Optimal control of engine controlled gearshift for a diesel-electric powertrain with backlash

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Abstract: Gearshift optimal control of a hybrid powertrain with a lumped/decoupled transmission model and backlash dynamics in the driveline is studied. A model is used for a heavy duty powertrain including a validated mean value diesel engine model with electric generator, transmission dynamics representing the dynamics of the automated manual transmission system and driveshaft flexibilities. Backlash dynamics are also included in the driveline model by introducing a switching function. By applying numerical optimal control methods and dividing the gearshift process into separate phases, optimization problems are solved to investigate the minimum time and low Jerk gearshift transients. The controls are also calculated with fuel penalties added to the minimum Jerk optimization and the transients are analyzed.

Keywords: powertrain and driveline modeling, switching dynamics, multi-phase optimal control

NOMENCLATURE

Symbol	Description	Unit
x	State variable	-
u	Control input	-
θ	Angle	deg
β	Maximum Backlash Angle	deg
$\psi_{1,2}$	Backlash function parameter	-
t	Time	s
F	Force	N
R	Gas constant	N·m/kg·K
p	Pressure	Pa
T	Temperature	K
M	Torque	N·m
k	Stiffness coefficient	N·m/rad
b	Damping coefficient	N·m·s/rad
ω	Rotational speed	rad· s^{-2}
α	Rotational acceleration	rad/s
m, $\dot{m}$	Mass, Mass flow	kg, kg/s
P	Power	W
E	Energy	J
u_{mf}, u_{wg}, P_{gen}	Control signals	mg/cycle, -, W
J	Inertia	kg · m^2
ρ	Density	kg · m^{-3}
r	Radius	m
A	Vehicle frontal area	m^2
BSR	Blade speed ratio	-
λ	Air/fuel equivalence ratio	-
ϕ	Fuel/air equivalence ratio	-
i	Gear ratio	-
η	Efficiency	-
Π	Compression ratio	-
c	Constant coefficient	-
$(A/F)_s$	Stoichiometric Air to fuel ratio	-

Table 1. Variables used in the paper.

1. INTRODUCTION

Applying engine control techniques in automated manual transmission (AMT) systems, a gearshift is performed without using a clutch. This is enabled by reducing the engine torque to zero level before disengaging the off-going gear, shifting into neutral while synchronizing the speeds on the two sides of the transmission, engaging the new gear at zero engine torque, and finally accelerating the driveline inertia to high level torques. This process is illustrated in Figure 1 for a case with backlash in the driveline.

Index	Description	Index	Description
im	Intake manifold	em	Exhaust manifold
gen	Generator	wg	Wastegate
e	Engine	a	Air
ds	Drive shaft	veh	Vehicle
i	phase i	mf	fuel mass
$mech$	Mechanical	tc	Turbocharger
w	Wheel	g, i	Gear i
fd	Final drive	tr	Transmission
c	Compressor	ac	Air into cylinder
d	Drag	r	Rolling
0	Initial	f	Final
co	Coupling	bl	Backlash
l	Load	gs	Genset

Table 2. Subscripts used for variables.

Especially in heavier vehicles, it is important to reduce the duration of the gearshift because in long periods of the gearshift, the vehicle is free rolling. A longer gearshift results in larger vehicle speed deceleration and the engine may not be able to recover from this situation for delivering the torque required at the end of the gearshift. An electrical power source in the powertrain can assist the engine during the gearshift. However, this increases the control complexity as the torque from both engine and generator should be controlled for the clutch free gearshift.

Another complexity in the control of powertrain systems is the backlash due to the lash in the gears of the gearbox. Backlash occurs when the driveline state changes from acceleration to deceleration or vice versa. According to Lagerberg (2004a) backlash size can be in the range of 20-40 crankshaft degrees or even more and therefore improper

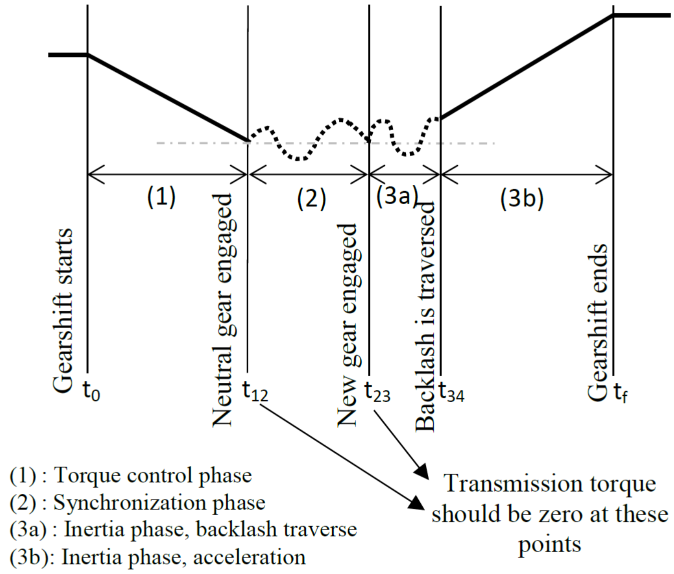

Fig. 1. Engine torque during a gearshift using engine torque control for gearshifting.

control of a powertrain with backlash results in undesirable driveline oscillations.

Optimal control can be used to identify the theoretical performance limits of a system and can be used to analyze such powertrains. The solutions can be used when designing feedback controllers. In this work, backlash nonlinearity is integrated into an already developed powertrain model in Nezhadali and Eriksson (2016) with purpose of gearshift optimal control analysis. The novelty is in the modeling of the backlash as a differentiable switching function enabling both backlash contact and traverse modes with a single equation. Moreover, the methodology for modeling and optimal control problem (OCP) formulations is insightful for researchers when solving similar industrial problems.

In the following sections, first the powertrain model is presented. Then details of OCP formulation are described followed by presenting the numerical optimal control results for min time and low Jerk gearshift transients.

2. POWERTRAIN MODEL

The powertrain model includes a mean value engine model (MVEM) with a generator (`genset`), transmission dynamics, driveshaft twist and vehicle speed dynamics. The powertrain dynamics are described by eight state variables, namely, $\omega_e(t)$, $p_{im}(t)$, $p_{em}(t)$, $\omega_{tc}(t)$, $\omega_{tr}(t)$, $\theta_{tw}(t)$, $\theta_{bl}(t)$ and $\omega_w(t)$. Three control inputs of the powertrain model are $u_{mf}(t)$, $u_{wg}(t)$ and $P_{gen}(t)$. For simplicity, the time dependence (t) is omitted in the rest of the paper.

2.1 `genset` model

Details of the `genset` model are described in Sivertsson and Eriksson (2014) while Figure 2 shows the internal relation between components of the `genset` model. The MVEM is comprised of submodels for ω_e, p_{im}, p_{em}, and ω_{tc} dynamics with u_{wg} and u_{mf} being the control inputs. The dynamics are described by the following system of differential equations:

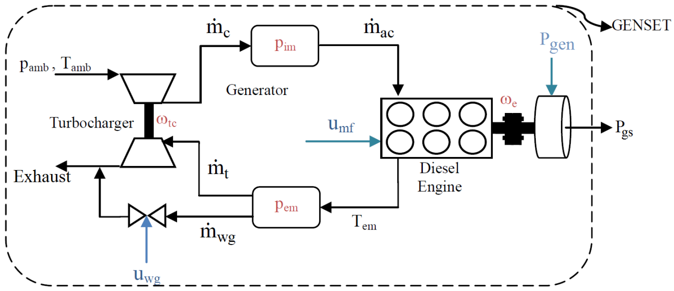

Fig. 2. `genset` model including models for generator losses and diesel engine.

$$\frac{d\omega_e}{dt} = \frac{1}{J_{gs}}(M_{gs} - M_{gs,l}) \tag{1}$$

$$\frac{dp_{im}}{dt} = \frac{R_{im}T_{im}}{V_{im}}(\dot{m}_c - \dot{m}_{ac}) \tag{2}$$

$$\frac{dp_{em}}{dt} = \frac{R_{em}T_{em}}{V_{em}}(\dot{m}_{ac} + \dot{m}_f - \dot{m}_t - \dot{m}_{wg}) \tag{3}$$

$$\frac{d\omega_{tc}}{dt} = \frac{P_t\eta_{mech} - P_c}{\omega_{tc}J_{tc}} \tag{4}$$

$M_{gs,l}$ for different gearshift phases is calculated and presented in section 2.4. The torque from the `genset` is calculated by:

$$M_{gs} = M_e(u_{mg}, u_{wg}) - \frac{P_{mech}(P_{gen})}{\omega_e} \tag{5}$$

The electrical energy from/to the generator is calculated by the following integral:

$$E_{gen} = \int_0^{t_f} P_{gen}\, dt \tag{6}$$

2.2 Powertrain configuration during gearshift

Three distinct phases can be identified during an upshift according to Figure 1. In driveline modeling, to calculate the optimal gearshift controls, it is important to account for the significant torsional flexibilities in the driveline. Pettersson and Nielsen (2000) show that, the main flexibility in the driveline is in the driveshaft, hence modeling the driveshaft as a damped torsional flexibility describes the main oscillations of the driveline. The transmission speed oscillations following the shift into the neutral gear, during the synchronization phase, are properly described using a decoupled transmission model according to Pettersson and Nielsen (2000), and same approach is followed here.

The general configuration of the powertrain during each of the gearshift phases is depicted in Figure 3. The main source of backlash in the driveline is the gear play in the transmission and final drive between the two major inertias of the powertrain. Considering that the inertia of gears and the driveshaft are negligible compared to `genset` and the inertia at wheels, and according to Lagerberg and Egardt (2007), all powertrain backlash contributions can be lumped into a single backlash.

2.3 Backlash modeling

According to Lagerberg (2001), commonly used backlash models are dead-zone and physical models. Nordin et al.

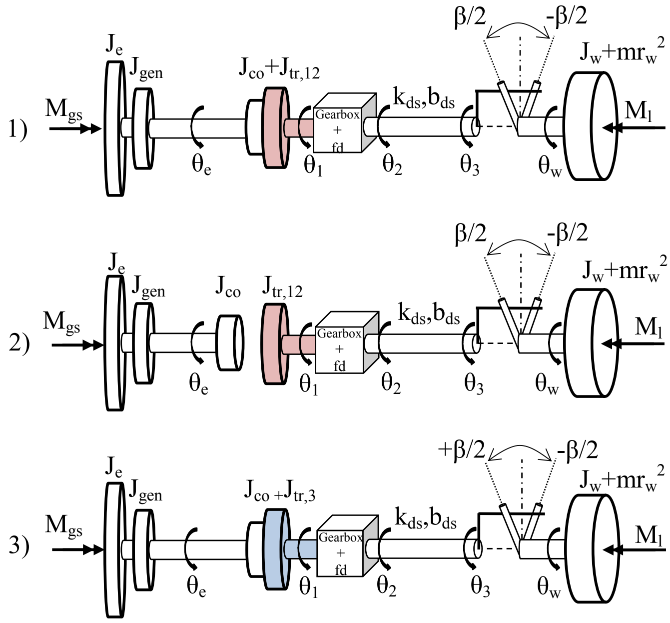

Fig. 3. The configuration of powertrain inertia in different phases of an engine controlled gearshift. The three phases are 1)Torque phase, 2)Synchronization phase, 3)Inertia phase.

(1997) shows that the dead-zone models are incorrect for the case of elastic shafts with internal damping, thus the physical modeling approach is followed in this article. In a physical model, both backlash angle and the remaining twist in the driveshaft are considered. Backlash dynamics depend on the backlash position with respect to the contact point between the driving and driven parts of the powertrain. In Figure 3, when backlash position is at $-\beta/2$ or $\beta/2$ the powertrain is considered to be in left or right contact mode, respectively. Depending on the backlash position, the backlash dynamics are described by:

$$\frac{d\theta_{bl}}{dt} = \begin{cases} \max\left(0, \frac{d\theta_{tw}}{dt} + \frac{k_{ds}}{b_{ds}}(\theta_{tw}-\theta_{bl})\right) & \theta_{bl} = -\frac{\beta}{2} \\ \frac{d\theta_{tw}}{dt} + \frac{k_{ds}}{b_{ds}}(\theta_{tw}-\theta_{bl}) & |\theta_{bl}| \leq \frac{\beta}{2} \\ \min\left(0, \frac{d\theta_{tw}}{dt} + \frac{k_{ds}}{b_{ds}}(\theta_{tw}-\theta_{bl})\right) & \theta_{bl} = +\frac{\beta}{2} \end{cases} \quad (7)$$

with θ_{bl} and θ_{tw} being:

$$\theta_{bl} = \theta_3 - \theta_w \quad (8)$$

$$\theta_{tw} = \theta_2 - \theta_w \quad (9)$$

According to (8), (9) and the powertrain geometry in Figure 3, the net driveshaft twist is $\theta_{tw,net} = \theta_{tw} - \theta_{bl}$.

Correct dynamics from (7) should be chosen for powertrain simulation. Lagerberg (2004b) defines separate operation phases when backlash is in contact $\theta_{bl} = |\frac{\beta}{2}|$ or traverse mode $|\theta_{bl}| \leq \frac{\beta}{2}$. Position of the backlash during a gearshift phase is not known beforehand and is highly dependent on other driveline dynamics. Moreover, for OC problem formulation, it is desired to always describe the backlash dynamics, at different positions, with a single differential equation covering all backlash modes. This is enabled by using a differentiable function κ which changes in [0,1] range enabling smooth transition between different backlash modes in (7) depending on θ_{bl}.

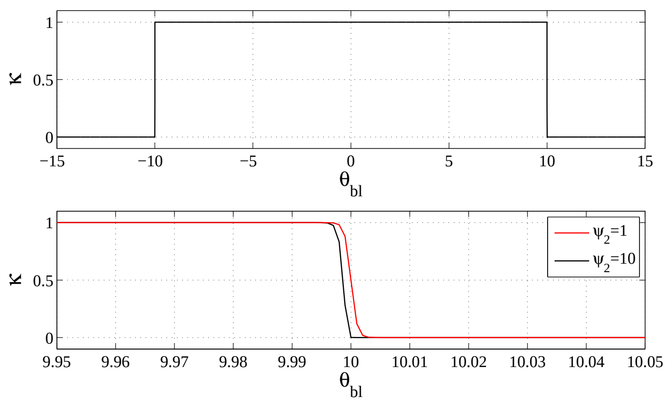

Fig. 4. A single differential equation for backlash dynamics is obtained using a switching function κ with ψ_1=1000 and two different values for ψ_2.

The backlash dynamics are then described by a single differential equation as follows:

$$\frac{d\theta_{bl}}{dt} = \kappa(\theta_{bl}) \times \left(\frac{d\theta_{tw}}{dt} + \frac{k_{ds}}{b_{ds}}(\theta_{tw}-\theta_{bl})\right) \quad (10)$$

$$\kappa(\theta_{bl}) = \left(0.5\tanh(\psi_1(\theta_{bl}+\beta)) - 0.5\tanh(\psi_1(\theta_{bl}-\beta))\right)^{\psi_2} \quad (11)$$

where $\kappa(\theta_{bl})$ is illustrated in Figure 4 for $\psi_1 = 1000$, $\beta = 20$ [deg] and two different ψ_2 values, to show the switching characteristics at left and right contact positions. A larger ψ_2 removes non-zero values of κ when $|\theta_{bl}| > |\beta|/2$.

The transferred torque by the driveshaft is calculated by:

$$M_{ds} = k_{ds}(\theta_{tw}-\theta_{bl}) + b_{ds}\left(\frac{d\theta_{tw}}{dt} - \frac{d\theta_{bl}}{dt}\right) \quad (12)$$

where $d\theta_{tw}/dt$ is the driveshaft twist dynamics calculated according to the gearshift phase as described in section 2.4.

At the beginning of the inertia phase, the new gear play should be traversed before any torque can be transferred to the wheels. This is because as long as $\theta_{bl} < +\beta/2$, solving (12) with $\kappa = 1$ in (10) results in $M_{ds} = 0$, thus the vehicle will be free rolling. Considering this, it is possible to divide the powertrain configuration during the inertia phase as `3a)` and `3b)` in Figure 5. In `3a)`, the backlash is traversed from the left contact side to the right contact side, while in `3b)`, $\kappa = 0$ implies that the backlash dynamics are zero. This improves the computational efficiency of the model in optimal control analysis as backlash nonlinearity will not exist during the larger part of the inertia phase.

2.4 Powertrain dynamics

The dynamics during each gearshift phase for transmission speed, driveshaft twist angle and wheel speed are described in the following sections. To describe transmission dynamics before/after and during the synchronization phase, the *lumped* and *decoupled* driveline models from Pettersson and Nielsen (2000) are used.

Torque phase dynamics The first phase of the gearshift starts after the driver commands a gearshift. Thereafter, engine control is transferred from the driver to the transmission control unit. During the torque phase, transmission components are connected to the `genset`,

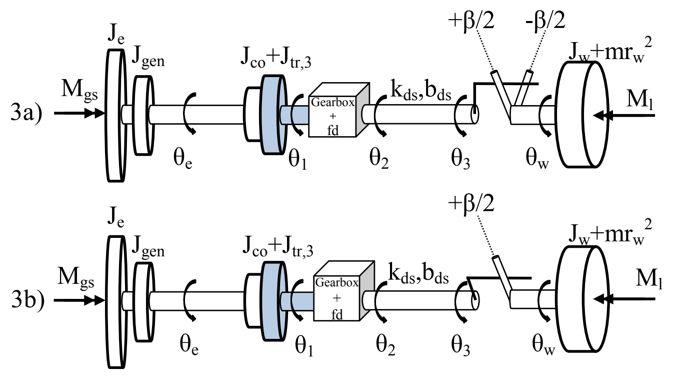

Fig. 5. Inertia phase is divided into *3a Inertia phase, backlash traverse* and *3b Inertia phase, acceleration*.

therefore they have same speed dynamics as ω_e only scaled with the initial gear ratio $i_{g,1}$. The rotational states of the driveline during this phase are described as follows:

$$\frac{d\omega_{tr}}{dt} = \frac{d\omega_e}{dt}\frac{1}{i_{g,1}} \tag{13}$$

$$\frac{d\theta_{tw}}{dt} = \frac{\omega_e}{i_{g,1}\, i_{fd}} - \omega_w \tag{14}$$

$$\frac{d\omega_w}{dt} = \frac{M_{ds} - M_l}{J_w + m_{veh} r_w^2} \tag{15}$$

with M_{ds} calculated by (12) and the load from the road calculated as:

$$M_l = (0.5\,\rho_{air}\, c_a\, A\, \omega_w^2 r_w^2 - m_{veh}\, g\, c_r)\, r_w \tag{16}$$

Assuming small driveshaft inertia, the lumped inertia of the masses rotating with the `genset` and $M_{gs,l}$ which are required in (1) are calculated as:

$$J_{gs} = J_e + J_{gen} + J_{co} + J_{tr,12} \tag{17}$$

$$M_{gs,l} = \frac{M_{ds}}{i_{g,1}\, i_{fd}} \tag{18}$$

Synchronization phase dynamics During this phase, the engine and wheel sides of the transmission are decoupled. Components of the transmission that are attached to the engine side, represented with J_{co} inertia, rotate with the engine. Rotational dynamics of the remaining parts of the transmission, $J_{tr,12}$, depend on the driveshaft twist and damping. The driveline dynamics can be described as:

$$\frac{d\omega_{tr}}{dt} = \frac{1}{J_{tr_{12}}}(-b_{tr}\omega_{tr} - \frac{M_{ds}}{i_{fd}}) \tag{19}$$

$$\frac{d\theta_{tw}}{dt} = \frac{\omega_{tr}}{i_{fd}} - \omega_w \tag{20}$$

$$\frac{d\omega_w}{dt} = \frac{M_{ds} - M_l}{J_w + m_{veh} r_w^2} \tag{21}$$

where M_l and M_{ds} are calculated with (16) and (12). $M_{gs,l} = 0$ and the engine side inertia is calculated by:

$$J_{gs} = J_e + J_{gen} + J_{co} \tag{22}$$

Inertia phase, backlash traverse dynamics In this phase, the transmission and `genset` are reconnected. The backlash has to be traversed from regions close to the left contact mode to the right contact mode until that $\kappa(\theta_{bl})$=1. The driveline dynamics are described the same as the torque phase dynamics with replaced $i_{g,2}$ instead of $i_{g,1}$ in (13) and (14). $J_{tr,12}$ in (17) should also be replaced with the $J_{tr,3}$ matching the new gear dimensions. M_{ds} is calculated by (12) using the backlash dynamics defined in (10), and the load on the `genset` is calculated by (18) using $i_{g,2}$.

Inertia phase, acceleration dynamics During this phase, backlash angle is constant at right contact position implying:

$$\frac{d\theta_{bl}}{dt} = 0 \tag{23}$$

The rest of the driveline dynamics, inertia and torques are calculated exactly same as the previous phase.

3. OPTIMAL CONTROL PROBLEM FORMULATION

The developed powertrain model is used to formulate OCPs for the different phases of the gearshift. In the following, first the choice of objective function and then the problem constraints are described for an up-shift. The same model can be used for a down-shift scenario after modification of the constraints.

3.1 Objective function formulations

Various properties can be considered for gearshift optimization such as gearshift duration, transmission output shaft oscillations and components' life length, see Haj-Fraj and Pfeiffer (2002) and Haj-Fraj and Pfeiffer (2001) for more details. Here the focus is mostly on the minimization of gearshift duration and oscillations in transmission output shaft. These oscillations are referred to as vehicle Jerk which is defined as follows:

$$Jerk = \int_{t_0}^{t_{12}} \dot{\alpha}_{tr}^2\, dt + \int_{t_{12}}^{t_{23}} \dot{\alpha}_{tr}^2\, dt + \int_{t_{23}}^{t_{34}} \dot{\alpha}_{tr}^2\, dt + \int_{t_{34}}^{t_f} \dot{\alpha}_{tr}^2\, dt \tag{24}$$

$$\alpha_{tr} = \frac{d\omega_{tr}}{dt} \tag{25}$$

where the times t are illustrated in Figure 1. Fuel consumption requirements affect hybrid powertrain transients. To investigate this, effects of including a fuel penalty term in the Jerk minimization problem are calculated where fuel consumption m_{mf} during the gearshift is calculated as follows:

$$m_{mf} = \int_{t_0}^{t_{12}} \dot{m}_{mf}\, dt + \int_{t_{12}}^{t_{23}} \dot{m}_{mf}\, dt + \int_{t_{23}}^{t_{34}} \dot{m}_{mf}\, dt + \int_{t_{34}}^{t_f} \dot{m}_{mf}\, dt \tag{26}$$

$$\dot{m}_{mf} = u_{mf}\, \omega_e\, n_{cyl} \frac{10^{-6}}{4\pi} \tag{27}$$

with n_{cyl} being the number of diesel engine cylinders.

3.2 Constraints

The constraints are originated from the component limitations or gearshift phase specifications. The constraints can be categorized into two categories where the first includes boundary properties at the beginning/end of each phase, whilst the second group are time varying.

3.3 Boundary conditions

At the beginning of the torque phase, t_0, initial engine speed, wheel speed and transmission speed are defined. To ensure that the gearshift starts and ends at stationary operating conditions, all state dynamics are set to zero at t_0 and t_f. To enable clutch-free gearshift, the torque in the transmission from the power source should be zero at the beginning and the end of the synchronization phase t_{12} and t_{23}. Moreover, the engine speed should be synchronized with the transmission speed at t_{12}. At the beginning of the backlash traverse phase, backlash is assumed to be close to the left contact mode and it has to reach close to the right contact mode at the end of the backlash traverse phase. The gearshift should end at predefined engine speed and wheel speed complying with the new gear ratio. Moreover, to ensure charge sustainability of the generator operation, it is required that the electrical energy and power be zero at the beginning and end of the gearshift. Finally, all states and controls of the system except the backlash between second and third phases of the gearshift are connected to each other to obtain continuous transients.

The complete set of boundary conditions during the gearshift are summarized as:

$$\begin{cases} \dot{x}(t_f) = \dot{x}(t_0) = 0, \quad \omega_{tr}(t_0) = \omega_{e,0}/i_{g,i} \\ \omega_e(t_0) = \omega_{e,0}, \quad \omega_w(t_0) = \omega_{e,0}/i_{fd} i_{g,i} \\ \omega_{tr}(t_{23}) = \omega_e(t_{23})/i_{g,i}, \quad M_{gs}(t_{12}) = M_{gs}(t_{23}) = 0 \\ M_{gs}(t_{12}) = M_{gs}(t_{23}) = 0 \\ E_{gen}(t_0) = E_{gen}(t_f) = P_{gen}(t_f) = P_{gen}(t_0) = 0 \\ \omega_e(t_f) = \omega_{e,t_f}, \quad \omega_w(t_f) = \omega_w(t_0) \\ \theta_{bl}(t_{23}) = -0.6\,\beta/2, \quad \theta_{bl}(t_{34}) > 0.99\,\beta/2 \\ (x,u)_{\text{phase 1,end}} = (x,u)_{\text{phase 2,start}} \\ (x,u)_{\text{phase 2,end}} = (x,u)_{\text{phase 3,start}} (\text{except for } \theta_{tw}) \\ (x,u)_{\text{phase 3,end}} = (x,u)_{\text{phase 4,start}} \end{cases} \tag{28}$$

3.4 Time varying and box constraints

The time varying constraints are originated from the engine torque limits, smoke limiter requirements and turbocharger constraints. Box constraints in the form of min and max values are also implemented representing the feasible operating range for various system states and control inputs. The practical constraints on how fast the wastegate can operate or generator torque can change are represented by c_{wg} and c_{gen} limits. These constraints can be summarized as follows:

$$\begin{cases} BSR_{min} \le BSR(x,u) \le BSR_{max}, \ \Pi_c = \Pi_{c,surge} \\ u_{min} \le u \le u_{max}, \ x_{min} \le x \le x_{max} \\ |\dot{u}_{wg}| \le c_{wg}, \ |\dot{P}_{gen}|/\omega_e \le c_{gen} \\ P_e(x,u) \le P_{e,max}(x), \ 0 < \frac{\dot{m}_{ac}}{\dot{m}_f}(A/F)_s \le \frac{1}{\lambda_{min}} \end{cases} \tag{29}$$

3.5 Optimal control problem formulation

OCPs are formulated for minimization of gearshift time and Jerk. Another case that is considered is the Jerk minimization problem including a penalty term for the fuel consumption. Knowing the powertrain dynamics $\dot{x} = f(x,u)$ from section 2.4, the OCP formulations for minimization of time, Jerk and Jerk with fuel penalty look as follows:

$$1)\begin{cases} \min\limits_{x,u} t_f \\ \text{subjected to} \\ \dot{x} = f(x,u) \\ t_{(1),min} \le t_{(1)} \\ t_{(2)} = t_{(2),fix} \\ t_{(3a),min} \le t_{(3a)} \\ t_{(3b)} \text{ free} \\ (28),(29) \end{cases} \quad 2)\begin{cases} \min\limits_{x,u} \text{Jerk} \\ \text{subjected to} \\ \dot{x} = f(x,u) \\ t_{(1)} = t_{(1),fix} \\ t_{(2)} = t_{(2),fix} \\ t_{(3a)} = t_{(3a),fix} \\ t_{(3b)} = t_{(3b),fix} \\ (28),(29) \end{cases} \quad 3)\begin{cases} \min\limits_{x,u} \text{Jerk} + \delta\, m_{mf} \\ \text{subjected to} \\ \dot{x} = f(x,u) \\ t_{(1)} = t_{(1),fix} \\ t_{(2)} = t_{(2),fix} \\ t_{(3a)} = t_{(3a),fix} \\ t_{(3b)} = t_{(3b),fix} \\ (28),(29) \end{cases} \tag{30}$$

where $t_{(1),(2),(3a),(3b)}$ denote the duration of gearshift phases, δ is the penalty constant, $t_{i,min}$ are minimum durations for each phase representing the dynamics in the transmission hydraulic actuation mechanism, and $t_{i,fix}$ are to avoid extremely long gearshifts when minimizing the Jerk. In all three OCPs it is assumed that the synchronization phase duration is a constant.

3.6 Solution the OCP

CasADi software package, Andersson (2013), is used for solving the formulated OCPs. System states, control inputs, time varying constraints and objective function are discretized according to the direct multiple shooting scheme where the dynamics in each shooting interval are integrated by fourth order Runge-Kutta method. Then, a Nonlinear Programming (NLP) problem is formulated and solved by IPOPT, Wächter and Biegler (2006).

4. RESULTS

Selected results from the numerical optimal control solutions are presented and analyzed in this section. At first, it is shown how the objective function formulation affects time-fuel-Jerk properties. Then, the powertrain transients are presented and analyzed for a gearshift case where there is a compromise between time-fuel-Jerk.

4.1 Compromise between time, Jerk and fuel consumption

Figure 6 shows the trade-off between gearshift duration and Jerk. The left-most point on the trade-off, which is not shown due to very large Jerk, is obtained by solving 1) in (30), while the rest of points are obtained by solving 2) in (30) by increasing the $t_{(1),fix}$, $t_{(3a),fix}$ and $t_{(3b),fix}$. The Jerk is drastically reduced when longer gearshift durations are chosen. However, the obtained trajectories, which are the focus of the next section, for the system transients are not proper in a sense that the control inputs are oscillatory specially during the torque phase. The transients corresponding to the low Jerk gearshift in Figure 6 are presented in Figure 9.

Figure 7 shows the effect of adding fuel penalty to the Jerk minimization problem, solving 3) in (30). Table 3 shows the changes in fuel consumption and Jerk in all gearshift phases for the two highlighted points in Figure 7. Compared to the case where only Jerk is minimized, by adding a fuel penalty into the problem formulation, fuel consumption is reduced up to 3.57 % in expense of only 0.29 % more Jerk. However, the trend in fuel consumption

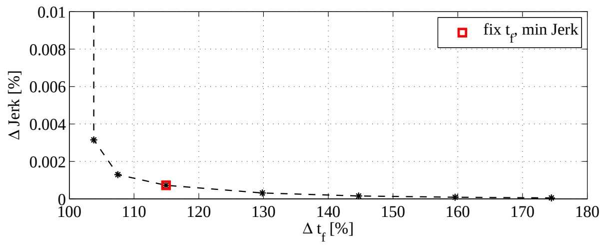

Fig. 6. Jerk is extensively reduced when gearshift duration increases slightly.

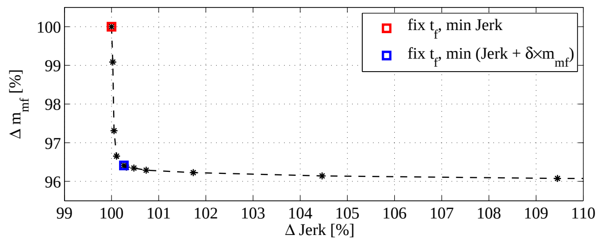

Fig. 7. Changes in fuel consumption and Jerk when increasing the fuel penalty in Jerk minimization problem.

reduction is not the same for all of the gearshift phases. Therefore, a more detailed comparison of the transients before and after including the fuel penalty is performed in the following section.

phase	Δm_{mf}(%)	Δ Jerk (%)
1	-3.2	5.8
2	27.9	16
3	-2.6	0.44
4	-12.6	-0.52
Total	-3.57	0.29

Table 3. Fuel consumption and Jerk changes in each phases of the gearshift when fuel penalty is added.

4.2 Jerk optimal transients, with and without fuel penalty

Figure 8 shows the electrical energy and fuel consumption comparison, and Figure 9 shows the states and controls. There is no big difference in terms of Jerk when fuel penalty is added into the problem formulation. In addition to the numbers in Table 3, this is also qualitatively verifiable by comparing ω_{tr} trajectories in Figure 9. The oscillations in u_{mf} and P_{gen} during the torque phase take place because the objective function is not sensitive to such oscillations. By adding the fuel penalty term into the problem, the oscillations become costly in terms of fuel consumption and thus are avoided. The resulting trajectories also become smoother which are preferred from the controller design perspective.

The second largest difference when adding the fuel penalty occurs during the synchronization phase. Fuel consumption increases by 27.9 %, ω_e is considerably lower and P_{gen} has a larger peak. This results in higher available electrical energy at the beginning of the backlash traverse and inertia phases. The extra available electrical energy reduces the need to the diesel engine power and lowers the fuel consumption.

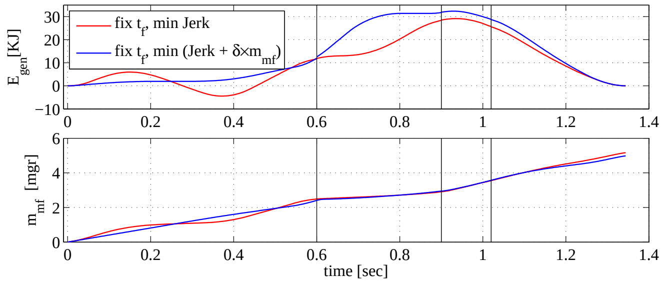

Fig. 8. Comparison of fuel consumption and electrical energy usage with and without including fuel penalty in Jerk minimization problem.

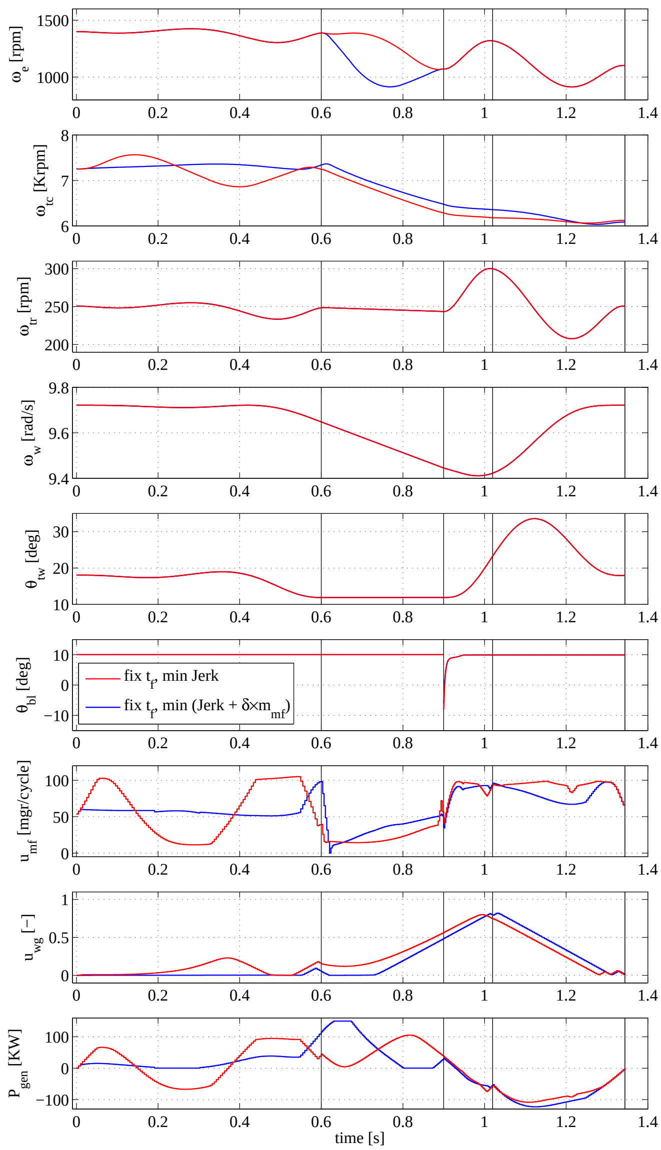

Fig. 9. Optimal state and control input transients with and without including fuel penalty in the Jerk minimization.

Considering ω_{tr} transients, for which Jerk is minimized, it is important that when the transmission is decoupled at the end of the torque phase, ω_{tr} reaches this point from a lower speed while θ_{tw} is also reduced to match the drive-

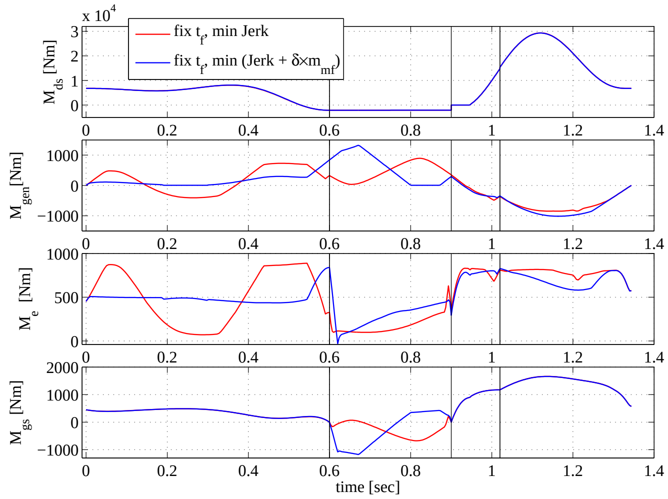

Fig. 10. Comparison of driveshaft torque M_{ds} and the `genset` torque M_{gs} in min t_f and low Jerk transients. The dashed lines mark the boundary times of the gearshift phases.

shaft twist at the beginning of the synchronization phase. Otherwise, the remained torsional energy in the driveshaft stored as a larger θ_{tw} twist, induces large oscillations in ω_{tr} when transmission sides become decoupled.

The backlash traverse phase is almost unchanged with or without including the fuel penalty. Since Jerk should be minimized in both problem formulations, backlash is smoothly traversed when close to the contact mode while it is rapidly traversed at the beginning of the phase. Moreover, the backlash remains at the right contact mode during the torque and synchronization phases which shows that the backlash dynamics could as well be removed during these phases in order to reduce the computation costs.

The torque transients compared in Figure 10 show that the delivered torque to the wheels M_{ds} are almost identical in both cases. The unchanged M_{ds} while there are large differences in the engine and generator torque transients yet again shows that the Jerk minimization problem without considering fuel consumption effects is insensitive to the torque oscillations inside the `genset`.

The point to consider here is that directly after having a small contribution of Jerk in time minimization or fuel penalty in Jerk minimization, the results are extensively affected, e.g consider the difference between first points and rest of the points in Figures 6 and 7, and also the big difference between transients. This makes it important to have a representation of all important properties in the formulation of the objective function such that the results are not representative of only extreme conditions.

5. CONCLUSIONS

A diesel-electric powertrain including backlash is modeled with the aim to perform gearshift optimal control analysis. Powertrain backlash nonlinearity is modeled using physical models. In order to avoid multiple backlash definitions at various backlash positions, a continuously differentiable switching function is utilized covering different backlash positions. The optimal transients for a compromise between transmission jerk, gearshift duration and fuel consumption are calculated using numerical optimal control methods.

The results show that the the choice of a switching function efficiently describes the backlash traverse dynamics. Moreover, it is shown that using only one property such as Jerk or time in the objective function of optimal control problem formulations, results in complex control trajectories representing only extreme cases. This can be avoided by proper formulation of the optimization criteria e.g by adding fuel penalty term into the Jerk minimization problem objective.

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