

## DYNAMIC MODELLING OF GAS TURBINES

G. LĂZĂROIU

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The gas turbine has occupied a strong position on the power plant market for a long time now, and this is very clearly reflected in the number of gas turbines ordered in recent years. Greater use is being made of dynamic simulation of energy systems as a design tool for selecting control strategies and establishing operating procedures. As it is a part of the power system, it affects overall system behaviour during dynamic conditions so that its dynamic model is of prime importance. The present paper endeavours to establish a theoretical way of finding a suitable dynamic model of a single shaft gas turbine, especially for power system stability studies.

## 1. INTRODUCTION

There are 50th years from the bringing into operation of the first power plant with gas turbines, in 1939 by Brown Boveri at Newchâtel, Swiss, with an output of 4 MW at a thermal efficiency of 17%, [1]. Meantime the unitary power of gas turbines raised over 240 MW and thermal efficiency at 38% (see table 1). Thermal efficiency, turbine inlet temperature and pressure ratio have doubled, having as result the increase of the specific power over 300 kW/(kg/s); for example for the turbine V94.3A Siemens the specific power is 375 kW/(kg/s).

Table 1

Progress in gas turbine development

	50 years ago	15 years ago	today
Power [MW]	4	80	240
Thermal efficiency [%]	17	30	38
Turbine inlet temperature [°C]	550	945	>1250
Pressure ratio [-]	6	10	≈14

The increase of the specific power allowed the increase of the unitary power and the construction of some installations more compact on a single shaft.

The plants with gas turbines have the smallest level of pollution with SO<sub>2</sub>, reduced level of CO<sub>2</sub>, the smallest specific investment costs and the reduced consume of cooling water.

The gas turbines have occupied a strong position on the power plant market for a long time now, and this is very clearly reflected in the number of gas turbines ordered in recent years. Gas turbines are widely used as sources of power generation especially during peak load conditions. Some features such as ease of installation and maintenance, high reliability, and quick response have made it an attractive means of producing mechanical energy, [2].

On the other hand, as a basic element in an electric power network, a gas turbine affects overall system behaviour during dynamic/transient conditions so that its accurate dynamic model is of definite requirement.

Although dynamic models for steam and hydroturbines have received special attention in the literature, [3,4], no specific reference is made to gas turbines so that the task of finding a suitable model is of prime importance in power system stability studies. The dynamic analysis of a plant with combined cycle is of recent date, [5].

The dynamic processes are very complex and are proceeding very quick therefore there are necessarily some simplifications of the models in order for them to be evaluated in real time.

2. PHYSICAL AND MATHEMATICAL MODELS

Let us consider an installation of gas turbines (ITG) on a single shaft, as in fig. 1, which consists of compressor, a combustion chamber and a turbine.

For this arrangement the physical model is shown in fig. 2, where nozzle 1 represents the compressor part, nozzle 2 represents the gas turbine part and the rectangular zone represents the combustion chamber unit.

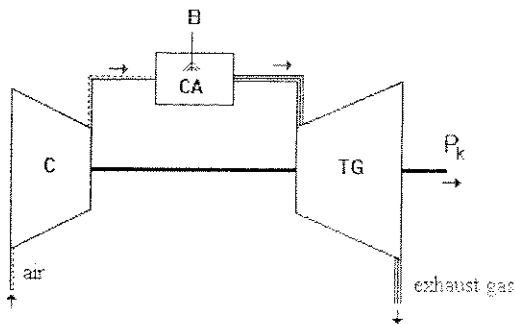


Fig. 1 - Installation of gas turbines.

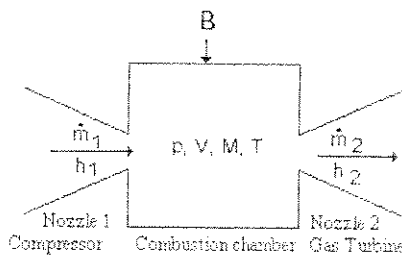


Fig. 2 - Single shaft gas turbine equivalent model.

The main parameters are:  $\dot{m}_2$  [kg/s],  $h_2$  [kJ/kg] - the mass flow rate, respectively the enthalpy of the gas turbine TG or the inferior calorific value;  $P_k$  [kW] - the mechanical power;  $p_a$  [bar] - the atmospheric pressure;  $T_c$  [K] - the temperature in the combustion chamber;  $p_a$  [bar] - the atmospheric pressure.

The mathematical model of the energy, and of the mass balance, is based on the equation of polytropic expansion. The balance equation is:

Considering small variations of the mass flow rate constant ( $\Delta \dot{m}_1 = 0$ ), the

where bar represents the average value of the ITG, calculable on the basis of the reference system.

The energy balance equation is:

$$\frac{d}{dt} [A]$$

For small variations of the mass flow rate of the turbine system to be considered, the operation is:

$$\Delta \bar{h}_2 = \frac{1}{1 + \dots}$$

where  $C_B$ ,  $C_h$ ,  $C_{Pk}$  are constants with the relations

$$C_B = 1$$

Considering the constant,  $\Delta \bar{m}_1 = 0$ , the

The main parameters of the model are:  $\dot{m}_1$  [kg/s];  $h_1$  [kJ/kg] – the mass flow rate, respectively the air enthalpy value of compressor C inlet;  $\dot{m}_2$  [kg/s],  $h_2$  [kJ/kg] – the mass flow rate, respectively the gas enthalpy value of the gas turbine TG outlet;  $B$  [kg/s],  $H_i$  [kJ/kg] – the mass flow rate, respectively the inferior caloric power of the fuel of combustion chamber inlet CA;  $P_k$  [kW] – the mechanical power in ITG;  $p$  [bar],  $T$  [K],  $V$  [m<sup>3</sup>],  $M$  [kg] – the pressure, the temperature, the volume and respectively the mass of the combustion chamber;  $p_a$  [bar] – the atmospheric air pressure.

The mathematical model is based on the balance equations of the mass, of the energy, and of the debit who flow through a nozzle, as well as on the equation of polytropic expansion, [6].

The balance equation of mass applied to this model leads to:

$$\frac{dM}{dt} = \dot{m}_1 - \dot{m}_2. \quad (1)$$

Considering small variations and assuming the mass flow rate,  $\dot{m}_1$ , to be constant ( $\Delta\dot{m}_1 = 0$ ), the equation under operational form and in *per unit* is

$$s \cdot \Delta\bar{M} = \frac{-1}{a_1} \Delta\bar{m}_2 \Rightarrow \Delta\bar{M} = \frac{-1}{s \cdot a_1} \Delta\bar{m}_2, \quad (2)$$

where bar represents *per unit* and  $a_1 = M_0/\dot{m}_0$  is the gas turbine constant ITG, calculable on the constructive data, while the indices 0 refers to a reference system.

The energy balance equation applied to the whole gas turbine ITG is:

$$\frac{d}{dt}[M \cdot h_2] = H_i \cdot B - P_k + \dot{m}_1 \cdot h_1 - \dot{m}_2 \cdot h_2. \quad (3)$$

For small variations and considering that the mass flow rate into the gas turbine system to be a constant ( $\Delta\dot{m}_1 = 0$ ), after some mathematical manipulations, the operational form of the energy balance equation per unit becomes:

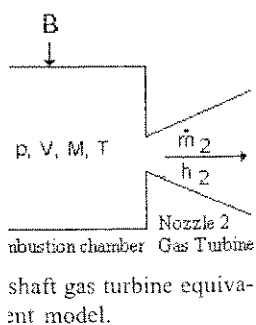
$$\Delta\bar{h}_2 = \frac{C_B}{1 + s \cdot a_1} \cdot \Delta\bar{B} + \frac{C_h}{1 + s \cdot a_1} \cdot \Delta\bar{h}_1 - \frac{C_{Pk}}{1 + s \cdot a_1} \cdot \Delta\bar{P}_k, \quad (4)$$

where  $C_B$ ,  $C_h$ ,  $C_{Pk}$  are the amplifications, calculable for the reference system with the relations

$$C_B = H_i \cdot \frac{h_{20} - h_{10}}{h_{20}}, \quad C_h = \frac{h_{10}}{h_{20}}, \quad C_{Pk} = \frac{P_{k0}}{\dot{m}_0 \cdot h_2}.$$

Considering that the ambient parameters and the power output remain constant,  $\Delta\bar{m}_1 = 0$ ;  $\Delta h_1 = 0$ ;  $\Delta P_k = 0$ , it results:

$$\Delta\bar{h}_2 = \frac{C_B}{1 + s \cdot a_1} \cdot \Delta\bar{B}. \quad (5)$$



The polytropic can one of the lower down forms:

$$T \cdot v^{\gamma-1} = p^{\frac{1}{\gamma}} \cdot v = p \cdot T^{\frac{-\gamma}{\gamma-1}} = T \cdot p^{\frac{-\gamma-1}{\gamma}} = p \cdot v^{\gamma} = \rho \cdot T^{\frac{-1}{\gamma-1}} \quad (6)$$

The flow equation who gives the flow through a nozzle is, [6],

$$\dot{m}_i = A \frac{p_i}{\sqrt{RT_i}} \left\{ \frac{2 \cdot \gamma}{\gamma-1} \left[ \left( \frac{p_e}{p_i} \right)^{\frac{2}{\gamma}} - \left( \frac{p_e}{p_i} \right)^{\frac{\gamma+1}{\gamma}} \right] \right\}^{\frac{1}{2}} \quad (7)$$

where: index "i" represent the entrance in the nozzle; index "e" signifies the exit from the nozzle;  $\gamma = \frac{k}{k-\eta \cdot (k-1)}$  is the real polytropic exponent;  $\eta$  is the average output of the steps group;  $k = \frac{c_p}{c_v}$  is the adiabatic average exponent of the steps group.

Assuming the ambient temperature to remain constant during the process,  $p_e = p_a = \text{constant}$ , after some mathematical manipulations, the operational form per unit becomes:

$$\Delta \bar{m}_2 = C_1 \cdot \Delta \bar{p} - 0.5 \cdot \Delta \bar{T} \quad (8)$$

where  $C_1 = 1 - 0.5 \frac{\frac{2}{k} \left( \frac{p_{a0}}{p_0} \right)^{\frac{2}{k}} - \frac{k+1}{k} \left( \frac{p_{a0}}{p_0} \right)^{\frac{k+1}{k}}}{\left( \frac{p_{a0}}{p_0} \right)^{\frac{2}{k}} - \left( \frac{p_{a0}}{p_0} \right)^{\frac{k+1}{k}}}$ .

Making the reasonable assumption that the combustion gas behaves like a perfect gas, then we may use the perfect gas equation

$$p \cdot V = M \cdot R \cdot T \quad (9)$$

Considering the small variations and taking into consideration that the volume  $V$  of the combustion chamber remains constant and the relation between the enthalpy and the temperature, per unit, it results the relation

$$\Delta \bar{p} = \Delta \bar{M} + \Delta \bar{h}_2 \quad (10)$$

### 3. DETERMINATION OF POWER OUTPUT

The net power output can be calculated, in this case, with the relation:

$$P_{net} = \dot{m}_2 \cdot (h_2 - h_1) \quad (11)$$

Transforming it that the ambient enthalpy is maintained

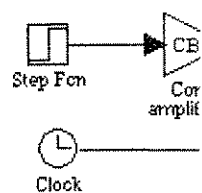
Taking into consideration the expression of the relation between the enthalpy and the temperature, the relation between the enthalpy and the temperature is

Therefore, the enthalpy at the exit from the nozzle is

where:  $a_2 = \frac{a_1}{C_1}$ ;  $a_3 = \dots$

The net power output is considered those

The block diagram, written in SIMULINK



A special impeller gas turbine, respect compressor rate, as well as analysed the adiabatic

Transforming it in per unit, considering the small variations and the fact that the ambient enthalpy is constant,  $\Delta \bar{h}_1 = 0$ , the following equation is obtained

$$\Delta \bar{P}_{net} = \Delta \bar{m}_2 + \bar{h}_2. \tag{12}$$

Taking into consideration the previous relations, (2) and (10), who give the expression of the mass  $M$  variations and of the pressure  $p$  as well as of the relation between the enthalpy and the pressure,  $\Delta \bar{T} = \Delta \bar{h}_2$ , the mass at the exit is

$$\Delta \bar{m}_2 = \frac{(C_1 - 0.5) \cdot a_1 \cdot s}{C_1 + a_1 \cdot s} \Delta \bar{h}_2. \tag{13}$$

Therefore, the net power variation depending on the gas enthalpy variation at the exit from the gas turbine is:

$$\Delta \bar{P}_{net} = \frac{1 + a_1 \cdot s}{1 + a_2 \cdot s} \cdot \Delta \bar{h}_2. \tag{14}$$

where:  $a_2 = \frac{a_1}{C_1}$ ;  $a_3 = \frac{C_1 + 0.5}{C_1} \cdot a_1$ .

The net power variation with the input combustibile variation, taking into consideration those which we established previous, is

$$\Delta \bar{P}_{net} = \frac{C_B}{1 + a_1 \cdot s} \cdot \frac{1 + a_3 \cdot s}{1 + a_2 \cdot s} \cdot \Delta \bar{B}. \tag{15}$$

The block diagram of the gas turbine installation, proposed for the simulation, written in SIMULINK language, [7], is shown in fig. 3.

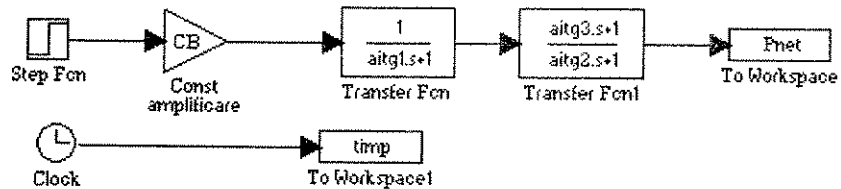


Fig. 3 – The simplified block diagram.

#### 4. SIMULATION RESULTS

A special importance in any simulation have the specific time constant gas turbine, respectively *aitg1*, as well as the adiabatic exponent; the compressor rate, as well as the gas turbine rate, have high values. Also it has been analysed the adiabatic exponent influence on the time constants *aitg2* and

aitg3, see tab. 2, from which it results that these decrease during the increasing of the adiabatic.

Table 2

The adiabatic exponent influence on the time constants

$k$	$a_2$	$a_3$
1.4	0.2359 $a_1$	0.2180 $a_1$
1.35	0.2484 $a_1$	0.2242 $a_1$
1.3	0.2633 $a_1$	0.2317 $a_1$
1.25	0.2814 $a_1$	0.2407 $a_1$
1.2	0.3039 $a_1$	0.2519 $a_1$
1.1	0.3700 $a_1$	0.2850 $a_1$

In fig. 4 is presented the exquisite importance,  $p_i/p_c$ , on the dynamic behaviour, from which we observe that a greater ratio is also favourable from a dynamic sight.

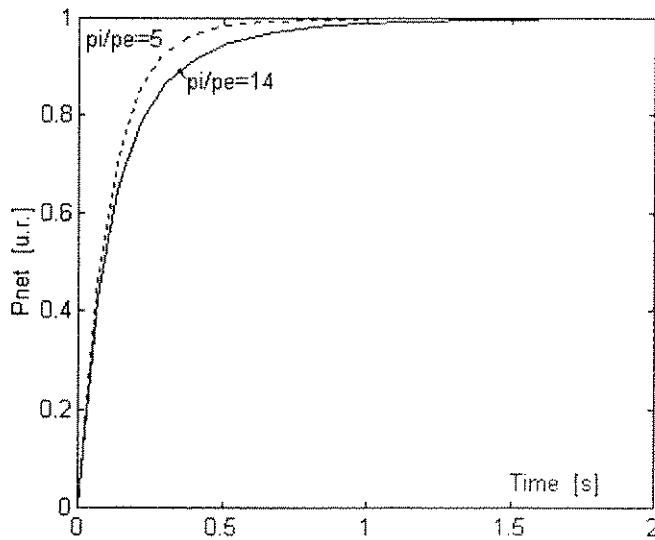


Fig. 4 – The influence of pressure ratio for dynamic.

The time constants' influence on the dynamic behaviour and on the net power is presented in fig. 5. The time constants used in simulation studies affect the performance of the derived gas turbine model and the fuel and governor system are the significant effect.

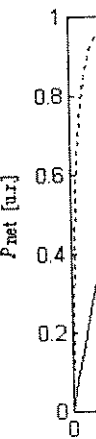


Fig. 5 -

The fuel system flow out of the fuel s and the valve position

$$\Delta \bar{B} = \frac{1}{(a \cdot k_t \cdot k_{ff}}$$

where:  $a, b, c$  – valve  $k_{ff}$  – fuel system gain fuel signal, specific;  $\alpha$   $\tau$  – fuel system pure t

The gas turbine bine generator. From can be stated that this for power system stab derived model was at

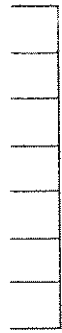
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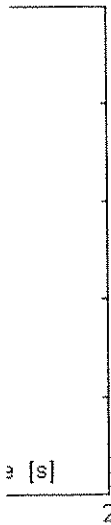
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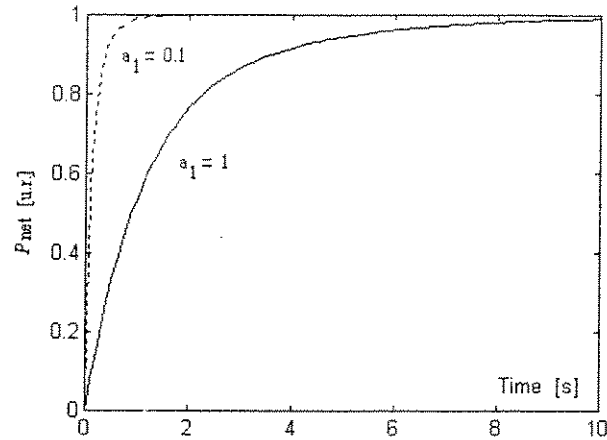


Fig. 5 - The time constants' influence on the dynamic.

The fuel system consists of the fuel valve and the actuator, [7]. The fuel flow out of the fuel systems results from the inertia of fuel system actuator and the valve positioner:

$$\Delta \bar{B} = \frac{a \cdot k_{ff}}{(a \cdot k_f \cdot k_{ff} + c) + (b + c \cdot \tau_f) \cdot s + b \cdot \tau_f \cdot s^2} \cdot (\text{MF} + \omega \cdot e^{-\tau \cdot s} \cdot \Delta \bar{F}), \quad (16)$$

where:  $a$ ,  $b$ ,  $c$  – valve parameters for positioner;  $k_f$  – feedback coefficient;  $k_{ff}$  – fuel system gain constant;  $\tau_f$  – fuel system time constant; MF – minimum fuel signal, specific;  $\omega$  – rotation speed of the turbine, from the generator model;  $\tau$  – fuel system pure time delay;  $F$  – fuel demand signal, from a controller.

The gas turbine controller regulates both the gas turbine and the gas turbine generator. From the dynamic performances shown for the derived model, it can be stated that this simple representation of single shaft gas turbine is valid for power system stability studies. Despite the different time constants used, the derived model was able to approximate quite closely the measured test results.

The paper established a theoretical way of finding a suitable model of a single shaft gas turbine, mainly for power system stability studies. The author are in the process of performing a similar task for the twin shaft arrangements of a gas turbines.

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"Politehnica" University of Bucharest

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## THERMAL CALCULATIONS

The conception of the energy, which is efficient with preheated air is preconized a new heaters with the wide presentation will be of tubular recuperator where the first variation can be perfected by

A heat pipe air heater specifically designed for these purposes. The HPAH has a high efficiency with heat transfer, from the flue gas to the air. Also the HPAH is the most advantageous character of the tubular pipe air heaters include

- very high efficiency;
- ability to function in a wide range of temperatures;
- design flexibility;
- ability to use different materials.

These basic characteristics are the most beneficial features of the HPAH:
 

- practical position in the flue gas stream;
- lack of moving parts;
- absence of metallic separation surfaces in a tubular recuperator.