

Smart Energy Usage for Vehicle Charging and House Heating

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Abstract: In northern Europe the electricity price is set by hourly rates one day in advance. The price fluctuates due to supply and demand, and these fluctuations are expected to increase when solar and wind power are increased in the energy system. The potential in cost reduction for heating a house and charging of an electrified vehicle by using a smart energy management system in a household is investigated. Dynamic programming is used and a simulation study of a household in Sweden comparing this optimal control scheme with a heuristic controller is carried out. The time frame in the study is one year and a novel way of handling the fact that the vehicle is disconnected from the grid at some times is developed. A plug-in hybrid electric vehicle is considered, but the methodology is the same also for pure electric vehicles. It is found that the potential in energy cost reduction for house heating and vehicle charging is significant and that using a smart energy management system is a promising path of cost reduction, especially with the introduction of electrified vehicles.

Keywords: Electricity price optimization, Electric vehicle, Plug-in electric vehicle, Household, Smart energy management

1. INTRODUCTION

Today, Swedish households can buy electricity with varying hourly rates, and these rates are determined for the coming day at lunchtime and are based on electricity producers' capacity to supply electricity and consumers' predicted demand. Since the electricity prices vary throughout the day and are known in advance, it is possible to reduce the cost of household electricity by moving a portion of the energy consumption to times when it is cheaper. Domestic heating and electric vehicles are examples of large electric loads where the energy consumption could be shifted in time. This paper studies how much electricity costs and energy consumption could be reduced for a household by introducing smart energy management based on, among other things, varying electricity prices and weather. How much the electricity price varies over the days differ, but in general the electricity price is cheaper during the nights and most expensive in the mornings and evenings, see Figure 1 where the average electricity price for each hour of the day during 2013 is presented.

Advanced energy management provides besides lowering the cost of electricity also other advantages. Electricity price and total power demand is strongly correlated as can be seen by comparing the electricity price in Figure 1 with the power consumption per hour in Figure 2. When electricity prices are high and demand for energy is high, back-up generators such as gas turbines and coal plants are used to a greater extent. A reduction of the energy output during these times makes the electricity production more environmentally friendly and less dependent on fossil fuels. In addition to lowering the cost of electricity the knowledge that electricity is produced in a more environmentally

friendly way could be an important incentive for consumers to invest in smart energy management. Furthermore, if many households start to use smart energy management it would have a peak shaving effect and the capacity of back-up power plants can be reduced.

A simulation study has been conducted to estimate the reduction in electricity costs for a household with an electric or plug-in hybrid electric vehicle. A large part of the energy consumption for a Swedish household is used for heating and the indoor temperature depends on several factors. Here we consider heating, household electricity, weather such as solar radiation and outside temperature, and the building's thermal inertia. In order to obtain a representative value that takes into account the effects of different seasons, the household has been simulated for one year with conventional heating and direct charging and compared it with the cost of electricity when smart energy management is used. Several approaches have been

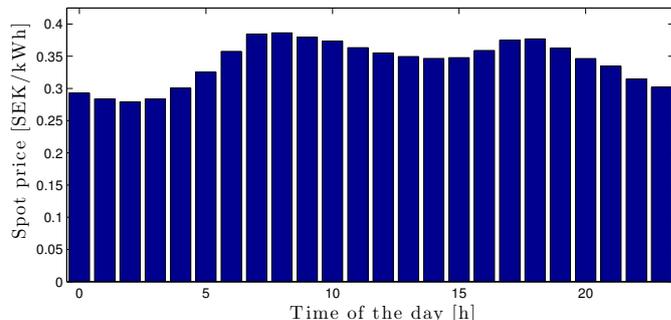


Fig. 1. Average spot price per hour of the day in the Swedish price area SE3 during 2013.

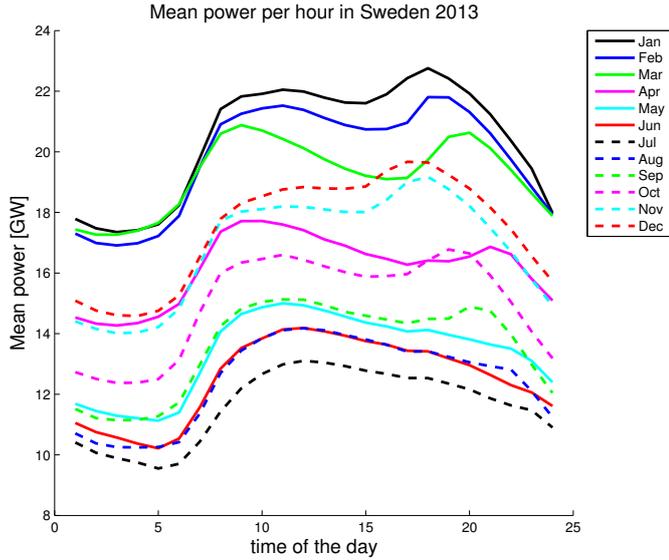


Fig. 2. Mean power consumption per hour in Sweden 2013 for different months.

used previously; In Logenthiran et al. (2012) heuristic optimization is used to control the domestic electricity demand, in Mohsenian-Rad et al. (2010) game theory is used to control the household loads, and in Sarabi and Kefsi (2014) dynamic programming is used to charge electric vehicles in order to reduce the overall peak power in the system. In this study dynamic programming is used to minimize the electricity cost, and to illustrate the optimization a typical Swedish house has been considered. Few houses in Sweden are equipped with air cooling systems and therefore cooling has not yet been considered.

The outline of the paper is as follows. First, Section 2 provides an overview of the household including an electric vehicle and a formalization of the energy management optimization problem to be solved. Section 3 describes a thermodynamic model of a house and an electric vehicle model designed with dynamic programming in mind. Section 4 describes and discusses how smart energy management is included in the dynamic programming framework such as vehicle charging, heating, and the choice of state and time discretization. The optimization assumes future electricity prices, weather, and household energy consumption to be known which gives an upper limit of the gain introducing smart energy management. Section 5 shows the results of simulations of a typical household with and without smart energy management. The optimal solution is shown and characteristics of it discussed. Finally some conclusions are drawn in Section 6.

2. PROBLEM FORMULATION

The purpose of the paper is to investigate possible electricity cost savings by introducing smart energy management to a household with an electric vehicle. This section provides a more detailed description of the studied optimization problem.

Figure 3 shows a control oriented view of a household including an electric vehicle. The control variables considered here are heating power P_{heat} used for controlling indoor

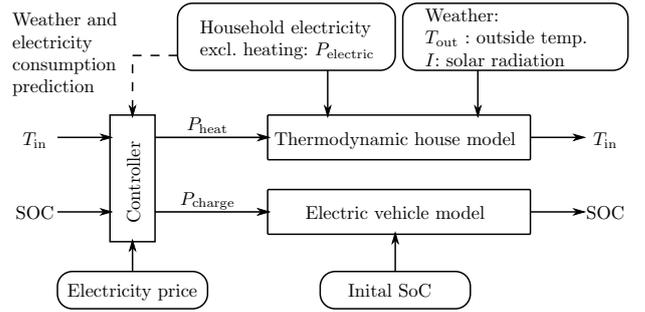


Fig. 3. Control oriented view of a household with an electric vehicle.

temperature T_{in} and vehicle charging power P_{charge} used for state of charge (SoC) control. The control objective is to minimize electricity cost while keeping the indoor temperature T_{in} above a specified minimum temperature and to reach a fully charged state of the electric vehicle before a user specified time. The total power consumption of the household is assumed to be bounded to a specified value P_{max} , i.e.,

$$P_{\text{max}} \geq P_{\text{heat}} + P_{\text{charge}} + P_{\text{electric}}$$

where P_{electric} denotes the total power of household electricity including all electric loads except for heating and vehicle charging.

Household electricity is from a control perspective considered as an uncontrollable disturbance both because it influences the power available for heating and charging, but also because it generates heat in the house. The weather condition is another disturbance influencing indoor temperature. Here two weather related signals are considered, the outside temperature T_{out} and the solar radiation intensity I .

The inputs to the controller are measured indoor temperature T_{in} , estimated state of charge SOC, future electricity price, measured household electricity power P_{electric} and outside temperature T_{out} . The energy management system could also benefit from using weather forecasts and predicted user behavior, such as predicted household electricity and vehicle charging behavior including when the vehicle is connected to the household and at what initial SoC and at what time the vehicle is expected to be fully charged.

In this paper the performance of an ideal control scheme is studied to investigate the potential savings for such a smart energy management system in terms of electricity cost. Therefore, dynamic programming is used to find the cost optimal solution given that weather, household electricity, and vehicle charging behavior is known by the controller in advance.

3. MODELS

This section describes the thermodynamic house model and the electric vehicle model that will be used for the electricity cost optimization of the household.

3.1 Thermodynamic house model

The thermodynamic house model is used to simulate the temperature variations with different heating control

systems. The computational complexity of dynamic programming is rapidly increasing with the number of states (Sniedovich, 2010). For that reason we use a thermodynamic house model with only one state, the indoor temperature T_{in} that is assumed to be equal throughout the house. The indoor temperature is affected by heating power P_{heat} , household electrical power P_{electric} , solar radiation power through windows $P_{\text{radiation}}$, heat losses due to transmission through walls etc P_{loss} , and the ventilation losses P_{vent} . If the thermal capacity of the house is R_{tot} we get

$$\frac{dT_{\text{in}}}{dt} = \frac{P_{\text{heat}} - P_{\text{loss}} + P_{\text{radiation}} + P_{\text{electric}} - P_{\text{vent}}}{R_{\text{tot}}} \quad (1)$$

The used model is similar to the model presented in (Muratori et al., 2012), but the model used here is easier to parametrize since the losses are modeled separately for different elements of the building.

3.2 Heat transfer losses

Heat loss P_{loss} through walls, windows, doors, ceilings, floors etc, is modeled as heat transfer by conductance. Different building elements have different transmission coefficient U_i and here we consider separate U-values for the roof, walls, windows, and floor. For a building element with conductance U_i and area A_i the heat transfer per unit time P_i is computed by Fourier's law as

$$P_i = A_i U_i (T_{\text{in}} - T_{\text{out}}) \quad (2)$$

where T_{in} is the indoor temperature and T_{out} is the outdoor temperature. When considering (2) for the floor the outdoor temperature is equal to the ground temperature. The total transmission heat loss is the sum of heat losses through each building block, i.e., $P_{\text{loss}} = \sum_i P_i$.

3.3 Ventilation losses

Heat loss due to ventilation with heat recovery can be expressed as

$$P_{\text{vent}} = V_{\text{air}} \gamma_{\text{vent}} \rho_{\text{air}} c_p (1 - \eta_{\text{vent}}) (T_{\text{in}} - T_{\text{out}}) \quad (3)$$

where V_{air} is the air volume in the house, γ_{vent} the air exchange rate, ρ_{air} the air density, c_p the specific heat capacity of air, and η_{vent} the heat recovery efficiency. Without heat recovery $\eta_{\text{vent}} = 0$ and for some heat recovery systems the efficiency can be greater than 0.8. Infiltration due to leakage in the building could partly be included in the ventilation losses. However, infiltration also depends on wind and this dependency has not been considered.

3.4 Solar radiation through windows

The total solar radiation through all windows in a house $P_{\text{radiation}}$ is the sum of radiation power through each window. The solar radiation power P through one window with area A directed with surface azimuth angle α measured clockwise from north can be computed as

$$P = IA \cos(\theta) g \quad (4)$$

where I is the solar radiation intensity, θ the angle of incidence, and g the fraction of radiation power transmitted through the window. The solar radiation intensity is dependent on the local weather conditions and historical data for locations in Sweden can be obtained from Swedish Meteorological and Hydrological Institute (SMHI).

The angle of incidence θ is a function of the surface azimuth angle α and can be computed as

$$\cos(\theta) = \sin(\delta) \cos(\phi) \cos(\alpha) - \cos(\delta) \sin(\phi) \cos(\alpha) \cos(\omega) - \cos(\delta) \sin(\alpha) \sin(\omega)$$

where ω is the hour angle in the local solar time, δ the current declination of the sun, and ϕ the local latitude.

The declination of Earth is computed by

$$\delta = 23.45^\circ \sin((n - 81)360^\circ/365) \quad (5)$$

where n is the day number of the year, such that $n = 1$ is the first of January. The hour angle is computed as

$$\omega = t \frac{360^\circ}{24} - 180^\circ \quad (6)$$

where t is the time of the day in hours.

The window transmission coefficient of radiation, g , depends on the inclination angle θ and on the design of the window. In (Caram et al., 2001; Karlsson and Roos, 2000) it is shown that the transmission coefficient is relatively constant for small angles and then drops fast to zero for large angles. A coarse model that capture this behavior has been adapted here. It assumes a constant transmission g_0 for inclination angles below a threshold θ_0 and zero transmission for angles above the threshold, i.e.

$$g = \begin{cases} g_0, & \theta < \theta_0 \\ 0, & \text{else} \end{cases} \quad (7)$$

3.5 Electric Vehicle Model

The electric vehicle is modeled with one state, the state of charge (SoC) of its battery pack x_{SOC} . The vehicle model describes the battery only when connected to the household energy management system, i.e., during charging. Let P_{charge} be the charging power of the vehicle and Q the energy capacity of the battery, then the SoC of the battery can be integrated by using

$$\frac{dx_{\text{SOC}}}{dt} = \eta \frac{P_{\text{charge}}}{Q} \quad (8)$$

where η is the charging efficiency.

4. METHOD

As stated earlier, the optimization method used to find the lowest energy cost for heating and electric vehicle charging is dynamic programming. The basic idea in this approach will first be described, and then some modifications to the standard algorithm to handle the fact the electric vehicle is not connected to the electric grid at all time points is described in more detail.

Using dynamic programming the time is discretized as well as the states and control signals. As stated in Section 3 the states $X \in \mathcal{R}^{m \times n}$ in the optimization are the indoor temperature T_{in} and the SoC x_{SOC} , and m and n are the number of discrete levels in respectively state. The control signals $U \in \mathcal{R}^{o \times p}$ are heating power P_{heat} and charging power P_{charge} and o and p the number of discretization points in the control signal respectively. The initial values of the states are denoted $T_{\text{in},0}$ and $x_{\text{SOC},0}$ and the final states should not be lower than these values.

The first step in dynamic programming is to find a minimal cost to go from each point in the grid of time points and

states. The algorithm starts at the end time point and the final cost, J_n , is assigned to avoid ending up in a too low state value. In the algorithm k is the time index, X_k the set of possible states at time index k , $g_k(u_k)$ the cost to go from time step k to time step $k+1$ where u_k is the control signal, $f_k(x_k^i, u_k)$ the state in time step $k+1$, and $J_{k+1}(f_k(x_k^i, u_k))$ is the cost to go from time instant $k+1$ to the end (i.e. time step N). To avoid ending up in a lower x_{SoC} than the initial SoC , denoted $x_{SoC,0}$ and in a lower indoor temperature T_{in} than the initial T_{in} , denoted $T_{in,0}$, J_N is assigned as in point 1 in the computations below

- (1) Let $J_N = \begin{cases} \infty, & x_{SoC} < x_{SoC,0} \text{ or } T_{in} < T_{in,0} \\ 0, & \text{else} \end{cases}$
- (2) Let $k = N - 1$.
- (3) For all $x_k^i \in X_k$:

$$J_k(x_k^i) = \min_{u_k \in U_k} \{g_k(u_k) + J_{k+1}(f_k(x_k^i, u_k))\}$$
- (4) Repeat 3. for $k = N - 2, N - 3, \dots, 0$.

where the value of $J_{k+1}(f_k(x_k^i, u_k))$ is found by bi-linear interpolation since this function only is known in the grid points of the state grids.

The cost to reach the final time step from each time and state grid point is known and stored in J . This information is now used to find the optimal state trajectory, x^* , and control signals, u^* , given an initial value of the states. This is done by starting at the first time step and proceed according the following algorithm

- (1) Let $k = 0$, $x_0^* = x_0$
- (2) Let $u_k^* = \operatorname{argmin}\{g_k(u_k) + J_{k+1}(f_k(x_k^*, u_k))\}$
- (3) Let $x_{k+1}^* = f(x_k^*, u_k^*)$
- (4) Repeat 2. and 3. for $k = 1, 2, \dots, N - 1$

where, once again, bi-linear interpolation is used to find the value of $J_{k+1}(f_k(x_k^*, u_k^*))$.

4.1 Constraints in SoC

There are several ways to implement that the vehicle should be fully charged before usage in the optimization. In e.g. Rotering and Ilic (2011) it is used that the vehicle is a plug-in hybrid vehicle since the difference between the electricity and gasoline costs, including powertrain efficiencies, is used to compute a final cost if the vehicle is not fully charged. Here it is instead stated that the vehicle must be fully charged at a time t_{fc} every day. This is implemented by that the cost to go to at time t_{fc} is infinite if $x_{SoC}(t_{fc}) < 1$. This results in that the only valid solution is that $x_{SoC}(t_{fc}) = 1$, but the discretization in the control signal, i.e. the charging power, leads to that this level most likely cannot be achieved in the optimization, see Figure 4. One possible solution is to charge the battery with a too high power to achieve a $x_{SoC} > 1$, but limit the state value so $x_{SoC}(t_{fc}) = 1$. However, this would result in that the cost to go to this time step is non-physical and too high. Therefore, the energy required to charge the battery to achieve $x_{SoC} = 1$ is computed if $x_{SoC} > 1$, even though the power required to achieve this is not included in the discretization set of the control signal. To illustrate this an example is used.

Example 1. Assume that $t_k = t_{fc}$, $P_{charge} \in \{0, 1, 2, 3\}$ kW, $x_{SoC,k-1} = 0.85$, $Q = 10$ kWh, and the sample time one hour. Charging with 1kW then results in $x_{SoC,k}^{1kW} = 0.95$ and charging with 2kW results in $x_{SoC,k}^{2kW} = 1.05$. Ending

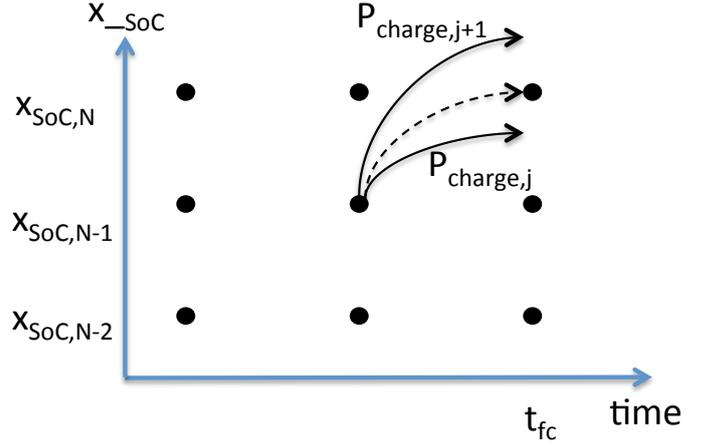


Fig. 4. Figure illustrating the aspect of achieving a fully charged vehicle when the control signals are discretized.

up in $x_{SoC,k}^{1kW} = 0.95$ is not a feasible solution since the battery is not fully charged. The solution $x_{SoC,k}^{2kW} = 1.05$ uses the lowest charging power to achieve $x_{SoC,k} \geq 1$, and the energy used in this time step is computed based on the difference between $x_{SoC,k-1}$ and $x_{SoC,k}$. In this case this results in a mean power of 1.5kW since the losses in the battery charging are constant.

The charging of the batteries in a vehicle can typically only be done at discrete powers. Therefore, it is assumed that the charging is made with the power $P_{charge,j+1}$ in Figure 4, where j is the index of possible charging powers, for some time in the time interval between t_{k-1} and t_k in the example above, and that the charging is switched off during the rest of the time. This may affect the solution since the maximum power in the household is limited. This is of course included both in the backward and forward computations in the algorithm.

4.2 Vehicle SoC after driving

At the time instant the car is coming back home after driving, t_{ad} , x_{SoC} needs to be reinitialized. The algorithm can handle a day to day variation of t_{ad} , but here a fixed value of t_{ad} is used for simplicity. The value of x_{SoC} at t_{ad} is assumed perfectly known, but to model different usage of the vehicle every day the value of $x_{SoC}(t_{ad})$ is set by using random numbers between zero and one that are available in the optimization. In the optimization $x_{SoC}(t_{ad})$ is set to the value of the SoC of the vehicle for that specific day.

5. RESULTS

The input data used in the simulations is from year 2013 and the location is Linköping. Measured outdoor temperature and solar radiation intensity are publically available and are collected from Swedish meteorological and Hydrological Institute (SMHI, 2015) and measured household electricity is provided by Tekniska verken that is a regional company providing electricity and district heating (Tekniska verken, 2015). The electricity spot price in Northern Europe is publicly available at Nord Pool Spot (Nordpool, 2015), and the model parameters used are given in Appendix A.

Two control schemes are compared. The first scheme is a heuristic controller and the second scheme is based on dynamic programming finding the globally minimal cost to fulfill the criteria on charging and indoor heating. Both these schemes are based on the same states, T_{in} and x_{SoC} , which are computed based on the model described in Section 3, i.e. f is computed by (1) and (8) respectively. In the case with the heuristic controller it is assumed there is a 10kW thermostatic controlled radiator for the heating of the house. This leads to that there is only two heating powers P_{heat}^h available, namely 0kW and 10kW for this control scheme. Further, the vehicle is assumed to be plugged in at t_{ad} every day and maximum charging power is used until the vehicle is fully charged, in this case 16A single phase.

The optimal solution minimizes the energy cost, i.e.

$$g(u_k) = (P_{heat} + P_{charge})(t_{k+1} - t_k)c_k \quad (9)$$

where t_k is the time at index k and c_k is the electric price per energy unit at time k . There are constraints that the indoor temperature is not to be below a minimal temperature, T_{min} , the vehicle should be fully charged at t_{fc} , and the total power is to be below P_{max} . The charging power, P_{charge}^o , and heating powers, P_{heat}^o , are discretized in optimal control scheme as

$$P_{charge}^o \in \{0, 1, 2, 3\}kW$$

$$P_{heat}^o \in \{0, 2, 4, 6, 8, 10\}kW$$

and the states T_{in} and x_{SoC} are discretized as

$$T_{in}^{grid} \in \{19, 20, \dots, 29, 30\}^{\circ}C$$

$$x_{SoC}^{grid} \in \{0, 0.1, \dots, 0.9, 1\}$$

The discretization in time differs in the two control schemes. In the heuristic controller the time step used is ten minutes and in the optimal controller one hour is used. The reason for this is that the fluctuations in indoor temperature is too high in the heuristic controller when one hour sample time is used since the heater is either fully on or off. In the smart controller there are more steps in power resulting in approximately the same indoor temperature fluctuations comparing the two controllers.

A zoom in of the results from the simulations are presented in Figure 5. This figure presents the different signals from the morning of January 2 till the morning of January 3. As can be seen the vehicle is charged when the spot price is at minimum in the optimal controller, but also that the indoor temperature is increased when the electricity price is low. This has the disadvantage that the heat losses increase and thereby also the energy consumption for heating. However, in this case it is more economical to increase the temperature of the house before the electricity price increases. Further, even though the sample time is lower in the heuristic controller, it can be seen that the indoor temperature fluctuates more compared to the optimal solution. This fluctuation decreases when the outdoor temperature is lower since the heat power required is higher.

The results presented in Figure 5 are general in the sense that the electric vehicle often is charged at night time using the optimal controller, as can be seen in Figure 6 where the accumulated energy consumption for every hour of the day during the year is presented. In this figure it can also be seen that the heater is used more between three o'clock

Table 1. Energy consumption for the household during one year using the heuristic controller and the smart energy management.

	Heuristic	Optimal	Cost reduction
Charging energy [kWh]	1805	1806	
Charging energy cost [SEK]	681	507	26%
Heating energy [kWh]	10573	10486	
T_{in} [$^{\circ}C$]	20.5	20.6	
Heating energy cost [SEK]	3839	3410	11%
Total energy cost [SEK]	4520	3917	13%

and five o'clock in the morning due to the lower electricity price and the thermal inertia of the house is used for energy storage.

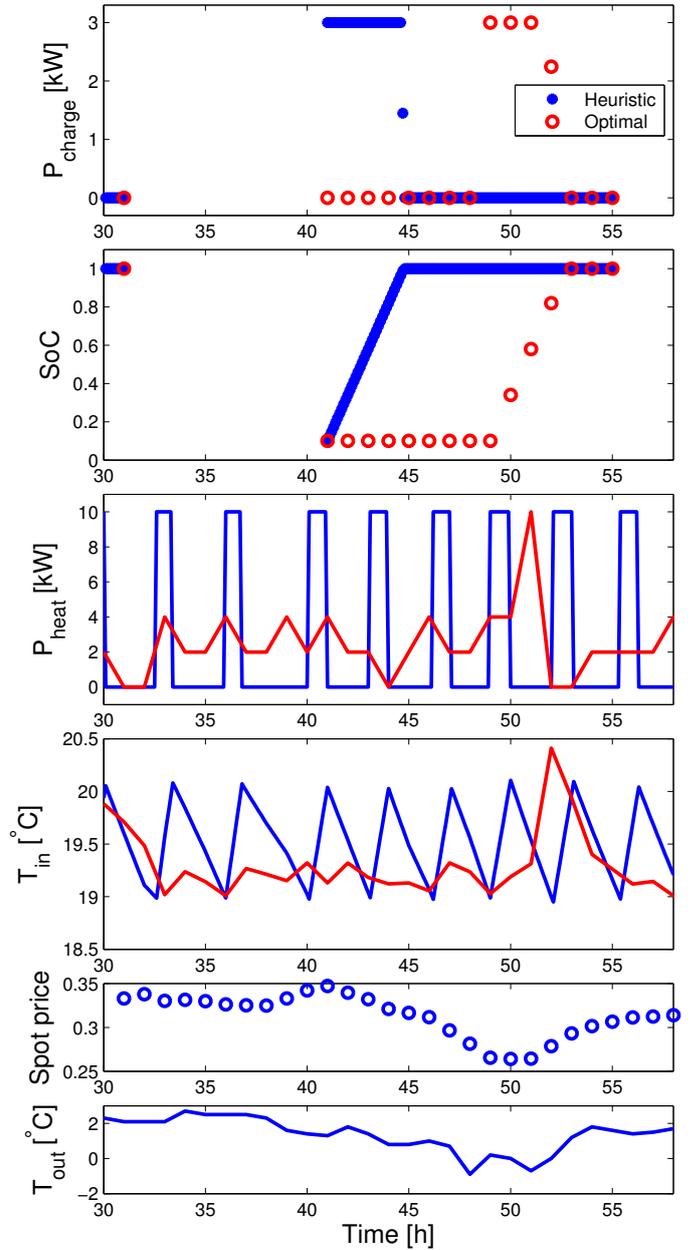


Fig. 5. Control signals and states compared during one day using the heuristic controller and the optimal controller.

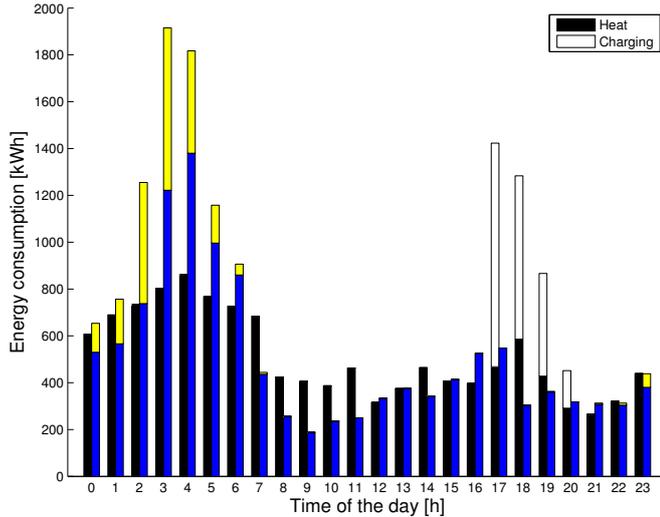


Fig. 6. The total energy consumption for heating and charging at different times of the day during the year. The left bars (black and white) represent the consumption using the heuristic controller, and the right bars (blue and yellow) represent the consumption using dynamic programming.

The results from the simulations are also presented in Table 1 where it can be seen that the cost for charging the electric vehicle is decreased by 26 % while the reduction in heating cost is lower. The reason for this is that in order to achieve a cost reduction for heating and still achieve an indoor temperature above the specified minimum temperature at all times, the heating power needs to be increased in advance when the electricity price is cheap. This results in a higher indoor temperature and the heating losses from the house increases by doing so, see (2) and (3). This disadvantage is not the case with the charging of the vehicle since the efficiency is modeled to be independent of the charging power, see (8), and there are no losses due to storing energy in the battery when there is no charging or discharging.

6. CONCLUSIONS

A method evaluating the potential in cost saving of using smart energy management for heating and electric vehicle charging is presented. The evaluation spans over an entire year and a novel way of handling the fact that the electric vehicle is disconnected to the grid at some times is developed. The simulations show that the cost reduction is significant using this smart energy management approach, in overall 13%. However, this potential is even higher using a heating system with an energy storage, e.g. water radiators in combination with a water tank, as well as including the water boiler for hot water production in the smart energy management for the household.

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Appendix A. MODEL PARAMETERS

Table A.1. Model parameters used in the simulation study.

Parameters	Value
U_{wall}	0.33 W/m ² K
U_{floor}	0.3 W/m ² K
U_{roof}	0.24 W/m ² K
U_{windows}	1.2 W/m ² K
A_{wall}	121.6 m ²
A_{floor}	125 m ²
A_{roof}	125 m ²
$A_{\text{windows,south}}$	7 m ²
$A_{\text{windows,west}}$	6 m ²
$A_{\text{windows,east}}$	2 m ²
$A_{\text{windows,north}}$	9 m ²
γ_{vent}	0.5
ϕ	58.41°
θ_0	70°
g_0	0.55
T_{ground}	8°C
R_{tot}	6 kWh/K
c_p	1.0 kJ/kg·K
ρ	1.3 kg/m ³
Q	10 kWh
η	0.8
P_{max}	15 kW