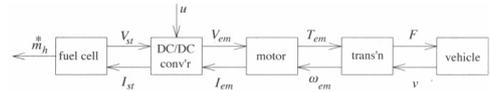
$$\lambda_1(t_f) = 0, \quad x(t_f) = x_f = v_m t_f$$

- ▶ Periodic route

$$x(0) = 0$$

$$\lambda_1(t_f) = \lambda_1(0), \quad x(t_f) = x_f = v_m t_f, \quad v(t_f) = v(0)$$

## Model Compilation 2 – Fuel Consumption



- ▶ Fuel flow,  $\dot{m}_H(t) = c_3 I_{fc}(t)$

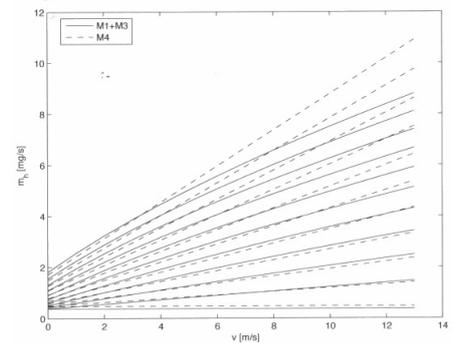
$$I_{fc}(t) = \frac{P_{aux}(I_{st}(t))}{U_{st}(I_{st}(t))} + \frac{K u(t)}{\eta_c U_{st}(I_{st}(t))} \left( K R_{em} u(t) + \kappa_{em} \frac{\gamma}{r_w} v(t) \right)$$

–Implicit nonlinear static function

- ▶ Simpler model

$$\dot{m}_H(t) = b_0 + b_1 v(t) u(t) + b_2 u^2(t)$$

## Simplified Fuel Consumption – Validation



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## PID Cruise Controller – Baseline for Comparison

Simple controller for the start

$$u(t) = K_p (f v_m - v(t)) + K_i \int_0^t (f v_m - v_i(t)) dt$$

$f$ -tuning parameter  $\approx 1$  to allow for matching the average speed

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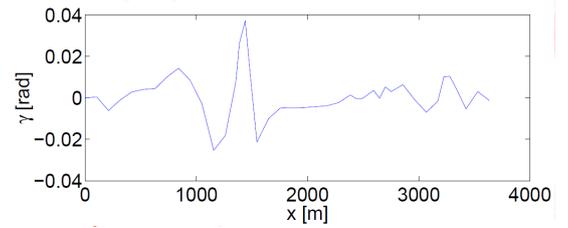
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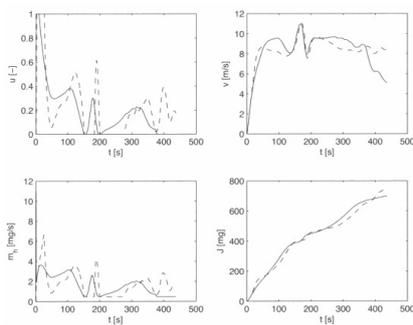
## Problem Setup – Road Slope Given

Road slope  $\gamma = \alpha(x)$



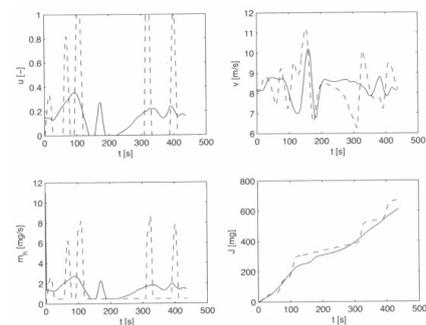
## Fuel Optimal Trajectory – Start

Fuel optimal trajectory has 7% lower fuel consumption



## Fuel Optimal Trajectory – Continuous Driving

Fuel optimal trajectory has 9% lower fuel consumption



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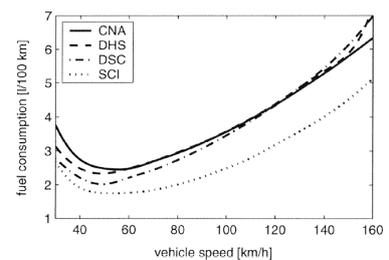
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## Fuel Optimal Speed for Normal Driving

ICE vehicle (light weight 800 kg)



## Engine Map an Gearbox Layout

CI engine (light weight 800 kg)

