Exam with solutions

TSFS06 Diagnosis and Supervision
June 5, 2020, kl. 14.00-19.00
(Note! 5 hours)

Responsible teacher: Erik Frisk

Solution language: Swedish or English

Total 40 points.

Preliminary grade limits

Grade 3: 18 points
Grade 4: 25 points
Grade 5: 30 points

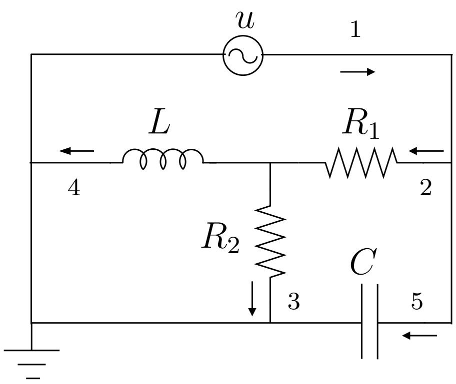
Information about this exam, read this carefully before starting with the tasks.

1. Submission of solutions is done in Lisam.
2. Note that the exam is one hour longer than normal, i.e., **deadline for submission is 19:00**. In case of technical difficulties when submitting, send per email (`erik.frisk@liu.se`).
3. The exam will not be graded anonymously.
4. Course material in this course and others are allowed aids. You can use any computer tools used in the course, but if you use computer tools be careful to explain intermediate steps. Just entering a command in Matlab is not a solution.
5. It is not permitted to consult, take help from, or in any way communicate about the exam with anyone else during the examination time.
6. Some tasks require that you write a little code and you will need access to Matlab. For those tasks, you shall also submit your code. Skeleton files, with pointers to any Matlab functions you need, will be provided.
7. You are responsible for ensuring that your submitted solutions are readable, so be careful if you scan/photo your solutions. If I cant read your solutions, I can't give any points.
8. During the examination time I will be available by Zoom on the link

   `https://liu-se.zoom.us/...`

   I have activated a waiting room so I will only let you in one at a time. If Zoom is not working, you can reach me on 013 - 28 57 14.

Task 1. Consider the small electrical circuit



which has 5 components: the signal source, two resistors, one capacitor and one inductance. The reference value u for the signal source is known and the system is also equipped with two current sensors measuring i_1 and i_3. The process can ideally be described by the equations

$$
\begin{aligned}
v_3 &= R_2 i_3 + f_{R2} & v_1 &= v_2 + v_3 \\
i_1 &= i_2 + i_5 & i_1 &= i_3 + i_4 + i_5 \\
v_1 &= u + f_u & v_2 &= R_1 i_2 + f_{R1} \\
v_3 &= L\frac{d}{dt} i_4 + f_L & i_5 &= C\frac{d}{dt} v_1 + f_C
\end{aligned}
$$

where $f = (f_u, f_{R1}, f_{R2}, f_L, f_C)$ are faults in the corresponding components.

a) Write the model in the form

$$H(p)x + L(p)z + F(p)f = 0 \tag{1}$$

and determine the model redundancy. Make any model manipulations and simplifications you like. (3 points)

b) For a model in the form (1) write down 1) the transfer function for any residual generator, and 2) the transfer function from faults to residual. (1 points)

c) Design a residual generator that isolates a fault in the capacitor from a fault in the inductor. The residual generator shall have a time constant of approximately 2 seconds and be written in state-space form. (4 points)

Hint: The observable canonical form of

$$G(p) = \frac{b_1 p^{n-1} + \cdots + b_{n-1} p + b_n}{p^n + a_1 p^{n-1} + \cdots + a_{n-1} p + a_n}$$

is given by

$$
\dot{x}(t) = \begin{pmatrix} -a_1 & 1 & 0 & \dots & 0 \\ -a_2 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -a_{n-1} & 0 & 0 & \dots & 1 \\ -a_n & 0 & 0 & \dots & 0 \end{pmatrix} x(t) + \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_{n-1} \\ b_n \end{pmatrix} u(t)
$$
$$
y(t) = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \end{pmatrix} x(t)
$$

Solution.

a) The model in matrix form, with no simplifications, is

$$H = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & -R_2 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 1 & 0 & 0 & 0 & -sL & 0 \\ 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & -1 & -1 \\ 0 & 1 & 0 & -R_1 & 0 & 0 & 0 & 0 \\ -sC & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{pmatrix}$$

$$L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad F = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

with $x = v_1, v_2, v_3, i_1, i_2, i_3, i_4, i_5)$, $z = (y_1, y_2, u)$, and $f = (f_u, f_{R1}, f_{R2}, f_L, f_C)$.

The model has redundancy 2 since the matrix H has 10 rows and rank 8. This also comes from the two sensors.

b)

$$R(p) = \frac{1}{d(p)}\gamma(p)N_H(p)L(p)$$
$$G_{rf}(p) = -\frac{1}{d(p)}\gamma(p)N_H(p)F(p)$$

c) Form a consistency relation from

$$i_5 = C\frac{d}{dt}v_1 = C\dot{u}$$

where

$$i_5 = i_1 - i_2 = y_1 - \frac{1}{R_1}v_2 = y_1 - \frac{1}{R_1}(u - R_2 y_2).$$

Thus, the consistency relation is given by

$$y_1 - \frac{1}{R_1}(u - R_2 y_2) - C\dot{u} = 0.$$

Compute residual according to the expression

$$\dot{r} + \alpha r = y_1 - \frac{1}{R_1}(u - R_2 y_2) - C\dot{u}$$

and with state variable $w = r + Cu$ we get the state-space realization of the residual generator

$$\dot{w} = -\alpha w + y_1 + \frac{R_2}{R_1}y_2 + (\alpha C - \frac{1}{R_1})u$$
$$r = w - Cu$$

Task 2. Assume 5 residuals has been designed to supervise 5 faults according to the decision structure

	f_1	f_2	f_3	f_4	f_5
r_1	X	X	X		
r_2	X			X	X
r_3	X		X	X	
r_4		X	X	X	
r_5		X	X	X	X

where fault f_i indicates fault in component C_i, $i = 1, \ldots, 5$.

a) Assume all 5 residuals has given an alarm, i.e., exceeded its corresponding threshold. Write down the generated conflicts and indicate which that are minimal conflicts. Write the conflicts with logic notation and let $OK(C_i)$ and $\neg OK(C_i)$ denote that component i is fault-free and faulty respectively. (2 points)

b) With the alarms from the a-task, compute all minimal diagnoses. Express the diagnoses using $OK(C_i)$ and $\neg OK(C_i)$. Describe under which assumptions all minimal diagnoses characterize all diagnoses. (4 points)

c) Compute the fault isolability matrix for the 5 residuals described above. (2 points)

Solution.

a) There are 5 conflicts

$$\begin{aligned}
\pi_1 &= OK(C_1) \wedge OK(C_2) \wedge OK(C_3) \\
\pi_2 &= OK(C_1) \wedge OK(C_4) \wedge OK(C_5) \\
\pi_3 &= OK(C_1) \wedge OK(C_3) \wedge OK(C_4) \\
\pi_4 &= OK(C_2) \wedge OK(C_3) \wedge OK(C_4) \\
\pi_5 &= OK(C_2) \wedge OK(C_3) \wedge OK(C_4) \wedge OK(C_5) \quad \text{(non-minimal)}
\end{aligned}$$

b) There are 6 minimal diagnoses

$$\begin{aligned}
\mathcal{D}_1 &= OK(C_1) \wedge \neg OK(C_2) \wedge OK(C_3) \wedge \neg OK(C_4) \wedge OK(C_5) \quad (\{C_2, C_4\}) \\
\mathcal{D}_2 &= OK(C_1) \wedge OK(C_2) \wedge \neg OK(C_3) \wedge \neg OK(C_4) \wedge OK(C_5) \quad (\{C_3, C_4\}) \\
\mathcal{D}_3 &= OK(C_1) \wedge OK(C_2) \wedge \neg OK(C_3) \wedge OK(C_4) \wedge \neg OK(C_5) \quad (\{C_3, C_5\}) \\
\mathcal{D}_4 &= \neg OK(C_1) \wedge \neg OK(C_2) \wedge OK(C_3) \wedge OK(C_4) \wedge OK(C_5) \quad (\{C_1, C_2\}) \\
\mathcal{D}_5 &= \neg OK(C_1) \wedge OK(C_2) \wedge \neg OK(C_3) \wedge OK(C_4) \wedge OK(C_5) \quad (\{C_1, C_3\}) \\
\mathcal{D}_6 &= \neg OK(C_1) \wedge OK(C_2) \wedge OK(C_3) \wedge \neg OK(C_4) \wedge OK(C_5) \quad (\{C_1, C_4\})
\end{aligned}$$

c) Isolability matrix

	f_1	f_2	f_3	f_4	f_5
f_1	X				
f_2		X	X		
f_3			X		
f_4				X	
f_5				X	X

Task 3. Consider the following system model

$$\begin{aligned}\dot{x}_1 &= x_1x_2 + u + f_1\\ \dot{x}_2 &= -(1+\delta)x_2 + x_1\\ y &= x_2 + f_2\end{aligned}$$

where x_i are the unknown dynamic states, y and u known measurement and control signal, and f_i the faults we want to detect. The parameter δ models uncertainty.

a) Discuss, briefly, why and how normalization/adaptive thresholds are useful when designing fault detectors. (2 points)

b) Assume that we know that the uncertainty parameter fulfills the constraint

$$|\delta| < 0.1$$

Design a simple residual generator (you can assume derivatives of known signals are known), and design an adaptive threshold. (3 points)

Solution.

a) See lecture 6 and chapters 4 and 5 in the course literature.

b) Insert y in the model and differentiating you get

$$\begin{aligned}\dot{y} &= -(1+\delta)y + x_1\\ \ddot{y} &= -(1+\delta)\dot{y} + x_1y + u\end{aligned}$$

Multiply the first equation with y and subtract the equations yields

$$y\dot{y} - \ddot{y} + y^2 - \dot{y} + u = (-y^2 + \dot{y})\delta$$

Thus, an adaptive threshold for

$$r = y\dot{y} - \ddot{y} + y^2 - \dot{y} + u$$

could be

$$J_{adp} = |\dot{y} - y^2| \cdot 0.1 + J_0$$

Task 4.

a) Consider a residual r with the internal form

$$r_t = f_t + e_t \tag{2}$$

where e_t is white Gaussian noise with mean 0 and variance 1. Assume we make a one-sided test, i.e., raise an alarm when $r > J$ (note: not $|r|$) for a specified threshold J

Let the cumulative distribution function for a $\mathcal{N}(0,1)$[1] be denoted by Φ, i.e.,

$$\Phi(r) = \int_{-\infty}^{r} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx$$

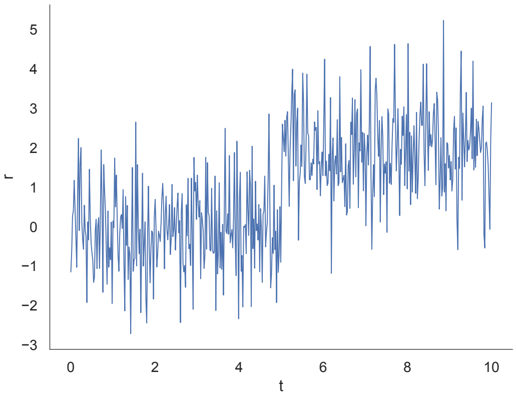
Write expressions, using Φ, for (2 points)

– the threshold J such that the probability of false-alarm equals α.

[1] The notation $\mathcal{N}(\mu,\sigma^2)$ represents Gaussian/Normal-distribution with expected value μ and variance σ^2.

– the power function, i.e., detection performance as a function of fault size.

b) In the provided file `task4.m`[2], there is code that generates a residual (2) with a fault of size $f = 2$ introduced at $t = 5$, looking like



Implement and compare performance of the following two detection algorithms

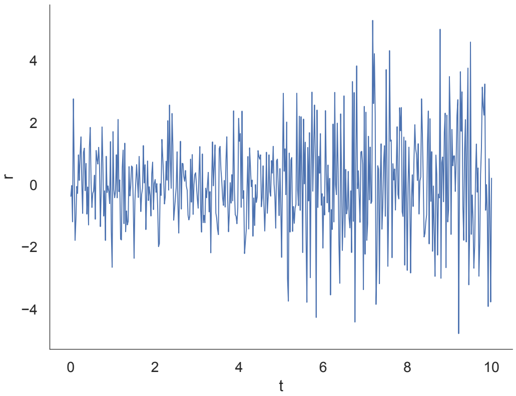
– low-pass filter + thresholding of the residual

– a CUSUM detector on the residual (no statistical knowledge assumed)

You do not have to select thrshold systematically/theoretically, it is enough to select theshold by visual inspection. Discuss how you choose design parameters and discuss consequences. Include relevant plots. (4 points)

c) Assume that you have full statistical knowledge of the residual, i.e., that the residual is distributed $\mathcal{N}(0,1)$ in the fault-free case and $\mathcal{N}(2,1)$ in the faulty-case. Design an optimal CUSUM-test with this knowledge and compare performance (visually) with the CUSUM from the b-task. (2 points)

d) Instead of a change in mean, consider a change in variance as below



In the file `task4.m`, there is code to generate this residual. Assume you know that the residual is distributed $\mathcal{N}(0,1)$ in the fault-free case and $\mathcal{N}(0,4)$ in the faulty-case. Modify the algorithm in the c-exercise to detect this change. Include relevant plots. (2 points)

Solution.

a) The expression for the threshold is given by

$$P(r > J|f = 0) = P(e > J) = 1 - \Phi(J) \Rightarrow J = \Phi^{-1}(1 - \alpha)$$

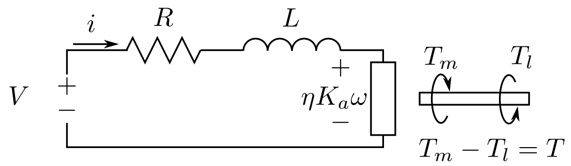
[2] A Python version, `task4.py`, is also included that you can use if you prefer.

and the power function

$$\beta(f) = P(r > J|f) = P(e + f > J) = 1 - P(e \leq J - f) = 1 - \Phi(J - f)$$

b-d) See Chapter 4.7 in the course material, parts of Chapter 2 in the book "Detection of Abrupt Changes", and Lecture 8.

Task 5. Consider an electric motor



A, somewhat ideal, model is given by the equations

$$e_1: V = iR + L\frac{di}{dt} + K_a i\omega \qquad e_4: T = T_m - T_l \qquad e_6: y_1 = T_l$$

$$e_2: T_m = \eta K_a i^2 \qquad e_5: \frac{d\theta}{dt} = \omega \qquad e_7: y_2 = i$$

$$e_3: J\frac{d\omega}{dt} = T - b\omega \qquad e_8: y_3 = \omega$$

where θ and ω are angle and angular velocity respectively, T net torque, T_m generated torque, T_l load torque, and i current. The known signals are the voltage V, the measurements y_1, y_2, and y_3 that measures T_l, i and ω. The known constants; b friction coefficient, K_a magnetization constant, η efficiency coefficient for torque generation, R resistance, and L inductance.

a) Model the following 5 faults: increased resistance in the motor (f_R), reduced torque generation efficiency (f_η), faults in sensors (f_1, f_2, and f_3) (1 points)

b) Design a residual generator that isolates fault f_2 from f_1. The residual generator shall be written in state-space form, with no derivatives of known signals included. (4 points)

Solution.

a) A model including faults is, for example, given by

$$e_1: V = iR(1 + f_R) + L\frac{di}{dt} + K_a i\omega \qquad e_4: T = T_m - T_l \qquad e_6: y_1 = T_l + f_1$$

$$e_2: T_m = (1 - f_\eta)\eta K_a i^2 \qquad e_5: \frac{d\theta}{dt} = \omega \qquad e_7: y_2 = i + f_2$$

$$e_3: J\frac{d\omega}{dt} = T - b\omega \qquad e_8: y_3 = \omega + f_3$$

b) A consistency relation is derived by substituting y_2 and y_3 into e_1. Introducing residual generator dynamics gives

$$\dot{r} + \alpha r = V - y_2 R - L\dot{y}_2 - K_a y_2 y_3$$

With state variable $w = r + Ly_2$, a state-space formulation of the residual generator is given by

$$\begin{aligned}\dot{w} &= -\alpha(w - Ly_2) + V - Ry_2 - K_a y_2 y_3 \\ r &= w - Ly_2\end{aligned}$$

This residual is sensitive to f_2 but not f_1.

Task 6. Assume you have a residual r that detects two faults f_1 and f_2. The detector has a false-alarm probability of $p_{fa} = 0.01$, probability to detect f_1, f_2 and a double fault $f_1\&f_2$ as $p_1 = 0.95$, $p_2 = 0.8$, and $p_{12} = 0.96$ respectively. Denote the a priori probabilities of faults, i.e., $p(f_1) = p_{f1} = 0.01$ and $p(f_2) = p_{f2} = 0.02$. The faults f_1 and f_2 can be assumed independent.

Assume we get an alarm in the residual and we want to rank faults f_1 and f_2. Therefore, derive the expression for

$$\frac{P(f_2|r)}{P(f_1|r)} \tag{3}$$

A main difficulty when doing Bayesian diagnostic inference is that the a priori probabilities p_{f1} and p_{f2} can be difficult to estimate. Assume we know their relative size, i.e., we know

$$\frac{p_{f2}}{p_{f1}} = c$$

for some constant c, but we do not know the true values for p_{f2} and p_{f1}. Make an argument why/when computing (3) with *incorrect* a prori probabilities, but with correct *relative* size, gives a good approximation for fault ranking. (4 points)

Solution.

$$\frac{P(f_2|r)}{P(f_1|r)} = \frac{p_2(1-p_{f1})p_{f2} + p_{12}p_{f1}p_{f2}}{p_1 p_{f1}(1-p_{f2}) + p_{12}p_{f1}p_{f2}} = \frac{1 + \frac{p_2}{p_{12}}\frac{1-p_{f1}}{p_{f1}}}{1 + \frac{p_1}{p_{12}}\frac{1-p_{f2}}{p_{f2}}} \approx 1.687$$

Assume now a factor k incorrect estimate of p_{f1}. Since the ratio c is known and correct, we have exactly the same factor in error for p_{f2}.

Using the incorrectly scaled a-priori, we can expand the expression above as

$$\frac{1 + \frac{p_2}{p_{12}}\frac{1-k\,p_{f1}}{k\,p_{f1}}}{1 + \frac{p_1}{p_{12}}\frac{1-k\,p_{f2}}{k\,p_{f2}}} = \frac{1 + \frac{p_2}{p_{12}}(\frac{1}{k\,p_{f1}} - 1)}{1 + \frac{p_1}{p_{12}}(\frac{1}{k\,p_{f2}} - 1)} \approx \frac{1 + \frac{p_2}{p_{12}}\frac{1}{k\,p_{f1}}}{1 + \frac{p_1}{p_{12}}\frac{1}{k\,p_{f2}}} = \frac{k\,p_{12} + \frac{p_2}{p_{f1}}}{k\,p_{12} + \frac{p_1}{p_{f2}}} \approx \frac{\frac{p_2}{p_{f1}}}{\frac{p_1}{p_{f2}}}$$

where the first approximation is valid if p_{f1} and p_{f2} are small and the second if $p_2/p_{f1} \gg k\,p_{12}$ and $p_1/p_{f2} \gg k\,p_{12}$.