# Three Way Catalyst Control using PI-style Controller with HEGO Sensor Feedback

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## Abstract

*Recent research has shown that control of the oxygen content in the catalyst has potential to further reduce the emissions from spark ignited engines. This gives rise to a cascade structure where an outer loop influences an inner loop. Different ways of augmenting the inner loop, a traditional PI-feedback controller based on feedback from the binary oxygen sensor, are studied. The SI-engine constraints on the control, such as low emissions and drive ability, are considered in the evaluation of the controllers.*

*The result is that a delayed switching of the sensor is needed to control the oxygen content in the TWC using binary sensor feedback.*

**Keywords:Air/fuel control, HEGO, TWC**

## 1 Introduction

Emissions from spark ignited (SI) engines are today effectively reduced by a three way catalysts (TWC) [5, 11, 7]. A TWC is most efficient when the air/fuel ratio, $\lambda$, is kept within a narrow region around $\lambda = 1$, see Figure 1. Feedback control of $\lambda$ is necessary for reaching this accuracy, due to unmeasurable disturbances and model errors. Therefore there is a $\lambda$-sensor before the TWC. There are also legislative demands that the catalyst must be diagnosed and this requires that a second $\lambda$-sensor is placed after the catalyst.

In most engine control systems of today the air/fuel ratio is controlled by a PI-style controller with feedback from the switch-type heated exhaust gas oxygen sensor (HEGO) located before the TWC [9]. Recent studies [13, 10] have shown that emissions can be reduced even further using feedback from the HEGO placed after the TWC for diagnostic reasons. Such a system gives a cascaded control structure, see Figure 2. In this case there are two controllers, the outer loop gives the set-point $\lambda_{\text{ref}}$ to the inner loop. During normal operation the catalyst removes and

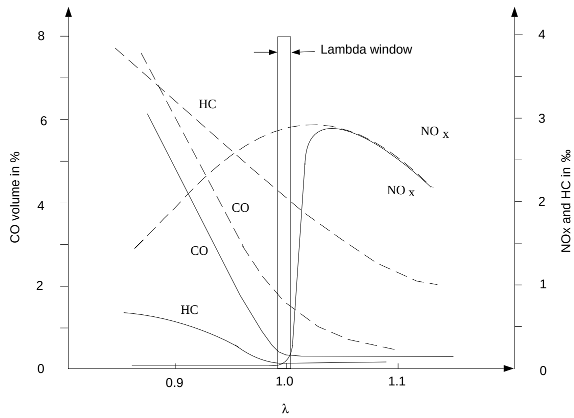


**Figure 1. Dashed: Emissions before the catalyst. Solid: Emissions after the catalyst. When $\lambda$ is close to unity the three way catalyst (TWC) effectively removes the three pollutants CO, HC, and $NO_x$.**

stores oxygen directly available in the gas and from $NO_x$ and then uses it to oxidize e.g. CO to $CO_2$. When the catalyst is full of oxygen $NO_x$ will pass through untreated and this is seen in the second HEGO as a voltage drop. The control goal is to maintain a sensor output of 0.6 V from the second sensor, which ensures that the catalyst is operating under optimal conditions.

An example of the effect of disturbances are shown in Figure 3, where only the inner air/fuel ratio controller is running. It can be seen that the output of the rear sensor drops from 0.6 V at several occasions. This experiment motivates the outer loop but the important issue is how the inner loop shall be designed to effectively control $\lambda$ to the catalyst.

In this study the focus is on the inner loop, which consists of a system with a PI-controller with relay feedback. And the aim is to study how the mean of $\lambda_{\text{engine}}$ and thereby the air/fuel ratio fed to the TWC can be changed with dif-

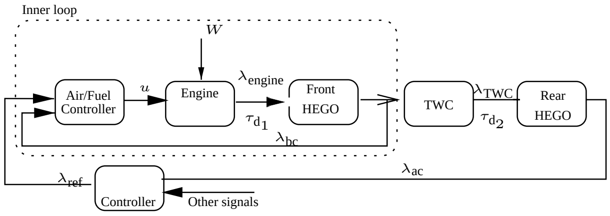


**Figure 2. The engine produces exhaust gases and its oxygen content is measured by a discrete sensor $\lambda_{\mathbf{bc}}$ which is fed back to the air/fuel ratio controller (inner loop). There are unmeasureable disturbances $W$ that offset the desired air/fuel ratio. After the TWC an additional discrete oxygen sensor $\lambda_{\mathbf{ac}}$ is placed for diagnostic. The transport delay to the first sensor is $\tau_{\mathbf{d}_1}$ and the transport time through the TWC is $\tau_{\mathbf{d}_2}$. The output of the air/fuel controller $u$ controls the mass of injected fuel.**

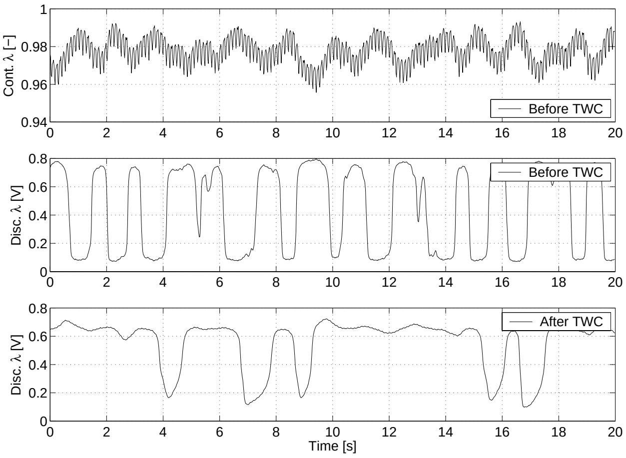


**Figure 3. Measured air/fuel ratio of an engine running at idle. Top: Continuous $\lambda$, which oscillates with an amplitude of around 1%. In the measurement there is a sensor offset that drives the measured value from 1. Center: Measured $\lambda$ in volts from an HEGO placed in front of the TWC. The oscillations are caused by the limit cycle of the controller. Bottom: Measured $\lambda$ in volts from the HEGO behind the TWC. Note the voltage fluctuations.**

ferent PI-controllers. The different proposed strategies are tested against four performance measures which takes into account the special requirements for SI-engines, such as low emissions and good driveability.

## 2 Performace Measures of Air/Fuel Ratio Controllers

For comparison of the different strategies four performance measures are introduced. To describe the first performance measure a brief introduction to TWCs is given.

Very simplified the TWC can be seen as an oxygen storage, which have modeled for control purposes [8]. As the rate of which the TWC is filled and emptied of oxygen depends on the net supply of oxygen, which can be expressed as $W_a\,(\lambda - 1)$, where $W_a$ is the air-mass flow. To further simplify the filling and emptying calculations the mean oxygen flow

$$W_a\left(\overline{\lambda} - 1\right) \tag{1}$$

is used instead. This simplification can be motivated by small net filling/emptying during the limit cycle of the air/fuel ratio controller before the TWC. In this study of mean air/fuel ratio, stationary conditions are assumed, that is constant speed and constant air-mass flow. Also the effects of Aquino style fuel dynamics [1] and sensor dynamics is not accounted for.

In Equation (1) the mean filling and emptying rate is expressed in terms of air-mass flow and mean air/fuel ratio. As the air-mass flow is assumed to be constant the filling/emptying rate depends on the mean air/fuel ratio $\overline{\lambda}$:

$$\overline{\lambda} = -1 + \frac{1}{T}\int_0^T \lambda_{\text{engine}}\mathrm{d}t \tag{2}$$

$T$ in Equation (2) is the limit cycle time of the air/fuel ratio controller. Therefore $\overline{\lambda}$ is the most important performance measure of how fast the TWC can be filled and emptied. In the performance measures $\overline{\lambda}$ is calculated through the areas of lean and rich mixtures:

$$\overline{\lambda}^{+} = \int_{\lambda_{\text{engine}}\geq 0,\, 0<t<T} \lambda_{\text{engine}}\mathrm{d}t \tag{3}$$

$$\overline{\lambda}^{-} = \int_{\lambda_{\text{engine}}< 0,\, 0<t<T} \lambda_{\text{engine}}\mathrm{d}t \tag{4}$$

For drive ability reasons it is desirable to keep the maximum deviations from stoichiometric as small as possible since the deviations results in torque fluctuations [10]. Torque fluctuations is a result of changes in the amount of energy supplied but also affected is the ratio of specific heats which changes the efficiency [7, pp. 182] and the air/fuel ratio dependency of burn angles [7, pp. 403].

$$\lambda_{\max} = -1 + \max_{0<t<T} \lambda_{\text{engine}}(t) \tag{5}$$

$$\lambda_{\min} = -1 + \min_{0<t<T} \lambda_{\text{engine}}(t) \tag{6}$$

# 3 Characterization of the HEGO

The HEGO is a discrete sensor and detects only the absence of oxygen in the exhaust gas. Its voltage jump is very narrow around stoichiometric, as can bee seen in Figure 4. In [2] a more detailed description is given of where the voltage jump occurs as a function of the air/fuel ratio. Using this knowledge of the voltage jump around stoichiometric the sensor output is thresholded which gives the following ideal relay characteristics:

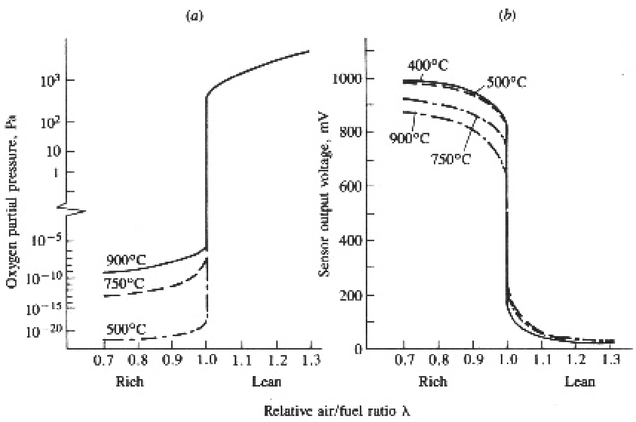


**Figure 4. Sensor characteristics as a function of air/fuel ratio, from [7]. Note the voltage jump at $\lambda = 1.0$.**

$$\lambda_{\text{bc}} = \begin{cases} -1, & \lambda_{\text{engine}}\left(t - \tau_{\text{d}_1}\right) > 1 \\ 1, & \lambda_{\text{engine}}\left(t - \tau_{\text{d}_1}\right) \leq 1 \end{cases} \quad (7)$$

The time-delay from the engine to the first sensor $\tau_{\text{d}_1}$ is also included.

# 4 Air/Fuel Ratio Control

This section focuses on the inner loop in Figure 2. Lets start by redefining $\lambda_{\text{ref}}$ by removing the bias of the control signal, that is $\lambda_{\text{ref}}$ =0 means stoichiometric conditions. This results in the following input to the PI-controller:

$$e(t) = \lambda_{\text{ref}} - \lambda_{\text{bc}}(t - \tau_{\text{d}_1}) \quad (8)$$

where $\lambda_{\text{bc}}$ follows the sensor characteristics given by Equation (7). The resulting controller equation is then:

$$u = K_p e(t) + \int K_i e(t) \mathrm{d}t \quad (9)$$

## 4.1 Stoichiometric Air/Fuel Ratio Control

Before the non-stoichiometric set-points are studied the controller parameters and the properties of the $\lambda_{\text{engine}}$ oscillation is studied as these parameter settings are necessary as default values for the non-stoichiometric controllers. A desired stationary oscillation amplitude $A$ under a time delay $\tau_{\text{d}_1}$ occurs when the controller parameters are set to:

$$K_I = \frac{A}{\tau_{\text{d}_1}} \quad (10)$$

$$K_p = \frac{1}{2} K_I \tau_{\text{d}_1} \quad (11)$$

With the selection above the controller minimizes the limit-cycle. The resulting output in $\lambda$ is shown in Figure (5). For discrete implementations of this structure some modifications can be made to improve stability [12]. The performance measures of this PI-controller are:

$$\overline{\lambda} = 0 \qquad T = 2\tau_{\text{d}_1}$$
$$\lambda_{\max} = K_I \tau_{\text{d}_1} \quad \lambda_{\min} = -\lambda_{\max}$$

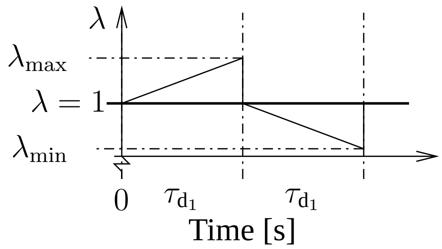


**Figure 5. $\lambda_{\text{engine}}$-output with PI-control and $K_p$ and $K_I$ calculated as described in Equations (10,11). $\lambda_{\max} = A = -\lambda_{\min}$. The time delay until the mixture reaches the sensor is $\tau_{\text{d}_1}$.**

## 4.2 Non-Stoichiometric Air/Fuel Ratio Control

Two methods of achieving non-stoichiometric air/fuel ratio control are tested. The first method is called the straight forward approach which means that the performance measures are calculated for different values of $\lambda_{\text{ref}}$ in Equation (8). The second approach is to modify Equation (8) and make the parameters $K_p$ and $K_I$ dependent of $\lambda_{\text{ref}}$. In the later case delayed switching is also introduced.

## 4.3 The Straight Forward Approach

By changing $\lambda_{\text{ref}}$ in Equation (8) to values between $-1 < \lambda_{\text{ref}} < 1$ it results in the following equations:

$$u = K_p(\lambda_{\text{ref}} + 1) + \int K_I(\lambda_{\text{ref}} + 1), \quad \lambda_{\text{engine}}\left(t - \tau_{\text{d}_1}\right) > 1 \quad (12)$$

$$u = K_p(\lambda_{\text{ref}} - 1) + \int K_I(\lambda_{\text{ref}} - 1), \quad \lambda_{\text{engine}}\left(t - \tau_{\text{d}_1}\right) \leq 1 \quad (13)$$

The input to the controller $e$ scales the parameters $K_p$ and $K_I$ differently depending on the direction of the step which results in the following performance measures when $\lambda_{\text{ref}} \geq 0$:

$$\begin{aligned}
\lambda_{\min} &= K_I(\lambda_{\text{ref}} - 1)\tau_{\text{d}_1} \\
\lambda_{\max} &= \lambda_{\min} + 2K_p + K_I(\lambda_{\text{ref}} + 1)\tau_{\text{d}_1} \\
\overline{\lambda}^{+} &= (\lambda_{\min} + 2K_p)\tau_{\text{d}_1} + \frac{K_I \tau_{\text{d}_1}^2}{2} + \frac{(\lambda_{\max} - 2K_p)\tau_{\text{d}_1})^2}{-2K_I(\lambda_{\text{ref}} - 1)} \\
\overline{\lambda}^{-} &= \frac{1}{2} K_I(\lambda_{\text{ref}} - 1)\tau_{\text{d}_1}^2 \\
T &= 2\tau_{\text{d}_1} + \frac{\lambda_{\max} - 2K_p)\tau_{\text{d}_1}}{-2K_I(\lambda_{\text{ref}} - 1)}
\end{aligned}$$

The case for $\lambda_{\text{ref}} < 0$ is similar.

### 4.4 Parameter Variations and Delayed Switching

Instead of feeding the controller with Equation (8), the relay signal $\lambda_{\text{bc}}$ is fed directly into the controller and the parameters depend on $\lambda_{\text{ref}}$. When Equation (9) is adapted for parameter change it can be summarized in two condition, when for $\lambda_{\text{bc}}(t - \tau_{\text{d}_1}) = -1$ as

$$u_u = K_{p_u}(\lambda_{\text{ref}}) + \int K_{I_u}(\lambda_{\text{ref}})\text{d}t \tag{14}$$

and secondly when $\lambda_{\text{bc}}(t - \tau_{\text{d}_1}) = 1$ the equation for the controller is

$$u_d = -\left(K_{p_d}(\lambda_{\text{ref}}) + \int K_{I_d}(\lambda_{\text{ref}})\text{d}t\right) \tag{15}$$

The second method to change the mean air/fuel ratio is introduction of the delayed switching. Here delayed switching means that the output of the controller is held constant during the delay, which is illustrated in Figure 6 with different delay times in lean and rich conditions.

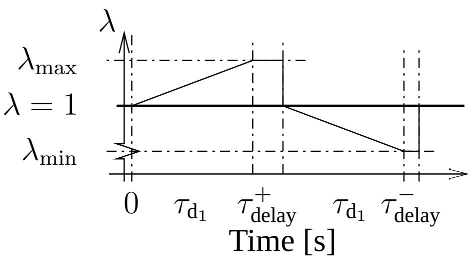


**Figure 6. Delayed switching with time $\tau^{+}_{\text{delay}}$ during lean conditions and $\tau^{-}_{\text{delay}}$ during rich conditions.**

The study of these methods starts with adjustment of the $K_I$ parameter and introduction of delayed switching in an I-style controller and continues with the PI-case.

When pure I-control is used there are two methods to control $\overline{\lambda}$, first the parameter $K_I$ can be changed into:

$$K_I(\lambda_{\text{ref}}) = \begin{cases} K_{I_u} & \text{When the mixture is leaned} \\ K_{I_d} & \text{When the mixture is enriched} \end{cases} \tag{16}$$

To further change $\overline{\lambda}$ a delayed switching can be introduced. With theses two modifications the performance measures becomes:

$$\begin{aligned}
\lambda_{\min} &= K_{I_d}\tau_{\text{d}_1} \\
\lambda_{\max} &= K_{I_u}\tau_{\text{d}_1} \\
\overline{\lambda}^{+} &= \frac{K_{I_u}\tau_{\text{d}_1}}{2} + \frac{K_{I_u}\tau_{\text{d}_1}}{2K_{I_d}} + \tau^{+}_{\text{delay}} K_{I_u}\tau_{\text{d}_1} \\
\overline{\lambda}^{-} &= \frac{-K_{I_d}\tau_{\text{d}_1}}{2} - \frac{K_{I_d}\tau_{\text{d}_1}}{2K_{I_u}} - \tau^{-}_{\text{delay}} K_{I_d}\tau_{\text{d}_1} \\
T &= 4\tau_{\text{d}_1} + \tau^{+}_{\text{delay}} + \tau^{-}_{\text{delay}}
\end{aligned}$$

$$\begin{aligned}
\overline{\lambda} = &\frac{1}{T}\left(\frac{(K_{I_u} - K_{I_d})\tau_{\text{d}_1}^2}{2}\right) + \\
&\frac{1}{T}\left(\frac{K_{I_u}\tau_{\text{d}_1}}{2K_{I_d}} - \frac{K_{I_d}\tau_{\text{d}_1}}{2K_{I_u}} + (\tau^{+}_{\text{delay}} K_{I_u} - \tau^{-}_{\text{delay}} K_{I_d})\tau_{\text{d}_1}\right)
\end{aligned}$$

In pure I-control different $K_I$ parameters changes the performance measures according to the equations above. What is the effect of changing the $K_p$-parameter? For the PI-controller the effects of changing the $K_p$ parameter is studied using a fix $K_I$.

$$K_p(\lambda_{\text{ref}}) = \begin{cases} K_{p_u} & \text{When the mixture is leaned} \\ K_{p_d} & \text{When the mixture is enriched} \end{cases}$$

To study this the result of different $K_p$-parameters Figure 5 is used. Just before $\tau_{\text{d}_1}$ the value of $\lambda$ is $\lambda(\tau^{-}_{\text{d}_1}) = K_{p_u} + \tau_{\text{d}_1} K_{I_u}$ and just after $\tau_{\text{d}_1}$ the value of the I-part is unchanged, $K_{I_u}\tau_{\text{d}_1}$, but $\lambda$ is $\lambda(\tau^{+}_{\text{d}_1}) = -K_{p_d} + \tau_{\text{d}_1} K_{I_u}$. The difference between the points is therefore always the sum of the parameters $-(K_{p_d} + K_{p_u})$. This also means that having separate proportional constants $K_{p_u}$ and $K_{p_d}$ give no advantages over one $K_p$ parameter and therefore only changes of $K_p$ is studied.

Here one way of selecting $2K_p = \min(K_{I_u}, K_{I_d})\tau_{\text{d}_1}$ $\lambda$ is studied. This selection results in following performance measures:

$$\begin{aligned}
\lambda_{\min} &= -K_{I_d}\tau_{\text{d}_1} \\
\lambda_{\max} &= K_{I_u}\tau_{\text{d}_1} \\
\overline{\lambda}^{+} &= \begin{cases} K_{I_u}\frac{\tau_{\text{d}_1}^2}{2} + \frac{(K_{I_u}\tau_{\text{d}_1} - 2K_p)^2}{2K_{I_d}} + K_{I_u}\tau_{\text{d}_1}\tau^{+}_{\text{delay}}, & K_{I_u} > K_{I_d} \\ K_{I_u}\frac{\tau_{\text{d}_1}^2}{2} + K_{I_u}\tau_{\text{d}_1}\tau^{+}_{\text{delay}}, & K_{I_u} \leq K_{I_d} \end{cases} \\
\overline{\lambda}^{-} &= \begin{cases} -K_{I_d}\frac{\tau_{\text{d}_1}^2}{2} - \frac{(K_{I_d}\tau_{\text{d}_1} - 2K_p)^2}{2K_{I_u}} - K_{I_d}\tau_{\text{d}_1}\tau^{-}_{\text{delay}}, & K_{I_d} > K_{I_u} \\ -K_{I_d}\frac{\tau_{\text{d}_1}^2}{2} - K_{I_d}\tau_{\text{d}_1}\tau^{-}_{\text{delay}}, & K_{I_d} \leq K_{I_u} \end{cases} \\
\overline{\lambda} &= \frac{1}{T}\left(\overline{\lambda}^{+} - \overline{\lambda}^{-}\right) \\
T &= 2\tau_{\text{d}_1} + \frac{K_{I_u}\tau_{\text{d}_1} - 2K_p}{K_{I_d}}
\end{aligned}$$

There are several more ways of selecting the parameters $K_p$ and $K_I$ as a function of $\lambda_{\text{ref}}$, but the example above serves more to show how different parameters can be used together with delayed switching.

## 5 Evaluation of Manipulated PI-control

Three methods have been described of how $\overline{\lambda}$ can be changed using different parameters in a PI-style controller. To evaluate each methods performance measures, the values of the parameters remains to be determined. Starting with the time delay, $\tau_{\mathrm{d}_1}$, which can be estimated using an approximation [9, pp. 77] as the summed time of one induction, compression and expansion event. With these assumptions $\tau_{\mathrm{d}_1} \approx \frac{3}{2N}$, where $N$ is the engine speed in revolutions per second. Given that normal engine operating speeds are between 800 RPM and 6000 RPM this corresponds to time delays between $15 \leq \tau_{\mathrm{d}_1} \leq 110$ ms. In [4] this approximation is supported but a dependency of the pressure ratio between the intake manifold and exhaust manifold can be included to improve the approximation.

Next the maximum deviation from the stoichiometric condition is to be selected, which also is a performance measure. High amplitudes can result in torque fluctuations and less good conversion efficiency of the TWC, therefore normally used amplitudes are less than 3% [3, pp. 493] off from stoichiometric.

To determine maximum delayed switching time the cycle time $T$ of the air/fuel ratio-controller is needed. To set this parameter Figure 2 is studied, where two controllers are cascaded. The air/fuel ratio controller is the inner loop and the reference value is set by the outer loop. As the inner loop has to be considerably faster than the outer [6, pp. 127] and the outer loop time is governed by the TWC and to fill or empty the TWC takes approximately between 3 and 15 seconds. A rule of thumb is to choose the inner loop 10 times faster than the outer which results in a maximum period time of at least 300 ms for the air/fuel-ratio controller. In [10] another method to determine the amplitude and cycle time is discussed that relies on the oxygen storage capacity of the TWC.

To set $K_p$ and $K_I$ the nominal values are used from Equation (10) and (11) respectively. An assumption made in the calculations of the maximum deviation from stoichiometric $\overline{\lambda}$, is when $K_{I_u}$and $K_{I_d}$are different then $K_{I_d} = \frac{1}{3}K_{I_u}$ or vice versa. These parameters can also be calculated using Equation (8) and Equation (9) in the following way given $\lambda_{\mathrm{ref}} > 0$: $K_{I_u} = K_I(\lambda_{\mathrm{ref}} + 1)$ and $K_{I_d} = K_I(\lambda_{\mathrm{ref}} - 1)$ with care taken that the limits $\lambda_{\min}$ and $\lambda_{\max}$ are not exceeded.

Calculations of $\overline{\lambda}$ with the assumptions that $\tau_{\mathrm{d}_1}$ is governed by the engine speed, a maximal desired deviation from stoichiometric, and a maximal loop time $T$ the resulting $\overline{\lambda}$ offset from stoichiometric has been made for the described methods. The offset from stoichiometric has only been calculated for the lean side as the same correction can be made in the rich direction.

First the straight forward approach is tested and the resulting performance measures are shown in Figure 7. In the middle plot it can be seen that the amplitude $\lambda_{\max}$ exceeds the desired maximum deviation from stoichiometric for $\lambda_{\mathrm{ref}} > 0$, which makes this method unsuitable for TWC control as the torque fluctuations would be too large. The maximum deviation from stoichiometric $\overline{\lambda}$ is also small even for large $\lambda_{\mathrm{ref}}$. In Figure 8 an I-controller with different $K_{I_u}$, $K_{I_d}$and delayed switching is shown, where the maximum amplitude is fulfilled all the time and delayed switching up to the period time T is used. With this method it is possible to reach mean deviations in $\overline{\lambda}$ of up to almost the amplitude of the oscillation. Finally in Figure 9 the deviation from stoichiometric has been accomplished using different $K_{I_u}$, $K_{I_d}$, $K_p$ together with delayed switching. This arrangement gives a further increase in $\overline{\lambda}$ at low engine speeds and short limit-cycle times $T$ compared to the previous methods. However the increase in $\lambda_{\mathrm{engine}}$ is small, less than $0.01$ and the advantage of having separate parameters is small compared to the delayed switching.

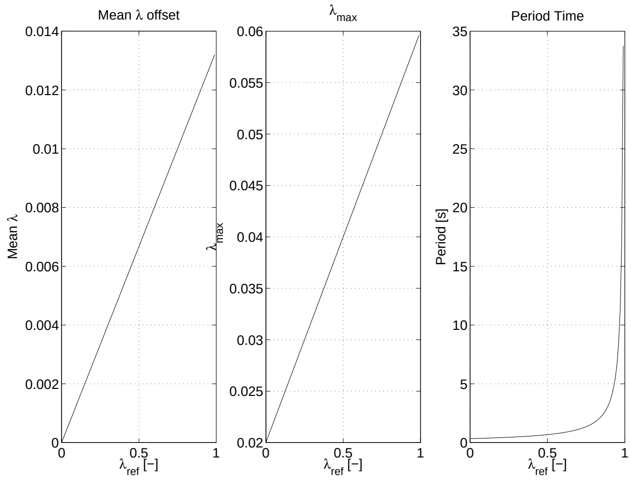


**Figure 7. Result using the straight forward approach with varying $\lambda_{\mathrm{ref}}$ using $\tau_{\mathrm{d}_1}$ = 110 ms, and A = 0.02. To the left the maximum obtainable $\overline{\lambda}$ is shown and in the middle $\lambda_{\max}$ is shown. The maximum amplitude exceeds the desired amplitude of 0.02 for $\lambda_{\mathrm{ref}} > 0$. To the right the period time $T$ is shown which is small for $\lambda_{\mathrm{ref}} < 0.8$.**

The straight forward approach for non-stoichiometric air/fuel ratio control with the introduction of delayed switching in the form of an output saturation to the desired maximum amplitude would improve that method significantly.

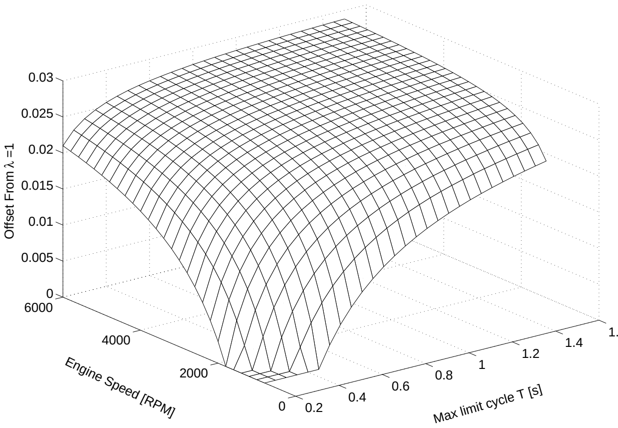


**Figure 8. Result using an I-controller with delayed switching. The figure shows the maximum mean deviation from stoichiometric with these settings.**

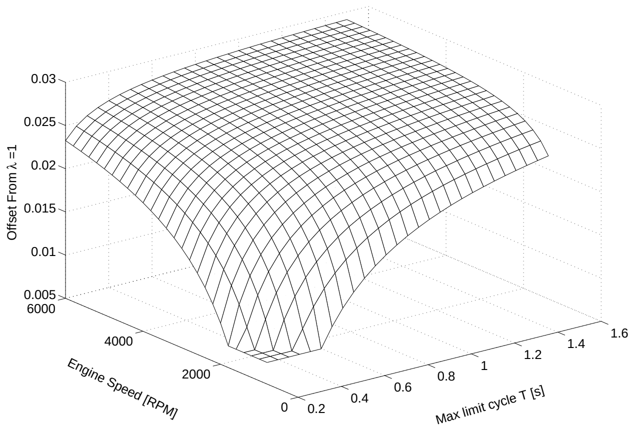


**Figure 9. Result using $2K_p = \min(K_{I_u}, K_{I_d})\tau_{\mathbf{d}_1}$ and delayed switching. Here $K_{I_d} = \frac{1}{3}K_{I_u}$. The figure shows the maximum mean deviation from stoichiometric with these settings.**

# 6 Conclusions

The mean air/fuel ratio $\overline{\lambda}$ into the TWC can be controlled using binary feedback from an HEGO to a modified PI-style controller. The straight forward approach with a standard controller only gives a negligible deviation from stoichiometric. To achieve a non-stoichiometric $\overline{\lambda}$, the best way to do this is to introduce a delayed switching. Also noteworthy is that using two different parameters $K_{p_u}$ and $K_{p_d}$ gives no additional advantages over having one single $K_p$.

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