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Optimal Model Predictive Acceleration Controller for a Combustion Engine and Friction Brake Actuated Vehicle

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Abstract: This paper investigates how model predictive control can be used to control the acceleration of an over actuated vehicle equipped with a combustion engine and friction brakes. The control problem of keeping appropriate comfort and low energy consumption and simultaneously follow an acceleration reference is described. Vehicle and actuator models are developed and the model predictive controller is tested for an adaptive cruise control cut in scenario in simulation. To be able to quantify the benefit of the proposed model predictive controller, the performance is analyzed and compared with a state of the art PID controller.

Keywords: Vehicle Dynamics, Model Based Control, Model Predictive Control, Actuator Coordination, Automotive Control.

1. INTRODUCTION

A modern conventional car is an example of a system that is overactuated. To change the speed in the longitudinal direction both friction brakes and combustion engine can be used simultaneously. The friction brake can generate a large negative torque while the combustion engine can generate both negative and positive torques. This makes the car overactuated since the negative torque can be generated by the two different actuators. In modern cars it is also common to have an electric machine which makes the car even more overactuated.

The three properties of the actuators that is of particular interest is the dynamics, controllability and the ranges of the actuators. The dynamics refer to how fast the system responds to a control signal. The meaning of controllability in this context is the expected difference between the requested and received torque on the system from the actuator. The range here refer to the range of torque an actuator can deliver to the system. An overview of the specifications of the actuators is given in Table 1.

Table 1. Overview of the characteristics of the two actuators.

	Combustion Engine	Friction Brake
Dynamics	Slow	Fast
Controllability	Mediocre	Good
Range	[-small , large]	[-large, 0]

Today the coordination part of control system in cars for the longitudinal propulsion is mostly rule based for the different actuators and the control of the individual actuators is typically done with a PID-controller. The benefits with that solution is the simplicity and the robustness, but the performance is not always optimal.

The goal with the paper is to investigate if it is possible to achieve the same or improved performance with a more sophisticated control structure, a model predictive controller (MPC). An MPC combines the possibility to predict the outcome through an open-loop controller with the stability of a closed-loop controller and gives the optimal solution for a finite horizon optimization problem. Another major benefit of MPC framework is that it can handles constraints in the control signals and states of the system in a very good way. The paper contributes with knowledge in how actuator redundancy should be utilized for best comfort using model-based control.

Many papers have been written about how to optimize the coordination of the actuators and find a global minimum using offline optimization methods. In Lorenzo Serrao [2011], Caiying Shen [2011] and Jinming Liu [2008] dynamic programming (DP) and Pontryagin's maximum principle (PMP) algorithms are presented to illustrate the possible benefits with hybrid electrical vehicles (HEV).

Lorenzo Serrao [2011] and Jinming Liu [2008] have also compared the offline solutions with equivalent consumption minimization strategy (ECMS) which is an instantaneous minimization method and the authors claims that it is possible to implement in real time.

An early MPC approach is used in M.J. West [2003] to control an electric vehicle with multiple energy storage units. The article also describe how zone control can be used in the MPC framework when it is desired to let a variable vary within a given interval. The performance of an MPC for a HEV is compared with both a DP and an ECMS approach in H.A. Borhan [2009]. The conclusion is that the performance is good and there is several advantages such as it is potentially real-time implementable and rather easy to tune.

In Chris Vermillion [2007] and Bjarne Foss [2013] model predictive control allocation (MPCA) is described, an approach to coordinate the actuators for an overactuated system when a specific behavior is desired. The focus in Chris Vermillion [2007] is on how to do this for a system with different limitations and dynamics for the actuators. Karin Uhln [2014] is studying how control allocation can be applied for controling the lateral dynamics for overactuated vehicles, although the papers don't handle any prediction horizon.

In Shengbo Li [2011] it is shown that MPC is used for adaptive speed control in order to minimize energy consumption without sacrificing tracking performance. The use of redundant actuator is however not adressed.

1.1 Outline

The paper is organized as follows. In section 2 the modeling of the vehicle and the actuators are presented. The problem formulation and the MPC algorithm are described in section 3. Section 4 presents the driving scenario and a comparison between the result from the developed MPC and existing PID controller. Finally in section 6 the conclusions are presented.

2. MODELING

2.1 Actuator modeling

The controllers internal model of the internal combustion engine (ICE) is represented by a first order system with time constant T_e from input u_e to the output force F_{eng}. A first order system is not optimal to describing the combustion engine but is chosen to keep the complexity of the system down. The dynamic of the combustion engine is also very dependent on the internal states of the engine and cannot be modeled with a higher order system that gives good fit for every case. However we assume that a second order model will improve the performance. The inertia of the powertrain, F_{pt}, is taken into account in the model as the expression in (2) where J_{eng} is the inertia of the engine, i is the transmission ratio and a is the longitudinal acceleration. The derivation of this is explained in detail in Lars Eriksson [2014].

$$F_{\text{eng}} = \frac{1}{sT_e + 1} u_e + F_{\text{pt,in}} \tag{1}$$

$$F_{\text{pt,in}} = -\frac{J_{\text{eng}} i^2}{r_{\text{w}}} a \tag{2}$$

The friction brake has the ability to convert kinetic energy to heat energy by friction. The brake system builds up a hydraulic pressure during braking, which engages the braking pads to generate a friction force and decelerate the vehicle. When the system requests less braking force on the other hand the pressure must be relieved. That is a significantly faster process than building the pressure. This is why the brake system is modeled as two separate processes as in (3). The model will however be kept the same under one prediction horizon and can only change in the beginning of each time step.

Table 2. Nomenclature used in the paper.

A_a	Cross sectional area of car
A_b	Contact area of braking pads
a_{ref}	Reference signal in acceleration
C_d	Aerodynamic drag coefficient
C_{rr}	Rolling resistance coefficient
F_{air}	Longitudinal force from air resistance
F_{brake}	Longitudinal force from brakes
F_{drag}	Air and rolling resistance
F_{eng}	Longitudinal force from engine
$F_{\text{pt,in}}$	Force from powertrain inertia
$F_{\text{w,in}}$	Force from wheels inertia
$F_{\text{road load}}$	Longitudinal force from road load
F_{roll}	Longitudinal force from rolling resistance
F_{slope}	Longitudinal force from slope
g	Gravity constant
i	Transmission ratio
J_{eng}	Inertia in the engine
J_{w}	Inertia in the wheels
j_{lim}	Jerk limit
k_{cone}	No. of sample before increasing factor
L_b	Time delay for brake model
m	Mass of car
p	Pressure in the braking system
Q_{cone}	Increasing cost for reference deviation
Q_{ref}	Cost for reference deviation
$Q_{\text{ref,total}}$	Total cost for reference
r_b	Wheel center to braking pad distance
r_w	Wheel radius
s	Time derivative operator
$T_{b,down}$	Time constant for brake pressure release
$T_{b,up}$	Time constant for brake pressure build
T_e	Time constant for engine model
T_s	Sampling time
u	Control signal vector
u_b	Control signal for brake
$u_{\text{b,max}}$	Maximum braking force
$u_{\text{b,min}}$	Minimum braking force
u_e	Control signal for engine
$u_{\text{e,max}}$	Maximum force from the engine
$u_{\text{e,min}}$	Minimum force from the engine
v	Longitudinal velocity of car
x	State vector
α	Slope of road
ϵ_j	Slack variable on the jerk state
η_e	Time delay for the engine (samples)
μ	Tire to ground friction coefficient
ρ	Density of air
τ_b	Torque generated by the brakes

$$F_{\text{brake}} = \begin{cases} \dfrac{e^{-sL_b}}{sT_{\text{b,up}} + 1} u_b & \text{if} \quad a_{\text{ref}} < a \\ \dfrac{e^{-sL_b}}{sT_{\text{b,down}} + 1} u_b & \text{if} \quad a_{\text{ref}} \geq a \end{cases} \tag{3}$$

The logic that determines which system to use is an estimate on which side of the reference value the actual acceleration is. If the actual acceleration is higher than the reference value it is highly probable that the brake, if used, is going to generate a larger negative torque. For

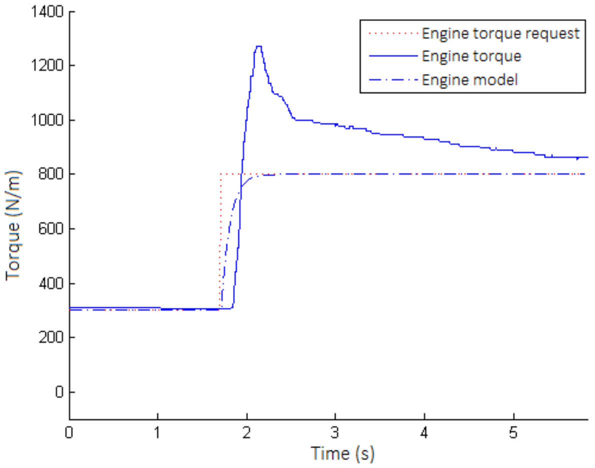

Fig. 1. Step response for the combustion engine. The force from the engine is plotted in blue and the reference signal in red. The blue dashed line shows the response from the combustion engine model.

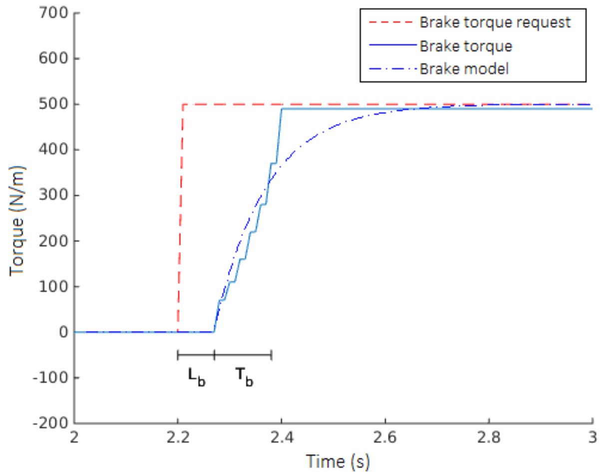

Fig. 2. Step response for the brake when building pressure. The brake force is plotted in blue and the reference signal in red. The blue dashed line shows the response from the brake model.

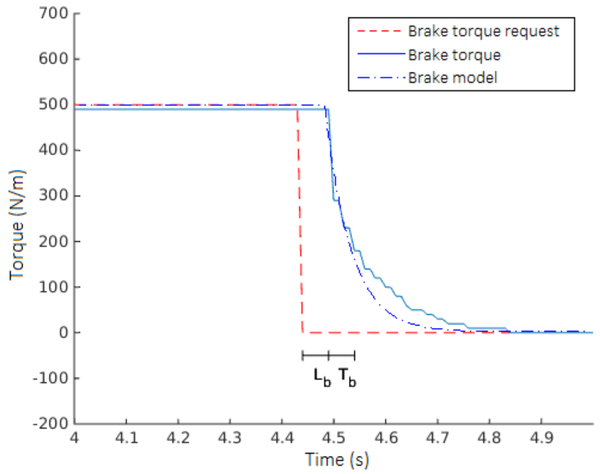

Fig. 3. Step response for the brake when releasing the pressure. The brake force is plotted in blue and the reference signal in red. The blue dashed line shows the response from the brake model.

the opposite case it is highly probable that the brakes are going to be released if used.

To identify the time constants of the first order systems that approximates the ICE, T_e, and the friction brake, T_b, along with the time delays for the friction brake L_b, step response experiments are performed on the test vehicle. The reason a simple first order system without any time delay is chosen for the combustion engine is that it gives a sufficiently accurate fit for the nonlinear system and not adding any unnecessary complexity. For the ICE the vehicle is allowed to run on the minimum torque keeping the speed constant. A fixed torque-request is then applied to the vehicle. A first order system is then fitted to the torque response. This is shown in Figure 1. The parameter is identified as $T_e = 0.1\,\mathrm{s}$.

The same procedure is applied on the friction brake. The difference is that the vehicle doesn't have to be in motion, a brake pressure is applied at standstill. The dynamics are different when the pressure is building and releasing. That is why step responses for both cases are made and analyzed. This is shown in Figures 2 and 3. The parameters are identified as $T_{\mathrm{b,upp}} = 0.1\,\mathrm{s}$ and $L_b = 0.05\,\mathrm{s}$ when building pressure and $T_{\mathrm{b,down}} = 0.05\,\mathrm{s}$ and $L_b = 0.05\,\mathrm{s}$ when releasing pressure.

2.2 Vehicle modeling

The vehicle is represented by a point mass where the longitudinal movement is modeled by Newton's second law according to (4). The inertia of the wheels is taken into account in the model in $F_{\mathrm{w,in}}$ where J_{w} is the inertia of all four wheels. It is assumed that the actuated longitudinal force will be significantly less than the maximum available force due to e.g friction and normal load, hence the tire longitudinal slip is low such that $r_w \cdot \omega \approx v_x$.

$$m\dot{v} = F_{\mathrm{eng}} + F_{\mathrm{brake}} + F_{\mathrm{road\ load}} + F_{\mathrm{w,in}} \tag{4}$$

$$F_{\mathrm{w,in}} = -\frac{J_{\mathrm{w}}}{r_{\mathrm{w}}} a \tag{5}$$

All the force components that is non-linear in v by the laws of physics will be linearized around $v = v_0$, where v_0 is the speed of the vehicle when the linearization is made. The same linearization will be applied during the prediction horizon and then update in the next time sample. If equation (4) is known to be linear in v it can be expressed in state space form.

The road load is modeled as the linearization of the expression derived in equation (6).

$$F_{\mathrm{road\ load}} = -\frac{1}{2}\rho C_d A v^2 - C_{rr} mg - mg \sin\alpha \tag{6}$$

The expression is linearized around $v = v_0$ using the first order Taylor expansion.

$$
\begin{aligned}
&\bar{F}_{roadload}\Big|_{v=v_0} \\
&= F_{\text{road load}}(v_0) + \frac{dF_{\text{road load}}}{dv}\Big|_{v=v_0}(v - v_0) \\
&= -\frac{1}{2}C_d A\rho v_0^2 - mg\sin(\alpha) \\
&- C_{\text{rr}}mg - C_d A\rho v_0 (v - v_0) \\
&= C_1 v + C_2
\end{aligned}
\tag{7}
$$

Where the constants C_1 and C_2 are

$$
\begin{aligned}
C_1 &= -C_d A\rho v_0 \\
C_2 &= \frac{1}{2}C_d A\rho v_0^2 - mg\sin(\alpha) - C_{\text{rr}}mg
\end{aligned}
\tag{8}
$$

The expression in (7) is now linear and can be used when expressing the full system in a state space form.

All the parts from the internal model is combined to a state space form. This is done by reshaping (4) when all the sub models are inserted.

$$
\begin{aligned}
m\dot{v} =& \frac{1}{sT_e + 1}u_e - \frac{J_{\text{eng}}i^2 + J_{\text{w}}}{r_{\text{w}}}\dot{v} + \\
&+ \frac{e^{-sL_b}}{sT_b + 1}u_b + C_1 v + C_2
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
&\left(m + \frac{J_{\text{eng}}i^2 + J_{\text{w}}}{r_{\text{w}}}\right)\dot{v}\,(sT_e + 1)\,(sT_b + 1) \\
&= (sT_b + 1)\,u_e + (sT_e + 1)\,e^{-sL_b}u_b \\
&+ (sT_e + 1)\,(sT_b + 1)\,(C_1 v + C_2)
\end{aligned}
\tag{10}
$$

To simplify further calculations the constant C_m is defined as in equation (11).

$$
C_m = m + \frac{J_{\text{eng}}i^2 + J_{\text{w}}}{r_{\text{w}}}
\tag{11}
$$

Since s is the Laplace transform of the time derivative operator, (10) can be rewritten.

$$
\begin{aligned}
&C_m v^{(3)} T_b T_e + C_m \ddot{v}(T_b + T_e) + C_m \dot{v} \\
&= T_b \dot{u}_e + T_e e^{-sL_b}\dot{u}_b + u_e + e^{-sL_b}u_b \\
&+ T_b T_e C_1 \ddot{v} + (T_b + T_e)C_1 \dot{v} + C_1 v + C_2
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
v^{(3)} =& \frac{T_b}{C_m T_b T_e}\dot{u}_e + \frac{T_e}{C_m T_b T_e}e^{-sL_b}\dot{u}_b + \frac{1}{C_m T_b T_e}u_e \\
&+ \frac{1}{C_m T_b T_e}e^{-sL_b}u_b - \frac{m\,(T_b + T_e) - T_b T_e C_1}{C_m T_b T_e}\ddot{v} \\
&+ \frac{(T_b + T_e)\,C_1 - C_m}{C_m T_b T_e}\dot{v} + \frac{C_1}{C_m T_b T_e}v + \frac{C_2}{C_m T_b T_e}
\end{aligned}
\tag{13}
$$

The state vector x and the input signal vector u is then defined as in (14). An advantage with the choice of this states is that all of them have a straight forward physical interpretation. The first state is the jerk, the second is the acceleration and the third is the velocity.

$$
x = \begin{bmatrix} \ddot{v} \\ \dot{v} \\ v \end{bmatrix} \qquad u = \begin{bmatrix} u_e \\ u_b \end{bmatrix}
\tag{14}
$$

From (13) a state space model can then be expressed.

$$
\dot{x} = Ax + Bu + D\dot{u} + c
\tag{15}
$$

where

$$
A = \begin{bmatrix} -\frac{C_m\,(T_b + T_e) - T_b T_e C_1}{C_m T_b T_e} & \frac{(T_b + T_e)\,C_1 - C_m}{C_m T_b T_e} & \frac{C_1}{C_m T_b T_e} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}
\tag{16}
$$

Table 3. A summary of vehicle specific parameters that are needed in the controller.

Parameter	Value
vehicle mass (m)	2200 kg
wheel radius (r_{w})	0.37 m
inertia in the engine (J_{eng})	0.25 kg· m^2
inertia in the wheels (J_{w})	1.2 kg· m^2
time constant, engine (T_e)	0.1 s
time constant, brake up ($T_{\text{b,up}}$)	0.1 s
time constant, brake down ($T_{\text{b,down}}$)	0.05 s
time delay, brake (L_b)	0.05 s

$$
B = \frac{1}{C_m T_b T_e} \cdot \begin{bmatrix} 1 & e^{-sL_b} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
\tag{17}
$$

$$
D = \frac{1}{C_m T_b T_e} \cdot \begin{bmatrix} T_b & T_e e^{-sL_b} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
\tag{18}
$$

$$
c = \begin{bmatrix} \frac{C_2}{C_m T_b T_e} \\ 0 \\ 0 \end{bmatrix}
\tag{19}
$$

To express this on a standard state space form, without derivatives on the input signals, a variable substitution is made. A fictive state vector x' is defined as

$$
x' = x - Du
\tag{20}
$$

The time derivative of the new state is then

$$
\begin{aligned}
\dot{x}' =& \dot{x} - D\dot{u} = Ax + Bu + c \\
=& A(x' + Du) + Bu + c \\
=& Ax' + (B + AD)u + c
\end{aligned}
\tag{21}
$$

This is now expressed on a state space form

$$
\dot{x}' = A'x' + B'u + c
\tag{22}
$$

where $A' = A$ and $B' = AD + B$.

The controller needs some parameters from the vehicle for its internal model. These are presented in Table 3. The parameters mass and wheel radius are not expected to be correct. The mass will be slightly different every time due to different fuel levels and load in the car while the wheel radius will change with different tires and tire pressures. The parameters that are used in the end are reasonable estimates that are close enough for the controller to function as intended. The time constant parameters from the different systems needs to be extracted from experiments as described earlier in this section.

3. PROBLEM FORMULATION

When solving an optimization problem the solution will be optimal for that specific problem formulation if the solver is allowed to run a large number of iterations and if the criteria for stopping is set to be very accurate. Since the computational power is a limited resource it is crucial for the implementation to work that the optimization problem is formulated in a computationally efficient way.

The optimization problem needs to be convex to make sure the globally optimal solution is found (see for example Stephen Boyd [2004]). For a non-convex problem the solver may find a locally optimal point but not the wanted globally optimal solution. Also for the solver to converge to a desired solution the problem cant be too flat either. The optimization problem can be formulated as equation (23).

$$\begin{aligned} \underset{u}{\text{minimize}} \quad & \sum_{k=0}^{N} f(x', u, k) \\ \text{subject to} \quad & Hu \leq b \end{aligned} \tag{23}$$

The design of the objective function, f, and the design of the constraints, H and b is what is going to be the design of the controller.

3.1 Objective Function

The decision to make here is what behaviors to be put a cost on in relation to all the behaviors that are considered to be of interest. These behaviors are listed below.

- How well the reference is followed.
- How much energy is consumed by the actuators.
- How quickly does the actuators signal energy change.
- How well the comfort criterion are fulfilled

A cost will be assigned to each of these properties as a quadratic cost term in the objective function since all of these are only dependent on the magnitude of the quantity and not on the sign.

The cost for deviation for the reference is expressed as equation (24). Here it is possible to assign a cost to deviation from the reference signal. Inspired by one of the methods to improve the stability, the terminal cost function is inconstant during the time horizon. Unlike that method with an extra cost on the last sample in the prediction horizon, the cost for the deviation is increased by a linear factor for every sample starting at a specified sample. This results in a smoother behavior compared to the terminal cost function. The idea is to make sure that the solver gets a larger freedom early in the prediction horizon, but later it gets more important to follow the reference. The parameters that can be changed for tuning the controller is Q_{ref}, Q_{cone} and k_{cone}. Q_{ref} is the start cost for deviations from the reference. Q_{cone} is the linear increasing factor, for which the cost for deviations from the reference increases every sample after k_{cone}.

$$\begin{aligned} f_r(x', u, k) &= (a_k - a_{\text{ref}})^T Q_{\text{ref,total}} (a_k - a_{\text{ref}}) \\ Q_{\text{ref,total}}(k) &= \begin{cases} Q_{\text{ref}}\,(1 + (k - k_{\text{cone}})\, Q_{\text{cone}}) & \text{when } k > k_{\text{cone}} \\ Q_{\text{ref}} & \text{when } k \leq k_{\text{cone}} \end{cases} \end{aligned} \tag{24}$$

Compared with other methods like *terminal equality constraint* and *contraction constraint* Alberto Bemporad [1999], this method never cause infeasibility and are very straightforward to implement and tune.

For this system a nonzero control signal is required in most cases to keep the desired acceleration. Therefore adding a cost for the signal energy to the objective function, the result is a trade-off between minimizing the signal energy and following the reference. By substituting the cost for the energy to a cost for change in the control signal, which is described in (25), one can avoid this behavior and integral action will be achieved.

$$f_{\Delta \text{U}} = \Delta U^T Q_{\Delta \text{U}} \Delta U \tag{25}$$

Another benefit with this term in the objective function is that the MPC gets a smoother behavior and it prevents the signals from become too noisy.

As already mentioned, energy efficiency have become more crucial for today's vehicles. For an overactuated vehicle as the one considered in this paper, where redundancy exists, it is therefore of great importance to find a solution to the optimization problem with a desired behavior. In this case the desired behavior of the controller is one that is not consuming more energy than needed to achieve the target behavior of the vehicle.

The method for control allocation is handled. The idea is to first calculate a desired virtual control signal, u_{ref}, and add the deviation from u_{ref} to the objective function as in (26).

$$f_{u_{\text{ref}}} = (u - u_{\text{ref}})^T Q_{u_{\text{ref}}} (u - u_{\text{ref}}) \tag{26}$$

To calculate the desired control signal for a stationary reference to follow is fairly straightforward. To minimize the energy consumption the brake should only be used when needed, i.e. the brake should only be used if the desired deceleration is greater than what the minimum engine torque and the road load can provide.

$$u_{\text{ref}} = \begin{cases} u_e = F\, r_{\text{w}} & \text{when} \quad F\, r_{\text{w}} \geq u_{\text{e,min}} \\ u_b = u_{\text{b,max}} & \\ u_e = u_{\text{e,min}} & \text{when} \quad F\, r_{\text{w}} < u_{\text{e,min}} \\ u_b = F\, r_{\text{w}} - u_{\text{e,min}} & \end{cases} \tag{27}$$

F in (27) is the force needed to achieve the required acceleration and is calculated as in (28) where $F_{\text{road load}}$ is the rolling resistance and aerodynamic drag and $F_{\text{pt,in}}$ describing the inertia in the powertrain.

$$F = m\, a_{\text{ref}} - F_{\text{road load}} - F_{\text{pt,in}} \tag{28}$$

3.2 Constraints

A large benefit with the MPC framework is the possibilities to include constraints on the control signals and the states in the optimization problem.

The constraints on the control signals may seems natural. The brake can never generate a propulsive force while the largest force it can generate is estimated as the friction coefficient times the normal force. For best performance it is assumed that the vehicle has a friction estimator on board. This is summarized in (29).

$$-\mu\, m\, g \leq u_b \leq 0 \tag{29}$$

For the powertrain the minimum and maximum torque that can be generated is calculated from a map depending on the engine speed. The map is visualized in figure 4.

One factor that contribute to discomfort for the driver is the derivative of the acceleration; jerk. For that reason there is a limit for the maximum level of jerk that are allowed. Unlike the constraints on the control signals a constraint on the jerk may not always be possible to fulfil. If the states initial violates the constraints it may not be possible to find a feasible solution given the dynamics of the system.

The solution to this is to use soft constraints. The soft constraints are used to ensure that a feasible solution exist. By adding a slack variable as in (31) and add a cost for the size of ϵ_j. To optimize the problem formulation and make sure that the slack variable only is used when the

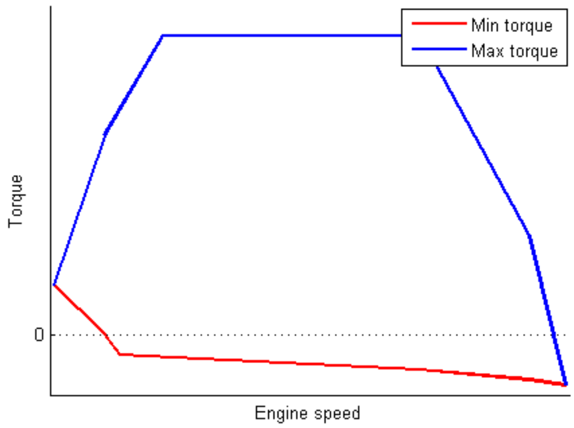

Fig. 4. The map that is used for setting the constraints on the engine control signals. The maximum and minimum torque is a function of the engine speed.

constraints already are violated, two different solvers are used as shown in (30). Otherwise the solver might find a solution to the optimization problem that violate the constraints even when it's not needed.

$$f_j = \begin{cases} f_{\text{j,hard constraints}} & \text{when} \quad |j| \leq j_{\text{lim}} \\ f_{\text{j,soft constraints}} & \text{when} \quad |j| > j_{\text{lim}} \end{cases} \tag{30}$$

$$\begin{aligned} -(j_{\text{lim}} + \epsilon_j) \quad \leq \quad j \quad \leq \quad j_{\text{lim}} + \epsilon_j \\ \epsilon_J \geq 0 \end{aligned} \tag{31}$$

3.3 Implementation

When implementing the MPC CVXGEN was used chosen to solve the optimization problem, described in Jacob Mattingley [2012]. CVXGEN is a web based tool that lets the user enter the optimization problem in a custom language. The syntax of language is very strict to ensure that the problem stated is a convex problem. The outcome of CVXGEN is a generated high speed solver optimized for the specific optimization problem that has been stated. The generated solver is available in both C code and Matlab code which allows the controller to run both in simulation and real time applications.

4. DRIVING SCENARIOS AND SIMULATION RESULTS

This section describes the driving scenario used to evaluate the controllers and compares simulation results between the MPC and PID controllers.

4.1 Scenario description

Consider driving in a multi-lane road as a host vehicle (H), following a target vehicle (T) using ACC. The controller will try to follow the estimated acceleration of T to keep a desired distance to it. This driving scenario considers another vehicle cutting in between the target vehicle, called cut in target (CIT), and the target vehicle from another lane. At some point the CIT will become the new T and the distance is changed instantly. This will result in a step in the acceleration reference shown in Figure

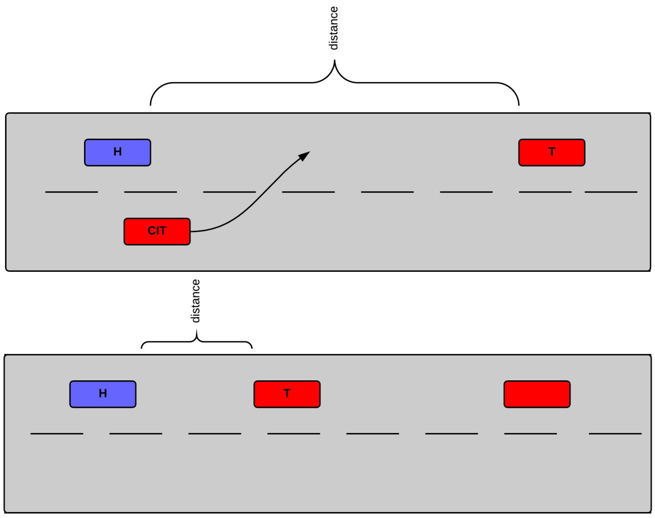

Fig. 5. In the start of the driving scenario the host vehicle is driving in a lane following a target vehicle when suddenly a cut in target is entering the lane between them. The cut in target has become the new target vehicle and the distance to the target changes momentarily.

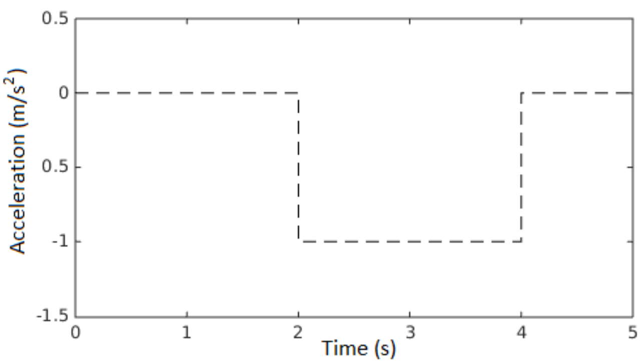

Fig. 6. The resulting request in acceleration in the driving scenario. The cut in target becomes the new target at time $t = 2$ and it has reached the desired distance at time $t = 4$.

6. Figure 5 is a visualization of the scenario where it is obvious how the distance changes instantly.

4.2 Simulation results

The driving scenario is simulated using the developed controller and presented in figure 7. The result is compared with the performance of a PID controller that exists in a series production premium vehicle.

The tests are performed with the same step sizes, jerk limits, initial states and a fixed gear to get a fair comparison. The steps sizes are $0.5\,\text{m/s}^2$, the jerk limit is set to $j_{\text{lim}} = 1,\text{m/s}^3$ and the initial state is set to $x = \begin{pmatrix} \frac{30}{3.6} & 0 & 0 \end{pmatrix}^T$. These are chosen such as the expected behavior of the controller is that both actuators will need to be used.

The prediction horizon used in the simulations was chosen large enough to cover the slowest dynamics in the system

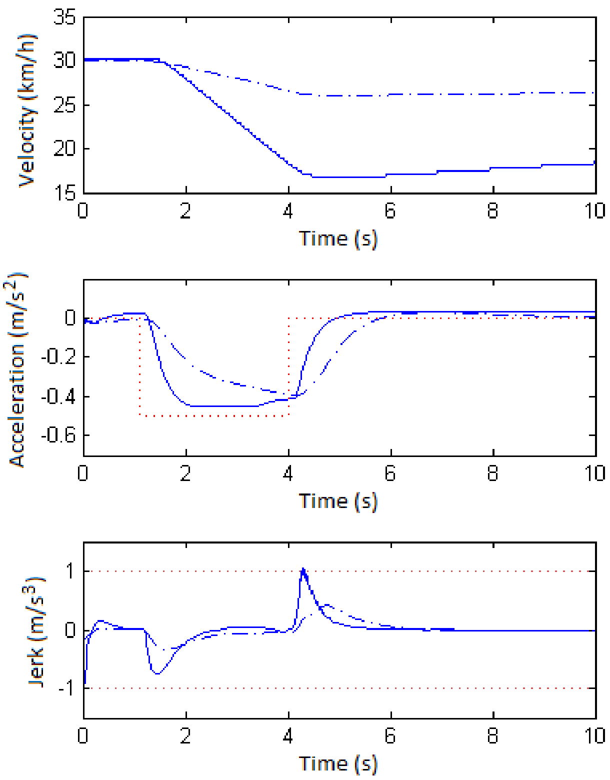

Fig. 7. The simulation result, the MPC with solid line and the PID with dashed line. The first plot is the vehicle velocity, the second is the requested acceleration plotted together with the resulting acceleration. In the third plot are the jerk and the jerk limits.

and increase the stability. In this case the prediction horizon was 1.25 seconds.

When verifying the controller a more advanced model than the internal model in the MPC was used as plant model. The dynamics used for the brakes and powertrain was of higher order systems with nonlinearities introduced to get a more realistic result. Instead of using the model in (6) a map was used to model the road load which cause some bias that most likely will occur if implementing in a car.

In figure 7 the plots shows the velocity, acceleration compared to reference and jerk with limits while figure 8 and 9 shows control signals and actual output from the actuators. The blue represent the control signal to the ICE and the red is the control signal to the friction brake. The dotted lines in each colour shows the actual output from each actuator. Figure 8 present the result from the PID and in figure 9 is the result from the MPC. In figure 7 are the two controllers compared.

When comparing the MPC with the PID in scenario there are obvious differences in the control signal. In figure 8 it can be seen that the control signals is controlled by a rule-set, specially when observing the control signal to the ICE. When the change in reference comes it directly sends the minimum force as control signal. In the MPC in Figure 9 on the other hand it has a different shape.

When analyzing the result on the states it can be seen that the MPC takes more advantage on the jerk limits

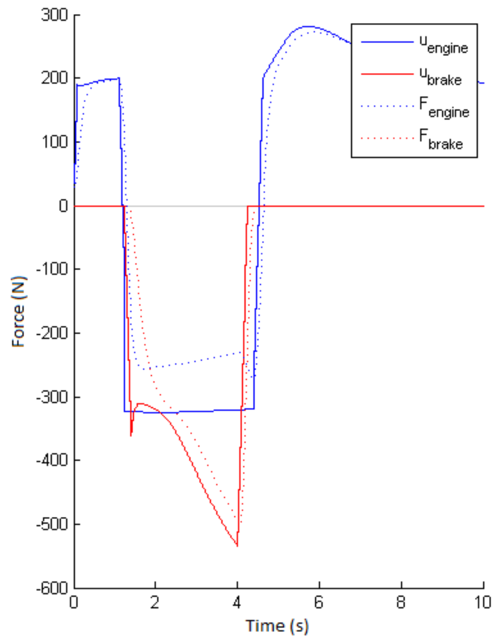

Fig. 8. The simulation result using the PID. The plot shows the requested force from the two actuators with the solid lines and the dotted lines are the measured force. The blue lines represent the force from the ICE and the red represent the force from the brakes.

then the PID. Followed by that is that the acceleration decreases faster in the MPC resulting in a lower final velocity. Since the MPC gives larger speed reductions it is also expected that the energy consumption is increased. Furthermore, the MPC uses larger friction brake forces and hence more actuator wear is expected. However, with the MPC framework this can be handled by modifying the jerk constraints. In this case lower jerk constraint would result in lower energy consumption.

To summarize the scenario it can be said that the MPC is better on using the jerk limit to maximize the performance. There is a problem with a steady state error that can be minimized by a better tuning of the controller. The acceleration is still closer to the reference for the MPC than the PID during the step, which is the important part of this scenario.

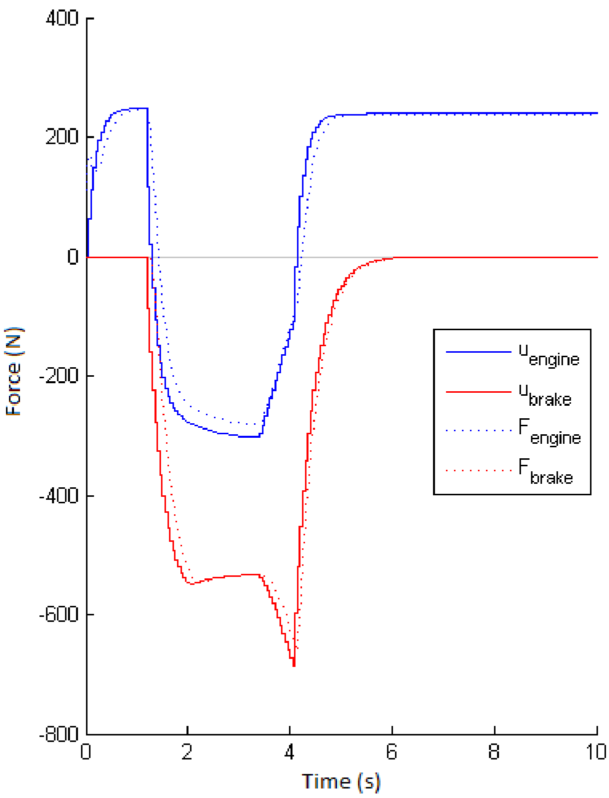

Fig. 9. The simulation result using the MPC. The plot shows the requested force from the two actuators with the solid lines and the dotted lines are the measured force. The blue lines represent the force from the ICE and the red represent the force from the brakes.

5. CONCLUSIONS

The MPC has been evaluated by comparison to the PID implementation similar to the one implemented in today's vehicles to answer the problem formulation. The main question in the problem formulation is whether it is possible or not to achieve similar or better performance using the MPC than the PID controller. The answer to this question is yes. However, there are several points that needs to be considered if the question had been if its a motivated switch.

A drawback with the MPC is that it is very dependent on a good model. If the internal model in the controller differs too much from reality the performance is quickly degraded. For example the controller might easily differs slightly from the reference when keeping zero acceleration if the internal models for air and rolling resistance differs too much from the plants corresponding models. In reality the rolling resistance is a parameter that is very difficult to get an accurate estimate of. Another example is that the MPC sometimes violates the jerk limits due to model errors. The advantage of the MPC is that it results in a faster response than the PID does in general. As soon as step in reference is known for the MPC it can prepare how to get there as fast as possible during its prediction horizon and will therefore be faster.

When summarizing the advantages and disadvantages of the MPC as it performs today, conclusions can be made that the MPC outperforms the PID on fast events like the one in the scenario where a rather fast deceleration is needed and still fulfil the comfort criteria. In longer events, when a constant acceleration needs to be kept the PID is a better choice since the correctness of the model becomes even more important.

To tackle engine model uncertainties mentioned in the paper it is suggested for future work to consider alternative engine models and robust MPC solutions. We also suggest to verify the developed MPC controller in a real world car.

In this paper the focus have been on comfort and performance. By extend the model and the constraints or use a more advanced control allocation other areas could also be considered for future work such as actuator wear and fuel consumption.

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