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Modeling and Simulation of a Gas Turbine Engine for Power Generation

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The gas turbine engine is a complex assembly of a variety of components that are designed on the basis of aerothermodynamic laws. The design and operation theories of these individual components are complicated. The complexity of aerothermodynamic analysis makes it impossible to mathematically solve the optimization equations involved in various gas turbine cycles. When gas turbine engines were designed during the last century, the need to evaluate the engines performance at both design point and off design conditions became apparent. Manufacturers and designers of gas turbine engines became aware that some tools were needed to predict the performance of gas turbine engines especially at off design conditions where its performance was significantly affected by the load and the operating conditions. Also it was expected that these tools would help in predicting the performance of individual components, such as compressors, turbines, combustion chambers, etc. At the early stage of gas turbine developments, experimental tests of prototypes of either the whole engine or its main components were the only method available to determine the performance of either the engine or of the components. However, this procedure was not only costly, but also time consuming. Therefore, mathematical modelling using computational techniques were considered to be the most economical solution. The first part of this paper presents a discussion about the gas turbine modeling approach. The second part includes the gas turbine component matching between the compressor and the turbine which can be met by superimposing the turbine performance characteristics on the compressor performance characteristics with suitable transformation of the coordinates. The last part includes the gas turbine computer simulation program and its philosophy. The computer program presented in the current work basically satisfies the matching conditions analytically between the various gas turbine components to produce the equilibrium running line. The computer program used to determine the following: the operating range (envelope) and running line of the matched components, the proximity of the operating points to the compressor surge line, and the proximity of the operating points at the allowable maximum turbine inlet temperature. Most importantly, it can be concluded from the output whether the gas turbine engine is operating in a region of adequate compressor and turbine efficiency. Matching technique proposed in the current work used to develop a computer simulation program, which can be served as a valuable tool for investigating the performance of the gas turbine at off-design conditions. Also, this investigation can help in designing an efficient control system for the gas turbine engine of a particular application including being a part of power generation plant. [DOI: 10.1115/1.2061287]

1 Modeling of Gas Turbine Components

Aero-derivative and industrial gas turbine engines are used for a variety of applications, such as electrical power generation, driving pumps, compressors on gas and liquid fuels, etc. The engine configuration may vary to suit the application. The common configurations are a single-, twin-, or triple-spool construction or a single-stage or multistage construction. In this study only the gas turbines used for electrical power generation are considered.

A gas turbine engine essentially consists of the following component parts: (i) intake, (ii) compressor(s), (iii) combustion chamber(s), (iv) turbine(s), and (v) engine auxiliaries, such as fuel pump, lubrication pump, electrical power supply, starting gear, and control system. A block diagram of the gas turbine engine showing these components is given in Fig. 1.

Overall performance of the complete gas turbine engine is mainly determind by the main components i, ii, iii, and iv. The mathematical model for each component was created using physical laws or emphrical data when available. The thermodynamic properties of combustion gases and air at various stages throughout the gas turbine are calculated by considering variation of temperature or instead universal gas constant (R) can be used since (R) is temperature independent.

Tables containing the values of the specific heats against temperature variation have been published in many references, such as Chappel and Cockshutt [1]. In the present work, to compute the values of specific heats at constant pressure and various temperatures for air and combustion gases, data from the tables were fitted with polynomial curves to obtain Eqs. (1)–(5). These equations provide details of the polynomials. Here, T_a and T_g refer to the average temperatures during the compression and expansion processes in the compressor and turbine, respectively.

For air at the low-temperature range of 200–800 K,

$$C_{\mathrm{Pa}} = 1.0189 \times 10^3 - 0.13784 T_a + 1.9843 \times 10^{-4} T_a^2 + 4.2399 \times 10^{-7} T_a^3 - 3.7632 \times 10^{-10} T_a^4 \tag{1}$$

For air at the high-temperature range of 800–2200 K,

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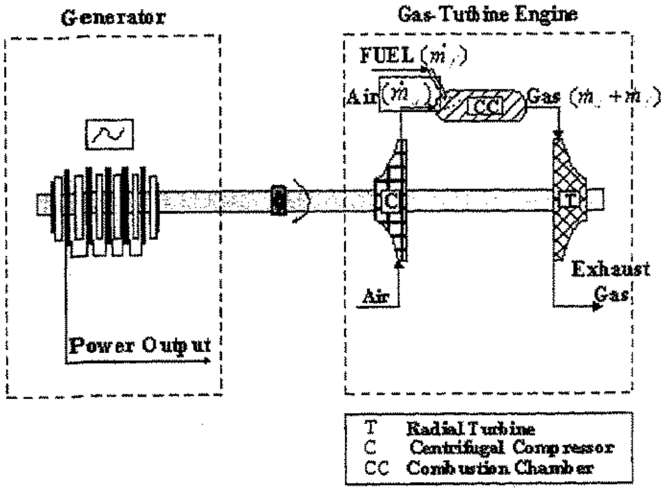

Fig. 1 Schematic diagram of simple gas turbine engine

$$C_{\mathrm{Pa}}=7.9865\times10^2+0.5339T_a-2.2882\times10^{-4}T_a^2+3.7421\times10^{-8}T_a^3 \tag{2}$$

For specific heats of products of combustion,

$$C_{Pg}=C_{\mathrm{Pa}}+[f/(1+f)]B_T \tag{3}$$

where B_T at the low-temperature range of 200–800 K,

$$B_T=-3.59494\times10^2+4.5164T_g+2.8116\times10^{-3}T_g^2-2.1709\times10^{-5}T_g^3+2.8689\times10^{-8}T_g^4-1.2263\times10^{-11}T_g^5 \tag{4}$$

and B_T at high-temperature range of 800–2200 K,

$$B_T=1.0888\times10^3-0.1416T_g+1.916\times10^{-3}T_g^2-1.2401\times10^{-6}T_g^3+3.0669\times10^{-10}T_g^4-2.6117\times10^{-14}T_g^5 \tag{5}$$

1.1 Compressor Modeling and Analysis. The performance of a compressor is fully described by a number of dimensionless parameters. The dimensionless parameters would be the same for every system of units. The dimensionless parameters are shown in Table 1.

Compressor performance, sometimes called compressor map, is usually represented by overall performance characteristics as shown in Fig. 2. These maps are in general, obtained experimentally but sometimes they can be predicted with reasonable accuracy using geometric properties of the components, i.e., intake, impeller, diffuser, and casing [2–4].

Mathematically, the compressor performance is described using the dimensionless parameters as given below

$$\frac{\tau_c}{d_2^2P_{o1}}=\frac{1}{2\pi}\frac{1}{\eta_c}\left(\frac{d_2N}{\sqrt{C_{\mathrm{Pa}}T_{o1}}}\right)^{-1}\frac{\dot m_a\sqrt{C_{\mathrm{Pa}}T_{o1}}}{d_2^2P_{o1}}\left[\left(\frac{P_{o2}}{P_{o1}}\right)^{(\gamma_a-1)/\gamma_a}-1\right] \tag{6}$$

$$\frac{\tau_c}{\delta}=f\left(\eta_c,\frac{N}{\sqrt{\theta}},\frac{\dot m_a\sqrt{\theta}}{\delta},\frac{P_{o2}}{P_{o1}}\right) \tag{7}$$

Equation (6) is in complete dimensionless form, and Eq. (7) is the general form.

The compression power W_c is given by

$$W_c=d_2^2P_{o1}\sqrt{C_{\mathrm{Pa}}T_{o1}}\frac{\dot m_a\sqrt{C_{\mathrm{Pa}}T_{o1}}}{d_2^2P_{o1}}\frac{1}{\eta_c}\left[\left(\frac{P_{o2}}{P_{o1}}\right)^{(\gamma_a-1)/\gamma_a}-1\right] \tag{8}$$

Using the compressor characteristics, if any two dimensionless parameters are known then the rest of the parameters can be determined easily.

Table 1 Dimensionless compressor parameters

PARAMETER	MEANINGS
$\left(\frac{\dot m_a\sqrt{C_{pa}T_{01}}}{d_2^2P_{01}}\right),\left(\frac{\dot m_a\sqrt{\theta}}{\delta}\right)$	Compressor dimensionless mass flow parameters
$\left(\frac{d_2N}{\sqrt{C_{pa}T_{01}}}\right),\left(\frac{N}{\sqrt{\theta}}\right)$	Compressor dimensionless speed parameters
$\left(\frac{\tau_c}{d_2^2P_{01}}\right),\left(\frac{\tau_c}{\delta}\right)$	Compressor dimensionless torque parameters
$\left(\frac{P_{02}}{P_{01}}\right)$	Compressor dimensionless pressure ratio parameters
$\left[\frac{(P_{02}/P_{01})^{\gamma_a-1/\gamma_a}}{(T_{02}/T_{01}-1)}\right]$	Isentropic compressor efficiency
$\left(\theta=\frac{T_{01}}{T_{0\_ref}}\right)$	Dimensionless temperature parameter
$\left(\theta=\frac{P_{01}}{P_{0\_ref}}\right)$	Dimensionless pressure parameter

The final stagnation temperature in the compression process T_{o2} can be calculated from the following equation:

$$T_{o2}=T_{o1}+\frac{T_{o1}}{\eta_c}\left[\left(\frac{P_{o2}}{P_{o1}}\right)^{(\gamma_a-1)/\gamma_a}-1\right] \tag{9}$$

In order to solve Eqs. (6)–(9) the needed input data can be obtained from the compressor performance map. This may require interpolation and will be dealt with in the section dealing with computer simulation.

1.2 Combustion Chamber Modeling and Analysis. The combustion chamber performance is normally given in terms of combustion efficiency η_{cc} and the factor for the loss of stagnation pressure ξ_{cc}. Using these parameters the fuel/air ratio F and the stagnation pressure at the exit of the combustion chamber P_{o3} can be determined from Eq. (10) and (11). However, there are many good programs that provide a more accurate determination of F, such as the NASA chemical equilibrium program of Gordon and McBride [5].

$$F=\frac{1}{\dfrac{\eta_{cc}(LCV)}{C_{Pg}(T_{o3}-T_{o2})}-1} \tag{10}$$

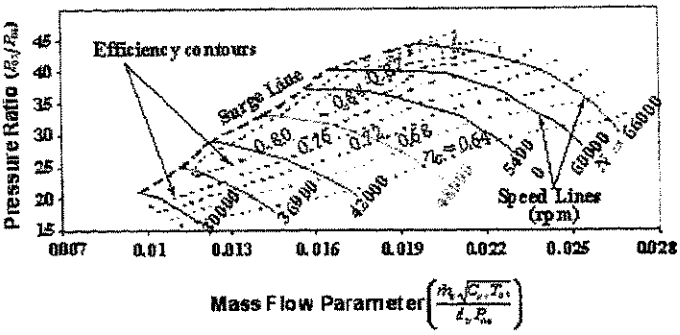

Fig. 2 General compressor characteristics (map)

Table 2 Dimensionless turbine parameters

PARAMETER	MEANINGS
$\left(\frac{\dot m_g\sqrt{C_{pg}T_{03}}}{d_2^2P_{03}}\right),\left(\frac{\dot m_g\sqrt{\theta}}{\delta}\right)$	Turbine dimensionless mass flow parameters
$\left(\frac{d_2N}{\sqrt{C_{pg}T_{01}}}\right),\left(\frac{N}{\sqrt{\theta}}\right)$	Turbine dimensionless speed parameters
$\left(\frac{\tau_c}{d_2^2P_{03}}\right),\left(\frac{\tau_c}{\delta}\right)$	Turbine dimensionless torque parameters
$\left(\frac{P_{03}}{P_{04}}\right)$	Turbine dimensionless pressure ratio parameter
$\left[\frac{(P_{03}/P_{04})^{\gamma_a-1/\gamma_a}}{(T_{03}/T_{04}-1)}\right]$	Isentropic Turbine efficiency
$\left(\theta=\frac{T_{03}}{T_{0\_ref}}\right)$	Dimensionless temperature parameter
$\left(\delta=\frac{P_{03}}{P_{0\_ref}}\right)$	Dimensionless pressure parameter

$$P_{o3}=(1-\xi_{cc})P_{o2} \tag{11}$$

1.3 Turbine Modeling and Analysis. The performance characteristics of a turbine, such as those of a compressor, are fully described by a number of dimensionless parameters. These parameters and their corresponding meanings are tabulated in Table 2 [6–11].

The turbine performance is represented by overall performance characteristics, also known as turbine map as shown in Fig. 3. This map is in general obtained experimentally but it can also be predicted with reasonable accuracy by using geometric properties and on the basis of previous experience [2–4].

In the current work, turbine cooling has not been accounted for and will be taken into consideration in future work. Using the turbine characteristics, if any two dimensionless parameters are known then the rest of the parameters can be determined easily. Mathematically, the turbine performance is described using the dimensionless parameters as follows:

$$\frac{\tau_t}{d_2^2P_{o3}}=\frac{1}{2\pi}\eta_t\left(\frac{d_2N}{\sqrt{C_{Pg}T_{o3}}}\right)^{-1}\frac{\dot m_g\sqrt{C_{Pg}T_{o3}}}{d_2^2P_{o3}}\left[1-\left(\frac{P_{o4}}{P_{o3}}\right)^{(\gamma_g-1)/\gamma_g}\right] \tag{12}$$

$$\frac{\tau_t}{\delta}=f\left(\eta_t,\frac{N}{\sqrt{\theta}},\frac{\dot m_g\sqrt{\theta}}{\delta},\frac{P_{o3}}{P_{o4}}\right) \tag{13}$$

Equation (12) is in complete dimensionless form, and Eq. (13) is the general form.

The expansion power W_t and the final stagnation temperature T_{o4} in the expansion process are calculated using Eqs. (14) and (15), respectively.

$$W_t=d_2^2P_{o3}\sqrt{C_{Pg}T_{o3}}\frac{\dot m_g\sqrt{C_{Pg}T_{o3}}}{d_2^2P_{o3}}\eta_t\left[1-\left(\frac{P_{o4}}{P_{o3}}\right)^{(\gamma_g-1)/\gamma_g}\right] \tag{14}$$

$$T_{o4}=T_{o3}-T_{o3}\eta_t\left[1-\left(\frac{P_{o4}}{P_{o3}}\right)^{(\gamma_g-1)/\gamma_g}\right] \tag{15}$$

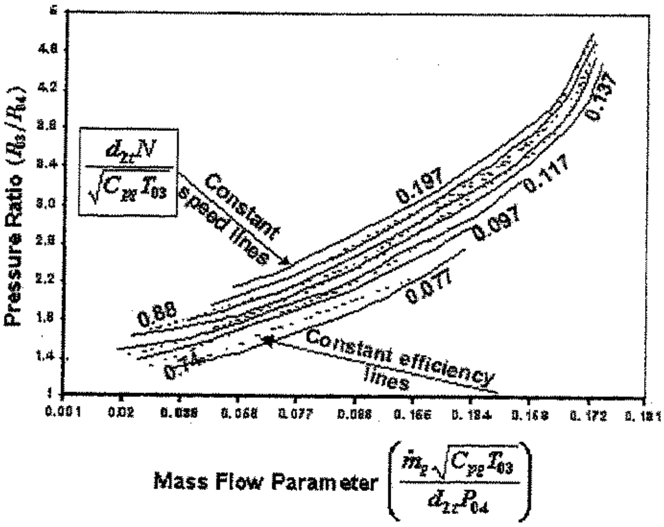

Fig. 3 Turbine Characteristics (Map)

2 Component Matching

Considering a simple gas turbine used for electrical power generation application schematically shown in Fig. 1. The performance of the compressor and the turbine are known by their characteristics maps as shown in Figs. 2 and 3. In this gas turbine engine, the component's matching should meet the following conditions:

i. The compressor shaft speed equals the turbine shaft speed,

$$N_c=N_t=N$$

ii. The gas mass flow through turbine is of the sum the air mass flow through compressor and the fuel mass flow,

$$\dot m_g=\dot m_a+\dot m_f$$

iii. Assuming that the pressure loss in the combustion chamber is a constant small percentage (ξ_{cc}) of the combustion chamber inlet pressure,

$$P_{o3}=(1-\xi_{cc})P_{o2}$$

iv. Assuming that the pressure loss in the compressor Inlet is a constant small percentage (ξ_c) of the atmospheric pressure.

$$P_{o4}=(1-\xi_c)P_{o2}\cong P_{o1}$$

v. Power flows, also, in balance.

It should be noted that the second condition is subject to modification in that it is common practice to bleed air from the compressor at various stations to provide cooling air for bearings and turbine blade cooling. Quiet often it is sufficiently accurate to assume that the bleed air equals the fuel flow, and therefore, the mass flow is the same throughout the compressor and the turbine, i.e., $\dot m_g=\dot m_a=\dot m$.

The steady-state or equilibrium operation of this gas turbine engine can be achieved by the matching of its compressor and turbine. Matching the compressor and the turbine can be done by superimposing the turbine performance map on the compressor map while meeting the components matching conditions. This matching procedure is schematically shown in Fig. 1.

Superimposing the turbine performance map on the compressor map can be achieved by applying Refs. [2–4] or by applying the new approach of making both map's axes (the abscissa and the ordinate) being identical. The main difficulty here is that of temperatures: T_{o1} for the compressor and T_{o3} for the turbine. The problem was solved by introducing a new dimensionless matching parameter $[\dot mN/d_{2c}P_{o1}]$ as presented in Sec. 2.1.

2.1 The Compressor. The abscissa of the compressor characteristics, i.e., the mass flow parameter is, $\dot{m}_a\sqrt{C_{pa}T_{o1}}/d_{2c}^2P_{o1}$. The matching parameter $[\dot{m}N/d_{2c}P_{o1}]$ was obtained by multiplying the mass flow parameter with the dimensionless speed parameter $d_{2c}N/\sqrt{C_{pa}T_{o1}}$ as follows:

$$\left[\frac{\dot{m}_a\sqrt{C_{pa}T_{o1}}}{d_{2c}^2P_{o1}}\right]\times\left[\frac{d_{2c}N}{\sqrt{C_{pa}T_{o1}}}\right]=\left[\frac{\dot{m}N}{d_{2c}P_{o1}}\right] \tag{16}$$

The ordinate, i.e., the pressure ratio parameter P_{o2}/P_{o1}, remains unchanged. Once these transformations had been made, the compressor characteristics map was plotted again where P_{o2}/P_{o1} as the ordinate and $[\dot{m}N/d_{2c}P_{o1}]$ as the abscissa.

2.2 The Turbine. The abscissa of the turbine characteristics, i.e., the mass flow parameter is $\dot{m}_g\sqrt{C_{pg}T_{o3}}/d_{2t}^2P_{o3}$. The matching parameter $[\dot{m}_gN/d_{2c}P_{o4}]$ was obtained by multiplying this parameter with the dimensionless speed parameter, turbine pressure ratio, and the ratio of the turbine rotor diameter to compressor impeller diameter as follows:

$$\left[\frac{\dot{m}_g\sqrt{C_{pg}T_{o3}}}{d_{2t}^2P_{o3}}\right]\times\left[\frac{d_{2t}N}{\sqrt{C_{pg}T_{o3}}}\right]\times\left[\frac{P_{o3}}{P_{o4}}\right]\times\left[\frac{d_{2t}}{d_{2c}}\right]=\left[\frac{\dot{m}_gN}{d_{2c}P_{o4}}\right] \tag{17}$$

To satisfy the compressor-turbine matching conditions specified previously, i.e., $\dot{m}_g=\dot{m}_u=\dot{m}$ and $P_{o4}\cong P_{o1}$. Then the matching parameter of the turbine is equal to the matching parameter of the compressor, i.e.,

$$\frac{\dot{m}_gN}{d_{2c}P_{o4}}=\frac{\dot{m}_aN}{d_{2c}P_{o1}}$$

For the turbine pressure ratio parameter, the ordinate axis of the turbine characteristics map $[P_{03}/P_{04}]$ for matching is developed into

$$\left[\frac{P_{o3}}{P_{o4}}\right]\times\left[\frac{1}{1-\xi_{cc}}\right]\times\left[\frac{P_{o4}}{P_{o1}}\right]=\left[\frac{P_{o2}}{P_{o1}}\right] \tag{13'}$$

Note that $P_{o3}=(1-\xi_{cc})P_{o2}$ and $P_{o4}\cong P_{o1}$.

Once these transformations had been made, the turbine characteristics map was plotted again in terms of these new parameters using Eqs. (17) and (18).

3 Gas Turbine Computer Simulation

A computer program for simulating a gas turbine engine would basically satisfy matching conditions analytically between the various components to produce the equilibrium running line. Representing this line either in the form of *lookup tables* or an *equation* is known as modeling and solving that equation with the help of a computer is computer simulation such that all energy and mass balances, all equations of state of working substances, and the performance characteristics of all components are satisfied.

Testing of the gas turbine engine is expensive and time consuming. Therefore, simulation can be an economic and fast tool for predicting its performance. The simulation of the gas turbine engine can be one of the following:

1. simulation at the design stage where no real gas turbine engine to meet the design specifications yet exists
2. simulation at the application stage where engine is already constructed
3. simulation at the application stage where the generation of design data for additional or auxiliary equipment, such as Lube oil requirements, blow off valve requirements, and limitations for transformers, etc., are needed
4. simulation for performance extrapolation of existing plant to meet higher output requirements

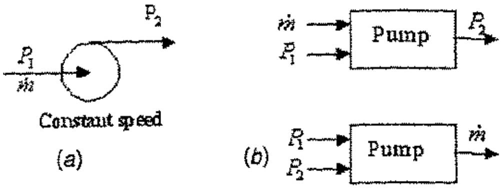

Fig. 5 (*a*) **Centrifugal pump in fluid-flow diagram, and** (*b*) **Possible information-flow blocks representing pump**

This study concentrates on the second purpose. The performance of the gas turbine plant at off-design conditions would always be of interest where the gas turbine simulation program for this purpose. This simulation may pin-point the cause of operating problems or shows how the effectiveness of the gas turbine plant may be improved.

Essentially, transient operation of a gas turbine plant is more difficult than the steady-state operation of a gas turbine plant. That field of study was considered to be outside the scope of present paper.

3.1 Information-Flow Diagrams. For system simulation, fluid-flow and energy-flow diagrams are standard engineering tools. An equally useful tool is the information-flow diagram, for example, a block diagram of a control system is an information-flow diagram wherein a block signifies that an output can be calculated when the input is known. A centrifugal pump might appear in a fluid-flow diagram, such as shown in Fig. 5(*a*), while in the information-flow diagram the blocks shown in Fig. 5(*b*). These figures represent functions or expressions that permit calculation of the outlet pressure for one block and the flow rate for the other. A block, as in Fig. 5(*b*), is called a transition function and may be an equation or may be tabular data to which interpolation would be applicable.

Figure 5 shows only one component. To illustrate how these individual blocks can build the information-flow diagram for a gas turbine plant, consider the simple gas turbine cycle in Fig. 1 shown earlier. The components in this cycle are the compressor, the combustion chamber and the turbine.

The information-flow diagram is arranged in Fig. 6 in a manner that might be used if the net power output W_{net} was to be calculated for the system with a given rate of fuel mass flow rate, i.e., heat input at the combustion chamber. Further input information includes the ambient conditions T_{o1}, P_{o1} and rotational speed N.

The compressor block diagram signifies that when the rotational speed N_c, inlet pressure P_{o1}, inlet temperature T_{o1}, and airflow rate $\dot{m}_a$ are specified, the outlet pressure P_{o2} and the compressor efficiency η_c can be determined from the compressor characteristics map as shown previously in Fig. 2 furthermore, power W_c required by the compressor and outlet temperature T_{o2} can be calculated from Eqs. (8) and (9), respectively.

The combustion chamber block diagram signifies that when the fuel flow rate $\dot{m}_f$, inlet temperature T_{o2}, and inlet pressure P_{o2} are specified, the outlet pressure P_{o3} and the outlet temperature T_{o3} can be calculated from Eqs. (10) and (11), respectively.

The turbine block diagram signifies that when the rotational speed N_t, inlet pressure P_{o3}, inlet temperature T_{o3}, and gas flow rate $\dot{m}_g$ are specified, the outlet pressure P_{o4} and the turbine efficiency η_t can be determined from the turbine characteristics map as shown previously in Fig. 3. Furthermore, power W_t delivered by the turbine and outlet temperature T_{o4} can be calculated from Eqs. (14) and (15), respectively.

Sometimes the arrangement of the system permits a direct numerical calculation for the first component of the system using input information. The output information for this first component is all that is needed to calculate the output information of the next component and so on to the final component of the system whose output is the output information of the system. Such a system

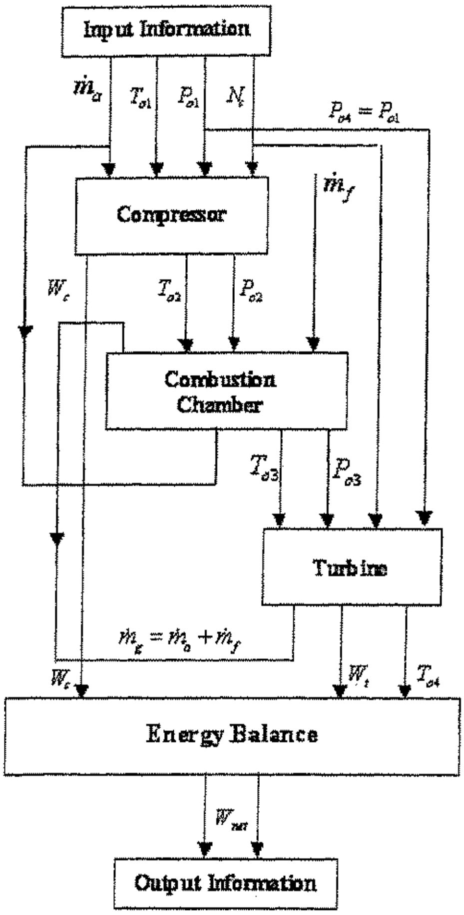

Fig. 6 Information flow diagram of a simple gas turbine engine

simulation consists of sequential calculations.

Before starting the design of the computer program for simulation purposes, it was necessary to identify its main features. Those features are summarized as follows:

i. The computer simulation should allow the user to simulate components individually or as a complete plant.
ii. The simulation program should be modeled for the linking with another program to finally simulate the CPP plant. The outputs of this computer program should contain the needed parameters to start simulating the steam power plant.
iii. The simulation program should be modular so that various modules may be assembled to represent different gas turbine plant configurations.
iv. The simulation program should be user friendly and written in such manner that data can be transferred from one module to another easily and efficiently.

To produce a running line, analytically, the computer simulation program will use the components mathematical models and the components characteristics. That running line will be essential to compute the various gas turbine performance parameters.

In order to use these maps in a computer program it was necessary to have them in a special form. This form can take one of the following shapes:

i. deriving an equation to describe the performance of the component and solving this equation to calculate the performance parameters for any selected point on the performance map
ii. storing the compressor characteristics in look-up tables and then using an interpolation or extrapolation technique to determine the values of the performance parameters for any selected point on the performance map
iii. using a neural network technique to teach the computer the pattern of the compressor characteristics then computing the performance parameters for any selected point on the performance map

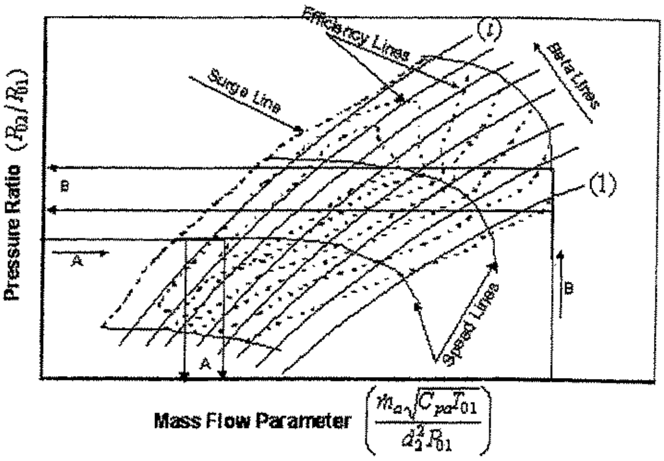

Fig. 7 Problems with reading compressor maps

In the present work, the second option was chosen as it produced a more reliable estimate of the parameters at any point on the component characteristics map.

3.2 Representation of Compressor and Turbine Maps in the Computer Program. The compressor or turbine characteristics in the standard format as shown in Figs. 2 and 3 cannot be used directly in the computer program; they require some processing to convert the maps into a good numerical representation. There are many problems and difficulties associated with the numerical representation process.

3.3 Representation of Compressor Maps in the Computer Program. It is not possible to read the compressor map parameters with given speed N_{Dim} and pressure ratio P_{o2}/P_{o1}, as there might be two values for the mass flow parameter $\dot{m}_{\mathrm{Dim}}$ at given pressure ratio (see point A in Fig. 7).

It is also not possible to determine the efficiency η_c from the compressor map with given speed N_{Dim} and mass flow parameter $\dot{m}_{\mathrm{Dim}}$, because at some parts of the compressor map the speed lines can be vertical. Hence, there might be two values for the pressure ratio P_{o2}/P_{o1} at a single value of mass flow parameter $\dot{m}_{\mathrm{Dim}}$ (see point B in Fig. 7).

Introducing a new coordinates, here called the BETA (β) [4] lines, was the solution for this problem. This allowed an independent map reading using the shape of the parameter lines with the β line and speed parameter N_{Dim}.

The auxiliary coordinates (β lines) can be selected arbitrarily with only two conditions. First, there are no intersections between the β lines within the range of interest, and second, the β lines are equally spaced. The β lines will have any numbers of lines with each line has a parameter number starting from 1. The β lines can be a parabolic lines or straight lines (straight lines are special parabolic lines).

Three-dimensional look-up tables (Tables 3–5) were created to represent the compressor characteristics. These tables represented the mass flow parameter $\dot{m}_{\mathrm{Dim}}$ versus rotational speed parameter N_{Dim}, the pressure ratio P_{o2}/P_{o1} versus rotational speed parameter N_{Dim} and the compressor efficiency η_c versus rotational speed parameter N_{Dim}.

If the values of any parameter with β line parameter are specified, the program searches and picks the other two parameters

Table 3 Mass flow parameter versus rotational speed parameter

		BETA Line (β_{line}) →				
		β_{line_1}	β_{line_2}	β_{line_3}	β_{line_4}	β_{line_i}
Speed Parameter (N_{Dim}) ↓	N_{Dim_1}					
	N_{Dim_2}		Equivalent Mass Flow Parameter ($\dot{m}_{Dim}$) ↓ →			
	N_{Dim_3}					
	N_{Dim_4}					
	N_{Dim_i}					

from these look-up tables. The other problem with the numerical representation of the compressor map is the surge line. One or both of the following can achieve the solution to this problem:

i. deriving an equation to describe surge line and solving this equation to check if the points are beyond the surge line
ii. storing the surge line in look-up table and then use an interpolation or extrapolation technique to check if the points are beyond the surge line

Either one of the two solutions can be efficient. In this program the second method was used for maintaining the consistency of the whole program (Table 6) shows the pressure ratio P_{02}/P_{01} versus mass flow parameter $\dot{m}_{\mathrm{Dim}}$ of the surge line.

A linear interpolation technique was used to estimate the parameters values lying at intermediate points. This method of interpolation is followed in most books of numerical analysis.

3.4 Representation of Turbine Maps in the Computer Program. As in the compressor case, the β lines must be introduced to the turbine maps to solve the problem of converting the maps into tabulated data. This can be seen clear in Fig. 3, where at the chocking condition and same speed parameter, more than one pressure ratio results from the same dimensionless mass flow parameter.

Table 4 Pressure ratio versus rotational speed parameter

		BETA Line (β_{line}) →				
		β_{line_1}	β_{line_2}	β_{line_3}	β_{line_4}	β_{line_i}
Speed Parameter (N_{Dim}) ↓	N_{Dim_1}					
	N_{Dim_2}		Equivalent Pressure Ratio Parameter (P_{02}/P_{01}) ↓ →			
	N_{Dim_3}					
	N_{Dim_4}					
	N_{Dim_i}					

Table 5 Compressor efficiency versus rotational speed parameter

		BETA Line (β_{line}) →				
		β_{line_1}	β_{line_2}	β_{line_3}	β_{line_4}	β_{line_i}
Speed Parameter (N_{Dim}) ↓	N_{Dim_1}					
	N_{Dim_2}		Equivalent Compressor Efficiency Parameter (η_c) ↓ →			
	N_{Dim_3}					
	N_{Dim_4}					
	N_{Dim_i}					

Table 6 Pressure ratio versus mass flow parameter of the compressor surge line

	Pressure Ratio (P_{02}/P_{01})	Mass Flow Parameter ($\dot{m}_{Dim}$)	
Pressure Ratio (P_{02}/P_{01}) ↓			Equivalent Mass Flow Parameter ($\dot{m}_{Dim}$) ↓

Three-dimensional look-up tables, as in the compressor case, have been developed and used to represent the turbine characteristics. These tables have the same forms as those of the compressor characteristics shown in Table 6. In these tables, linear interpolation technique was also used to compute the values lying at intermediate points.

3.5 Computer Simulation Program. The computer simulation program uses the components models based on either mathematical equations or performance characteristics to achieve matching between the various components in the gas turbine plant. This matching produces the engine equilibrium running line. The equilibrium running line can be used to calculate the different gas turbine performance parameters.

The principal advantages of gas turbine simulation program would be as follows

1. The computer simulation program can help in investigating the effects of the components performance characteristics on the performance of the complete engine. This investigation can be carried out at the design stage without bearing the cost of manufacturing and testing an expensive prototype.
2. The conceptual designs of the engine can be studied and the choice of particular concept can be made to suit the specified operational requirements.
3. The matching of the components can be explored for the design, off-design, and transient conditions.
4. The simulation program can serve as a valuable tool for investigating the performance of the gas turbine at off-design conditions. This investigation can help in designing an efficient control system for the gas turbine engine for a particular application, such as being a part of the combined power and power (CPP) plant.

The information flow diagram for the simple gas turbine cycle shown in Fig. 6 was used to create a computer simulation program. The flow chart for the program logic is shown in Fig. 8. The program is also suitable for dealing with the simulation of other configurations. But for the sake of brevity the flow charts for each configuration have been omitted.

4 Results and Discussion of Modeling, Matching, and Simulation

The output of the new methodology presented in this work is illustrated graphically in Figs. 9 and 10, which show complete typical performance characteristics of a centrifugal compressor and complete typical performance characteristics of a radial turbine [3], respectively.

In order to match the turbine with the compressor, Figs. 9 and 10 have to be reproduced by introducing the matching parameter

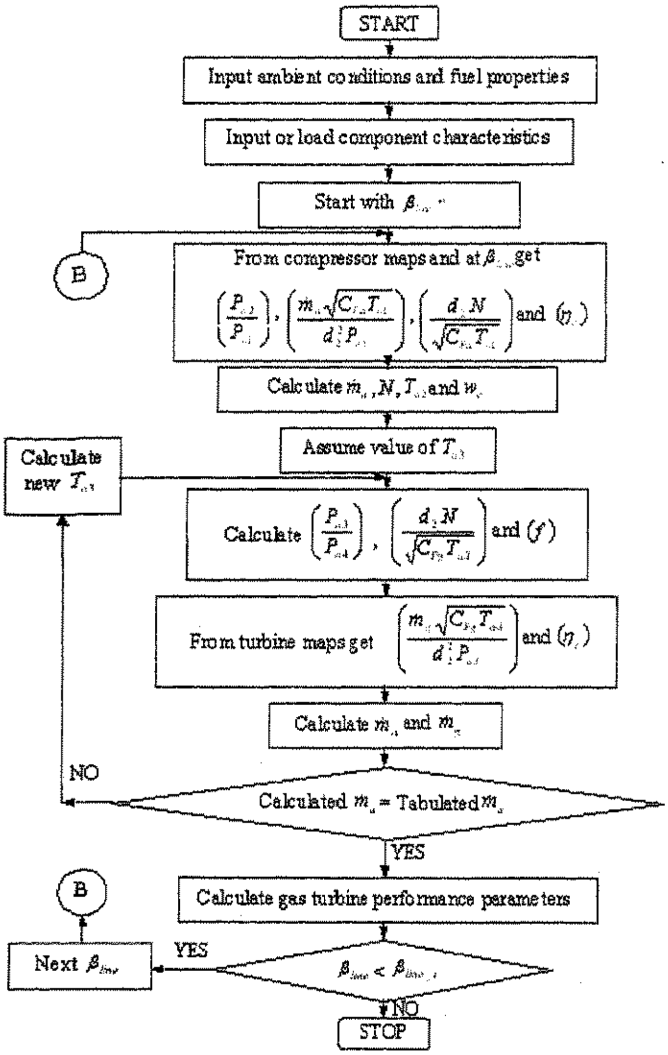

Fig. 8 Computer simulation flowchart

$(\dot{m}N/d_{2c}P_{o1})$. The transformation is shown in Figs. 11 and 12. For the compressor it is worth noting that the constant speed lines were shifted apart, nevertheless the trends stay the same. For the turbine, the trend of the constant speed lines has changed. The reason is because the turbine inlet temperature T_{o3} is not constant along any constant speed line while for the compressor case; the compressor inlet temperature T_{o1} is constant.

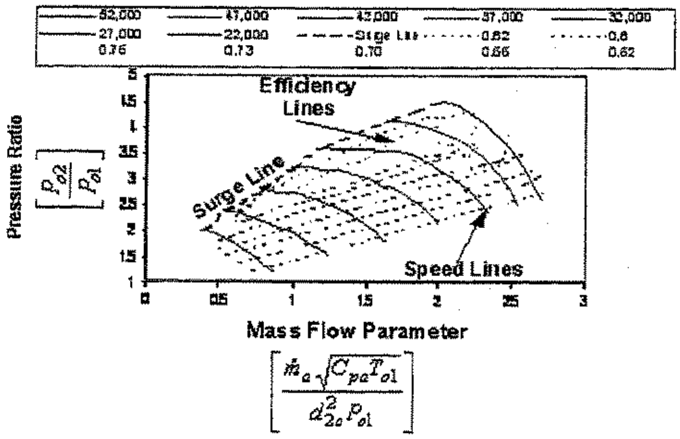

Fig. 9 Complete performance characteristics of a centrifugal compressor [3]

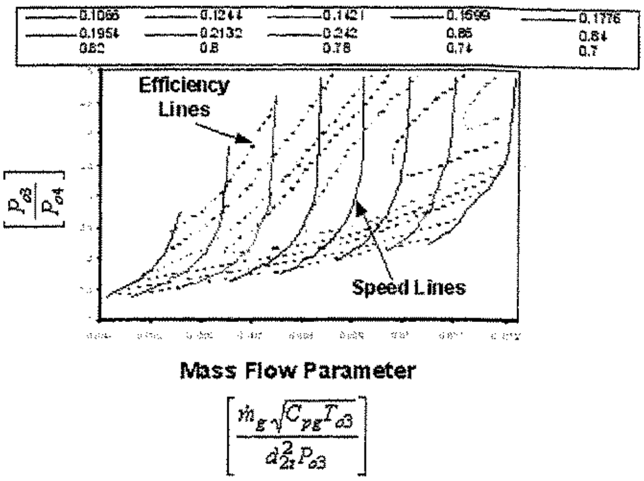

Fig. 10 Complete performance characteristics of a radial turbine [3]

Superimposing Fig. 11 on Fig. 12 produces the complete matching characteristics of the gas turbine performance as depicted in Fig. 13. At any point within the matching range the following parameters can be computed as given in Table 7.

The turbine inlet temperature T_{o3} lines of 650 and 1400 K were computed and drawn in Fig. 13. It can be seen that the changes of constant T_{o3} lines at various pressure ratios are linear and showed

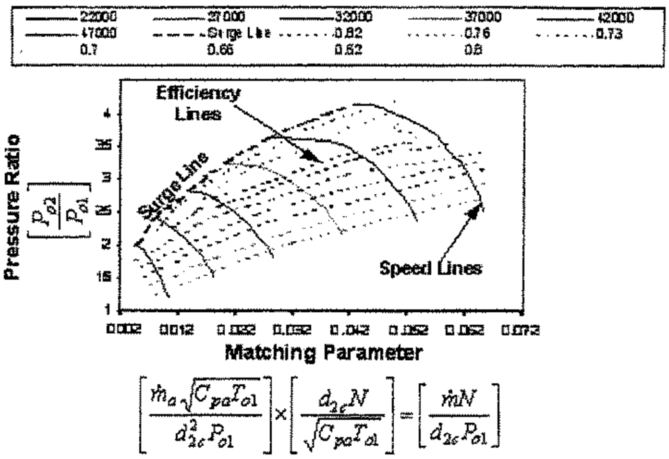

Fig. 11 Transformed performance characteristics of centrifugal compressor

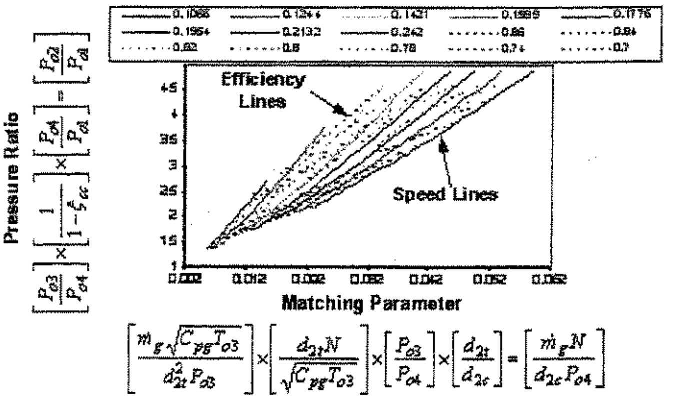

Fig. 12 Transformed performance characteristics of radial turbine

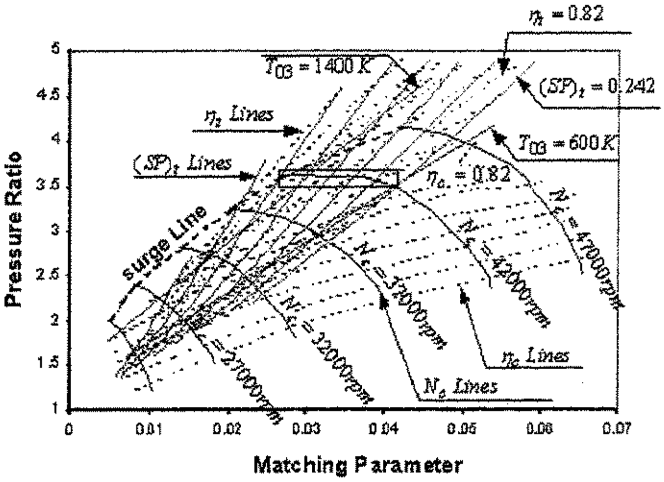

Fig. 13 Complete matching characteristics of the gas turbine performance

divergence at higher values of speed and pressure ratios. The area between these two lines represents the accepted working range for the gas turbine engine.

For a power generation driven by a gas turbine engine, let us consider any running line, for example, a speed of 42,000 rpm. For clearity, part of this running line enclosed by a rectangular box in Fig. 13 was enlarged to produce Fig. 14. Based on the graphical analysis, the above parameters can be calculated within the specified working range. The output results are given in Tables 8 and 9.

Using the results in Tables 8 and 9, the relationship between thermal efficiency and specific fuel consumption with the net power output are drawn in Fig. 15. It shows that the maximum thermal efficiency of 17.57% is attained, which corresponds to a net power output of 228 kW and minimum specific fuel consumption of 0.3327 kg/kW.hr.

Note that the low thermal efficiency is due to the fact that this selected gas turbine engine has a pressure ratio (r) of 4 and turbine inlet temperature ($To3$) of 1000 K.

Figure 16 shows that the gas mass flow rate is decreasing with an increasing net power output. At the same time the gas exhaust temperature is increasing under the same condition. This can be explained because the turbine is considered a constant volumetric flow component. Increasing the turbine work output at constant speed can be achieved by raising the turbine inlet temperature T_{o3}. In order to accommodate the same amount of volumetric mass flow at this higher T_{o3}, the mass flow rate must decrease. Consequently, the gas exhaust temperature increases.

Figure 17 shows the variation of temperatures T_{o2}, T_{o3}, and T_{o4} with the net power output. It can be seen that the variation of T_{o2} is fairly small because T_{o2} depends on the compressor pressure ratio and within the working range of the constant speed of 42,000, the pressure ratio variation is small (see Fig. 14).

Figure 18 shows the relationship between the turbine and compressor torque with net power output of the gas turbine engine at a constant speed of 42,000 rpm. It can be noted that the variation of torque and net power output is linear and increases at a constant rate.

Table 7 Gas turbine parameters

Turbine inlet temperature (T_{o3})	Gas turbine thermal efficiency (η_{gt})
Compressor power (W_c)	Net output torque (τ_{net})
Turbine power (W_t)	Turbine torque (τ_t)
Net power output (W_{net})	Specific fuel consumption (SFC)
Compressor torque (τ_c)	

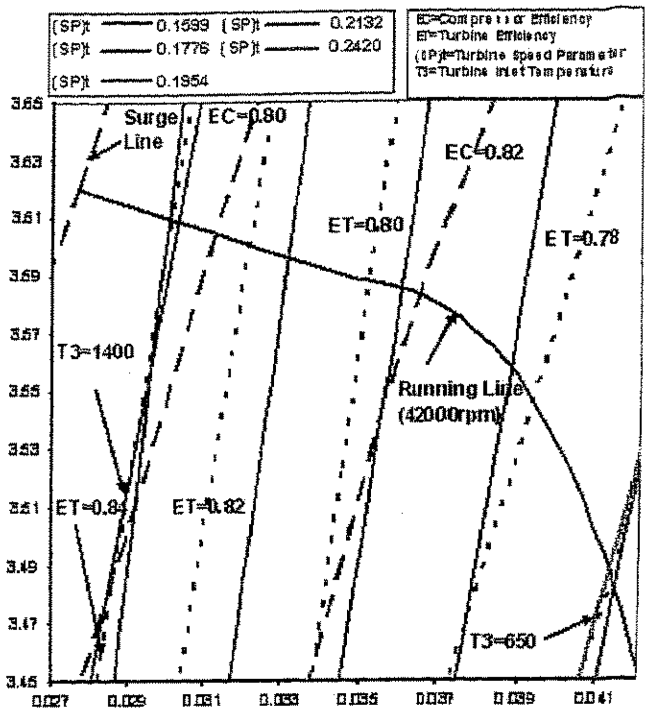

Fig. 14 Matching characteristics at running line of 42,000 rpm

5 Closure

Modeling, matching, and simulation of a gas turbine engine for power generation has been presented. A computer program for simulating a gas turbine engine has been developed that can satisfy the necessary matching conditions analytically and, thus, achieve matching between the various components in order to produce the equilibrium running line. Representing the data for this line either in the form of lookup tables or an equation is known as modeling; solving that equation with the help of a computer is computer simulation. Thus, modeling and simulation together satisfy all energy and mass balances, all equations of state of the working fluids, and the performance characteristics of all components.

Table 8 Calculated parameters within the specified working range of 42,000 rpm

#	Matching Parameter	r	Turbine Speed	η_c
1	0.0413	3.483	0.242	83
2	0.0387	3.558	0.2132	83.2
3	0.0359	3.585	0.1954	81.8
4	0.0331	3.597	0.1776	80.8
5	0.03	3.61	0.1599	79.5
#	W_t	W_{net}	Fuel/Air Ratio	$\dot{m}_f$
1	296.79	28.55	0.00611	0.01095
2	374.59	118.74	0.01143	0.01921
3	415.78	172.66	0.01596	0.02488
4	455.72	228.08	0.02205	0.03169
5	493.48	283.08	0.03036	0.03955

Table 9 Calculated parameters within the specified working range of 42,000 rpm

#	η_t	T_{o2}	T_{o3}	T_{o4}	W_c
1	82	436.82	651.97	508.69	268.24
2	84.5	439.48	840	647.02	255.85
3	84.5	443.17	1000	769.1	243.12
4	82.8	445.58	1210.52	936.02	227.64
5	80	448.7	1493.34	1165.4	210.4
#	η_{gt}	*SFC*	τ_c	τ_t	τ_{net}
1	6.37	0.7699	203.39	225.04	21.65
2	15.1	0.3464	194	284.04	90.04
3	16.95	0.3327	184.35	315.27	130.92
4	17.57	0.3481	172.61	345.55	172.94
5	17.48	0.3861	159.54	374.19	214.65

The results of component matching, modeling, and simulation are presented in this paper and lead to the following concluding remarks:

i. Matching conditions between the compressor and the turbine may be met by superimposing the turbine performance characteristics on the compressor performance characteristics with suitable transformation of the coordinates.

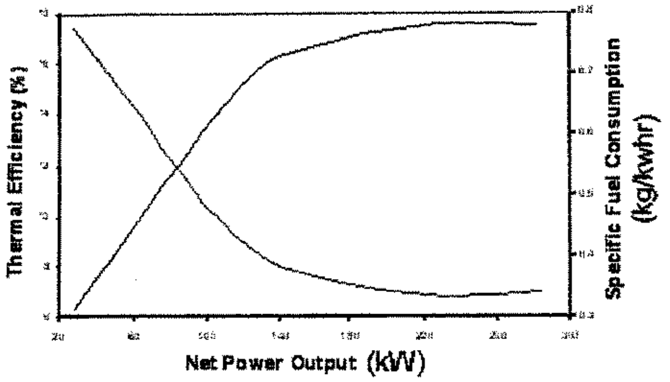

Fig. 15 Variation of thermal efficiency and specific fuel consumption at 42,000 rpm

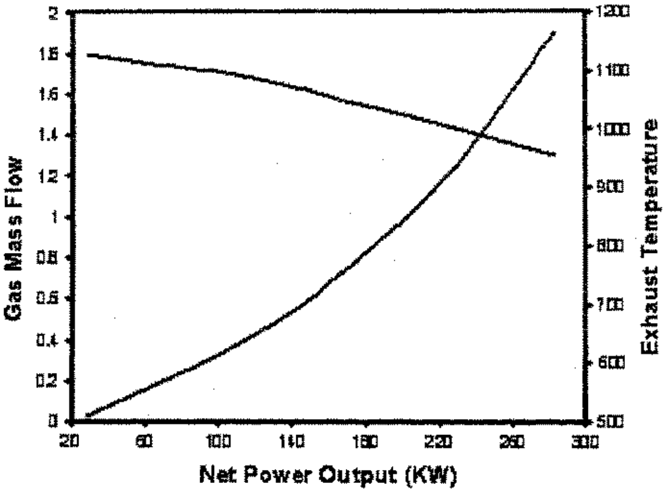

Fig. 16 Variation of exhaust flow and temperature at 42,000 rpm

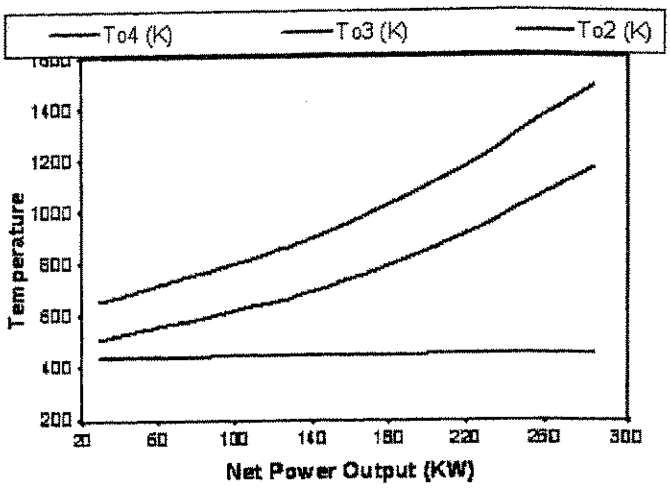

Fig. 17 Variation of various temperatures at 42,000 rpm

ii. Matching technique can be used to determine the following:

- the operating range (envelope) and running line of the matched components
- the proximity of the operating points to the compressor surge line
- the maximum operating point at the maximum turbine inlet temperature (T_{o3})
- most importantly, (from the figures) whether the gas turbine engine is operating in a region of adequate compressor and turbine efficiencies

iii. A computer program has been written for simulating a gas turbine engine. This program basically satisfies the matching conditions analytically between the various components to produce the equilibrium running line. Hence, it can serve as a very useful tool for simulating gas turbine engines. The principal advantages of the gas turbine simulation program are summarized as follows:

- The computer simulation program can help in investigating the effects of the components' performance characteristics on the performance of the complete engine. This investigation can be carried out at the design stage without bearing the cost of manufacturing and testing an expensive prototype.

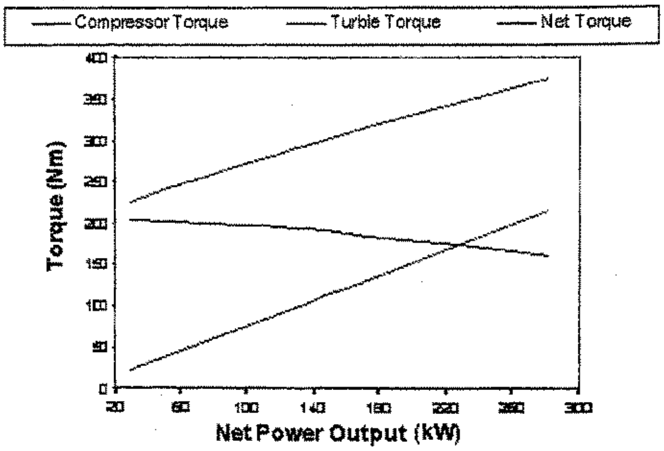

Fig. 18 Variation of torque values at 42,000 rpm

- The conceptual designs of the engine can be studied, and the choice of a particular concept can be made to suit the specified operational requirements.
- The matching of the components can be explored for the design, off-design, and transient conditions.
- The simulation program can serve as a valuable tool for investigating the performance of the gas turbine at off-design conditions. This investigation can help in designing an efficient control system for the gas turbine engine of a particular application, including being identical to the combined power and power (CPP) plant.

Acknowledgment

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Nomenclature

C_P = specific heat at constant pressure
C_V = specific heat at constant volume
γ = ratio of specific heats
$\dot{m}$ = mass flow rate
w, W = specific work output, work output
P = pressure
T = temperature
r = pressure ratio
η = efficiency
LCV = lower calorific value
τ = torque
F = fuel-to-air ratio, function
ξ = pressure loss in combustion chamber
d = diameter

Subscripts

1,2,3 = state points in the cycles
gt = gas turbine
s = isentropic
o = stagnation
g = gas
a = air
c = compressor
t = turbine
cc = combustion chamber

Superscript

$\cdot$ = Rate

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