Vehicle Propulsion Systems
Lecture 7
Non Electric Hybrid Propulsion Systems

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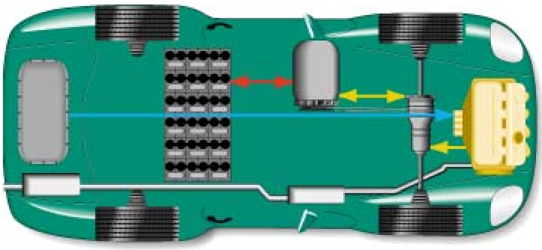
May 6, 2018

Outline

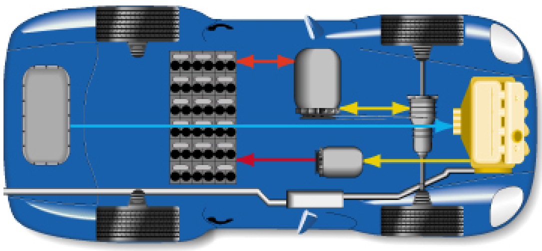
Hybrid Electrical Vehicles – Parallel

- Two parallel energy paths
- One state in QSS framework, state of charge



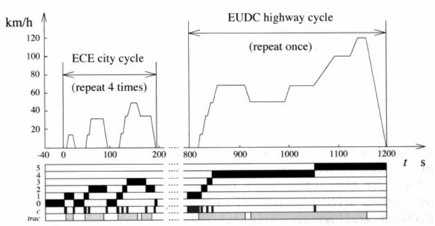
Hybrid Electrical Vehicles – Serial

- Two paths working in parallel
- Decoupled through the battery
- Two states in QSS framework, state of charge & Engine speed



Optimization, Optimal Control, Dynamic Programming

What gear ratios give the lowest fuel consumption for a given drivingcycle?
–Problem presented in appendix 8.1



Problem characteristics
- Countable number of free variables, $i_{g,j}$, $j \in [1,5]$
- A "computable" cost, $m_f(\cdots)$
- A "computable" set of constraints, model and cycle
- The formulated problem

$$\min_{i_{g,j},\, j\in[1,5]} \quad m_f(i_{g,1}, i_{g,2}, i_{g,3}, i_{g,4}, i_{g,5})$$
$$\text{s.t.} \quad \text{model and cycle is fulfilled}$$

Optimal Control – Problem Motivation

Car with gas pedal $u(t)$ as control input:
How to drive from A to B on a given time with minimum fuel consumption?

- Infinite dimensional decision variable $u(t)$.
- Cost function $\int_0^{t_f} \dot m_f(t)dt$
- Constraints:
  - Model of the car (the vehicle motion equation)

$$\begin{aligned} m_v \tfrac{d}{dt}v(t) &= F_t(v(t),u(t)) \quad -(F_a(v(t)) + F_r(v(t)) + F_g(x(t))) \\ \tfrac{d}{dt}x(t) &= v(t) \\ \dot m_f &= f(v(t),u(t)) \end{aligned}$$

  - Starting point $x(0) = A$
  - End point $x(t_f) = B$
  - Speed limits $v(t) \le g(x(t)$
  - Limited control action $0 \le u(t) \le 1$

General problem formulation

- Performance index

$$J(u) = \phi(x(t_b), t_b) + \int_{t_a}^{t_b} L(x(t), u(t), t)dt$$

- System model (constraints)

$$\frac{d}{dt}x = f(x(t), u(t), t), x(t_a) = x_a$$

- State and control constraints

$$u(t) \in U(t)$$

$$x(t) \in X(t)$$

Dynamic programming – Problem Formulation

- Optimal control problem

$$\min J(u) = \phi(x(t_b), t_b) + \int_{t_a}^{t_b} L(x(t), u(t), t)dt$$
$$\begin{aligned} s.t.\ \ \frac{d}{dt}x &= f(x(t), u(t), t) \\ x(t_a) &= x_a \\ u(t) &\in U(t) \\ x(t) &\in X(t) \end{aligned}$$

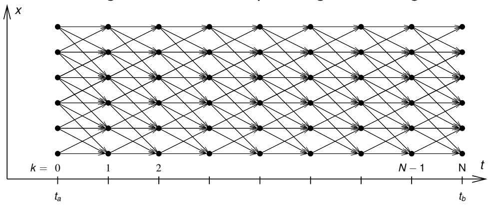
- $x(t)$, $u(t)$ functions on $t \in [t_a, t_b]$
- Search an approximation to the solution by discretizing
  - the state space $x(t)$
  - and maybe the control signal $u(t)$

  in both amplitude and time.
- The result is a combinatorial (network) problem

Deterministic Dynamic Programming – Basic algorithm

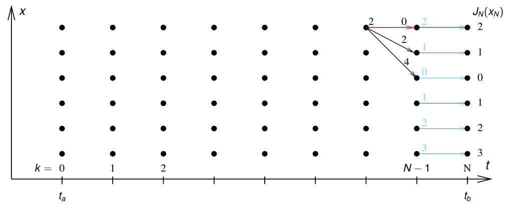
$$J(x_0) = g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k)$$
$$x_{k+1} = f_k(x_k, u_k)$$

Algorithm idea:
Start at the end and proceed backwards in time to evaluate the optimal cost-to-go and the corresponding control signal.



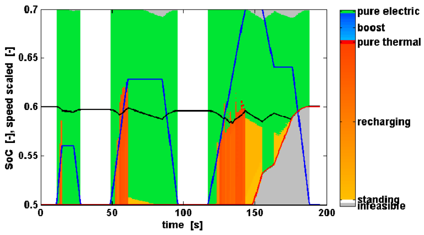
Deterministic Dynamic Programming – Basic Algorithm

Graphical illustration of the solution procedure



Parallel Hybrid Example

- Fuel-optimal torque split factor $u(SOC, t) = \frac{T_{e-motor}}{T_{gearbox}}$
- ECE cycle
- Constraints $SOC(t = t_f) \geq 0.6$, $SOC \in [0.5, 0.7]$



Analytical Solutions to Optimal Control Problems

- Core of the problem

$$\min J(u) = \phi(q(t_f), t_f) + \int_0^{t_f} L(t, u(t))dt$$
$$s.t.\ \dot{q}(t) = f(t, q(t), u(t))$$

- Hamiltonian from optimal control theory

$$H(t, q(t), u(t), \mu(t)) = L(t, u(t)) + \mu(t)\, f(t, q(t), u(t))$$

Analytical Solutions to Optimal Control Problems

- Hamiltonian

$$H(t, q(t), u(t), \mu(t)) = L(t, u(t)) + \mu(t)\, f(t, q(t), u(t))$$

- Solution (theory from Appendix B)

$$u(t) = \arg\min_u H(t, q(t), u(t), \mu(t))$$

with

$$\dot{\mu}(t) = -\frac{\partial}{\partial q} f(t, q(t), u(t))$$
$$\dot{q}(t) = f(t, q(t), u(t))$$

- If $\frac{\partial}{\partial q} f(t, q(t), u(t)) = 0$ the problem becomes simpler μ becomes a constant μ_0, search for it when solving

ECMS

- Given the optimal λ^* (cycle dependent exchange rate between fuel and electricity) .
- Hamiltonian

$$H(t, q(t), u(t), \lambda^*) = P_f(t, u(t)) + \lambda^* P_{ech}(t, u(t))$$

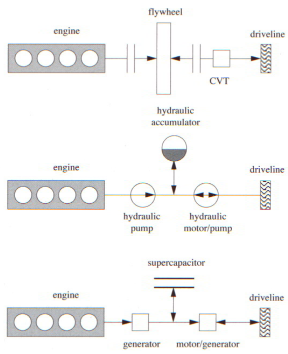
- Optimal control action

$$u^*(t) = \arg\min_u H(t, q(t), u, \lambda^*)$$

- Guess λ^*, run one cycle see end SOC, update λ^*, and iterate until $SOC(t_f) \approx SOC(0)$.

Outline

Examples of Short Term Storage Systems



Short Term Storage – F1

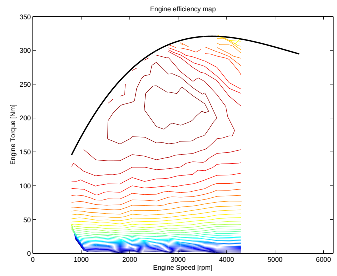
2009 FIA allowed the usage of 60 kW, KERS (Kinetic Energy Recovery System) in F1.
Technologies:

- Flywheel
- Super-Caps, Ultra-Caps
- Batteries

2014, will allow KERS units with 120 kilowatts (160 bhp).
–To balance the sport's move from 2.4 l V8 engines to 1.6 l V6 engines.

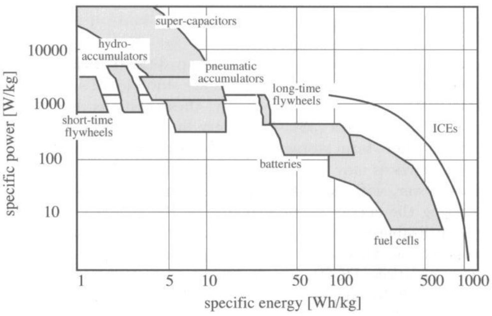
Basic Principles for Hybrid Systems

- Kinetic energy recovery
- Use "best" points – Duty cycle.
  - Run engine (fuel converter) at its optimal point.
  - Shut-off the engine.



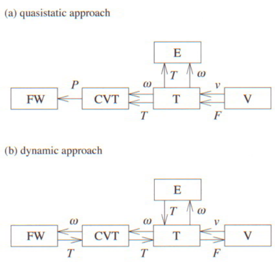
Power and Energy Densities

Asymptotic power and energy density – The Principle

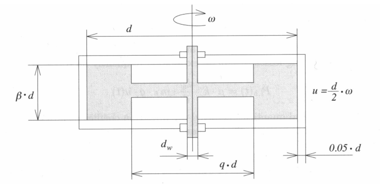


Outline

Causality for a hybrid-inertial propulsion system



Flywheel accumulator



- Energy stored ($\Theta_f = J_f$):

$$E_f = \frac{1}{2}\,\Theta_f\,\omega_f^2$$

- Wheel inertia

$$\Theta_f = \rho\,b \int_{Area} r^2\,2\,\pi\,r\,dr = \ldots = \frac{\pi}{2}\,\rho\,b\,\frac{d^4}{16}\,(1-q^4)$$

Flywheel accumulator – Design principle

- Energy stored (SOC):

$$E_f = \frac{1}{2}\,\Theta_f\,\omega_f^2$$

- Wheel inertia

$$\Theta_f = \rho\,b \int_{Area} r^2\,2\,\pi\,r\,dr = \ldots = \frac{\pi}{2}\,\rho\,b\,\frac{d^4}{16}\,(1-q^4)$$

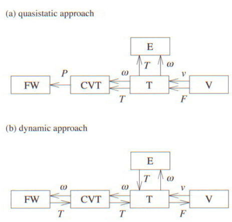
- Wheel Mass

$$m_f = \pi\,\rho\,b\,d^2\,(1-q^2)$$

- Energy to mass ratio

$$\frac{E_f}{m_f} = \frac{d^2}{16}(1+q^2)\omega_f^2 = \frac{u^2}{4}\,(1+q^2)$$

Quasistatic Modeling of FW Accumulators

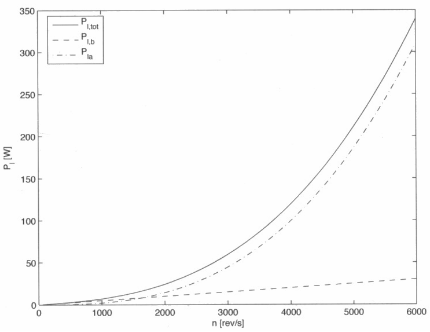


Flywheel speed (SOC) $\quad P_2(t)$ – power out, $P_l(t)$ – power loss

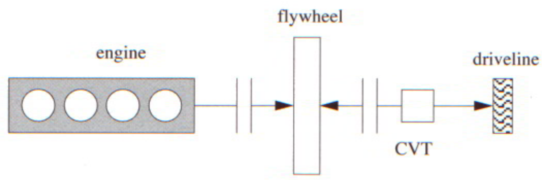
$$\Theta_f\,\omega_2(t)\,\frac{d}{dt}\omega_2(t) = -P_2(t) - P_l(t)$$

Power losses as a function of speed

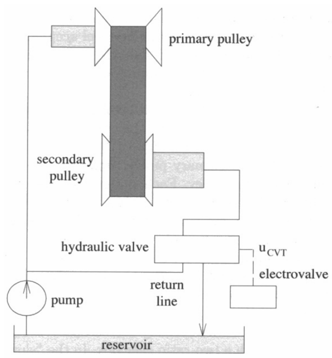
Air resistance and bearing losses



Continuously Variable Transmission (CVT)



CVT Principle



CVT Modeling

- Transmission (gear) ratio ν, speeds and transmitted torques

$$\omega_1(t) = \nu(t)\,\omega_2(t)$$
$$T_{t1}(t) = \nu\,(T_{t2}(t) - T_l(t))$$

- Newtons second law for the two pulleys

$$\Theta_1\,\frac{d}{dt}\omega_1(t) = T_1(t) - T_{t1}(t)$$
$$\Theta_2\,\frac{d}{dt}\omega_2(t) = T_2(t) - T_{t2}(t)$$

- System of equations give

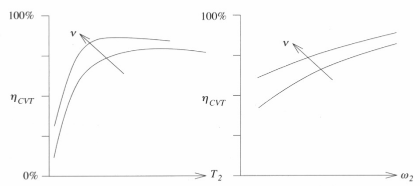
$$T_1(t) = T_l(t) + \frac{T_2(t)}{\nu(t)} + \frac{\Theta_{CVT}(t)}{\nu(t)}\frac{d}{dt}\omega_2(t) + \Theta_1\,\frac{d}{dt}\nu(t)\,\omega_2(t)$$

CVT Modeling

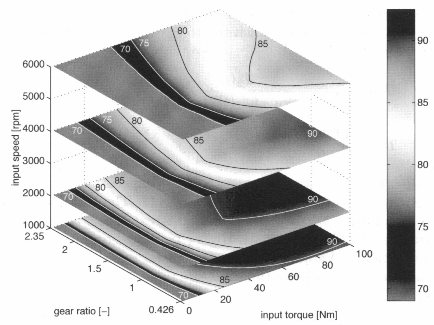
- Transmission (gear) ratio ν, speeds and transmitted torques

$$\omega_1(t) = \nu(t)\,\omega_2(t)$$
$$T_{t1}(t) = \nu\,(T_{t2}(t) - T_l(t))$$

- An alternative to model the losses, is to use an efficiency definition.



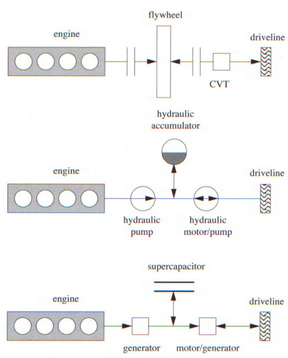
Efficiencies for a Push-Belt CVT



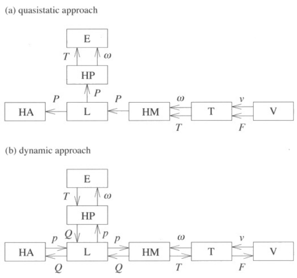
Outline

- Repetition
- Short Term Storage
- Hybrid-Inertial Propulsion Systems
  - Basic principles
  - Design principles
  - Modeling
  - Continuously Variable Transmission
- Hybrid-Hydraulic Propulsion Systems
  - Basics
  - Modeling
- Hydraulic Pumps and Motors
- Pneumatic Hybrid Engine Systems
- Case studies

Examples of Short Term Storage Systems



Causality for a hybrid-hydraulic propulsion system



Modeling of a Hydraulic Accumulator

Modeling principle
–Energy balance

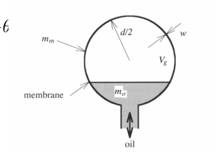
$$m_g\, c_v \frac{d}{dt}\theta_g(t) = -p\frac{d}{dt}V_g(t) - h\, A_w\,(\theta_g(t) - \theta$$

–Mass balance
(=volume for incompressible fluid)

$$\frac{d}{dt}V_g(t) = Q_2(t)$$

–Ideal gas law

$$p_g(t) = \frac{m_g\, R_g\, \theta_g(t)}{V_g(t)}$$



Power generation

$$P_2(t) = p_2(t)\, Q_2(t)$$

Model Simplification

Simplifications made in thermodynamic equations to get a simple state equation.

- Assuming steady state conditions.
  –Eliminating θ_g and the volume change gives

$$p_2(t) = \frac{h\, A_w\, \theta_w\, m_g\, R_g}{V_g(t)\, h\, A_w + m_g\, R_g\, Q_2(t)}$$

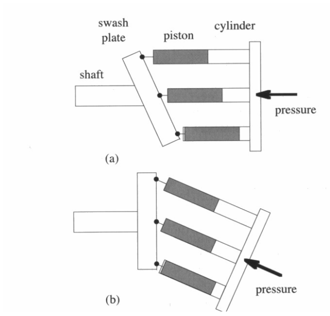
- Combining this with the power output gives

$$Q_2(t) = \frac{V_g(t)}{m_g}\, \frac{h\, A_w\, P_2(t)}{R_g\, \theta_w\, h\, A_w - R_g\, P_2(t)}$$

- Integrating $Q_2(t)$ gives V_g as the state in the model.
- Modeling of the hydraulic systems efficiency, see the book.
- A detail for the assignment
  –This simplification can give problems in the simulation if parameter values are off. (Division by zero.)

Outline

Hydraulic Pumps



Modeling of Hydraulic Motors

- Efficiency modeling

$$P_1(t) = \frac{P_2(t)}{\eta_{hm}(\omega_2(t), T_2(t))}, \qquad P_2(t) > 0$$
$$P_1(t) = P_2(t)\, \eta_{hm}(\omega_2(t), -|T_2|(t)), \qquad P_2(t) < 0$$

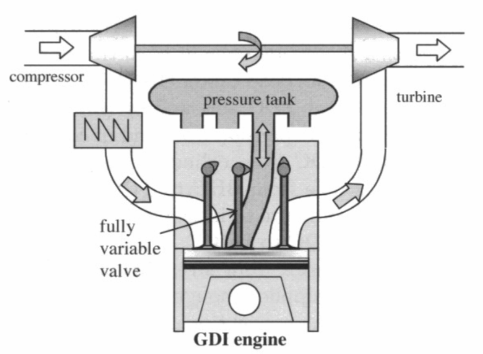
- Willans line modeling, describing the loss

$$P_1(t) = \frac{P_2(t) + P_0}{e}$$

- Physical modeling
  Wilson's approach provided in the book.

Outline

Pneumatic Hybrid Engine System

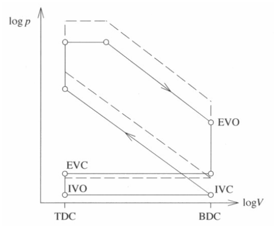


Conventional SI Engine

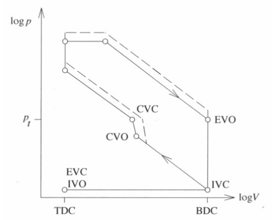
Compression and expansion model

$$p(t) = c\,v(t)^{-\gamma} \quad \Rightarrow \quad \log(p(t)) = \log(c) - \gamma \log(v(t))$$

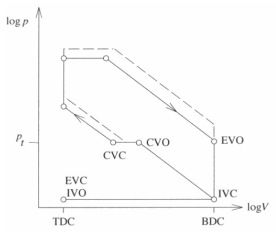
gives lines in the log-log diagram version of the pV-diagram



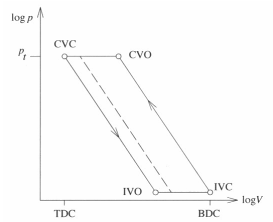
Super Charged Mode



Under Charged Mode



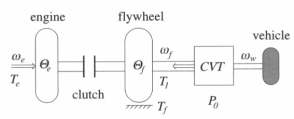
Pneumatic Brake System



Outline

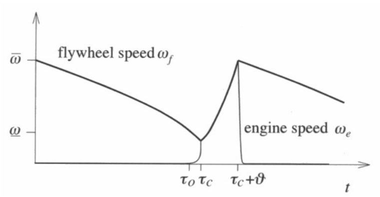
Case Study 3: ICE and Flywheel Powertrain

- Control of a ICE and Flywheel Powertrain
- Switching on and off engine



Problem description

For each constant vehicle speed find the optimal limits for starting and stopping the engine
–Minimize fuel consumption



–Solved through parameter optimization ⇒ Map used for control

Case Study 8: Hybrid Pneumatic Engine

- Local optimization of the engine thermodynamic cycle
- Different modes to select between
- Dynamic programming of the mode selection