A framework for modeling and optimal control of automatic transmission systems

V. Nezhadali * **L. Eriksson** *

* *Electrical Engineering Department, Linköping University, SE-581 83 Linköping, Sweden (e-mail: {vaheed.nezhadali, lars.eriksson}@liu.se)*

Abstract: Development of efficient control algorithms for the control of automatic transmission systems is crucial to maintain passenger comfort and operational life of the transmission components. An optimization framework is developed by state space modeling of a powertrain including a nine speed automatic transmission, diesel engine, torque converter and a model for longitudinal vehicle dynamics considering drive shaft as the only flexibility of the driveline. Emphasis is set on the kinematics of the automatic transmission with the aim of modeling for gearshift optimal control during the inertia phase. Considering the interacting forces between planetary gearsets, clutches and brakes in the transmission, kinematic equations of motion are derived for rotating transmission components enabling to calculate both transmission dynamics and internal forces. The model is then used in optimal control problem formulations for the analysis of optimal control transients in two up-shift cases.

Keywords: Automatic transmission, optimal control, modeling and simulation framework

1. INTRODUCTION

Various driveline configurations are utilized in todays cars while Automatic Transmission (AT) is one of the commonly used systems, Wagner (2001). Such transmission systems consist of planetary gearsets where various gear ratios are achieved by coupling different components of planetary gearsets with each other. The coupling takes place via engagement of oil immersed clutches and brakes, referred to as shift elements, at the time of gearshift. Using combinations of planetary gearsets in the transmission, wide range of gear ratios become available while the compact structure of such transmission makes it favorable for application in both normal passenger cars, Greiner et al. (2004), and heavy duty machinery, A40G (2015).

However, as power is transferred through transmission components during gearshifts, the difference between inertia and rotational speed of the coupled bodies introduces discontinuities in the powertrain dynamics. Apart from the long-term destructive impacts of the induced oscillations on powertrain components, passenger comfort is also greatly dependent on the gearshift smoothness, see Huang and Wang (2004) and Horste (1995).

Simultaneous control of off-going and on-going shift elements during gearshifts determines the shift characteristic which in its turn has direct effects on other vehicle properties such as drive comfort, fuel efficiency, vehicle operability and durability of gearbox components. In addition to the improvement of vehicle properties, proper AT control strategies also open up the design space for new transmission concepts with lesser components resulting in lower total vehicle mass and consequently fuel consumption reduction.

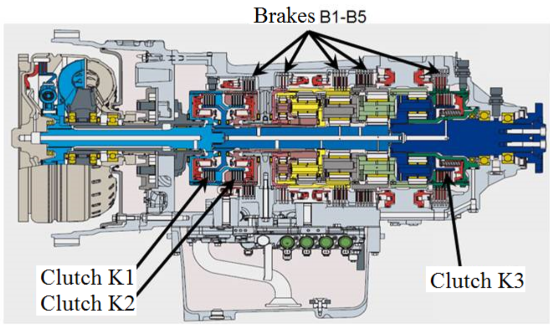

Fig. 1. Cutaway drawing of a nine speed automatic gearbox and torque converter.

Extensive research has been carried out on the topic of AT control for ride quality and operating life of shift elements, see Goetz et al. (2005), Sun and Hebbale (2005), Han and Yi (2003), Minowa et al. (1999) and Haj-Fraj and Pfeiffer (2002), concluding that the main criteria for the evaluation of shift quality are the duration of gear shift process and emerging oscillations in the powertrain. An efficient and cost effective approach to obtain suitable strategies for AT control is to utilize models with a time dependent structure where the dynamics of different components are well described during gearshift. Previous AT modeling efforts can be classified into two different categories. In the first the focus is on the dynamics of hydraulic actuation system initiated by the change of actuator lever position ending in the rise of hydraulic pressure at the brakes and clutches, see Minowa et al. (1999), Thornton et al. (2013) and Gao et al. (2010). In the second category, the kinematics of planetary gearsets before, during and after gearshifts are of interest while the hydraulic pressures are considered as control input signals, see Haj-Fraj and Pfeiffer (2002) and Kim et al. (2003).

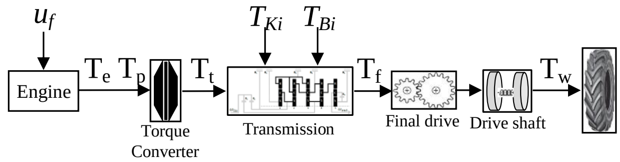

Fig. 2. Schematic view of the powertrain and driveline components with focus on transmission inputs and outputs.

In this work, the aim is to develop a framework enabling optimal control analysis of desirable AT gearshifts during the inertia phase. A nine speed AT as shown in Figure 1 comprised of a torque converter, five planetary gearsets and eight shift elements will be considered. Analyzing the kinematics of rotating bodies and calculating the internal forces/torques between various components, transmission dynamics at different gears are obtained as a state space system which is utilized for optimal control problem formulation. Then the AT transients during the inertia phase are calculated and analyzed for two example gearshifts to illustrate the framework applicability in different gearshifting scenarios.

2. DRIVELINE MODEL

Figure 2 shows a schematic overview of the developed powertrain model where a diesel engine generates the required power transferred to the wheels via torque converter, transmission, final drive and a drive shaft. Considering the large number of AT components and the corresponding state variables, which will be later introduced, in order to reduce the number of required state variables for the complete powertrain kinematics and for simplicity, the following assumptions and simplification are made during powertrain modeling:

(1) The turbocharger lag and engine pumping losses are neglected in diesel engine model assuming that the engine has already reached high speed when a gearshift starts.
(2) All components except the drive shaft are considered without torsional damping or flexibility.
(3) Friction and viscous losses in the planetary gearsets are neglected.
(4) Component properties are chosen according to the information available in A40G (2015).
(5) Perfect clutch fill and coordination is assumed. Only the inertia phase of the gearshift is considered and therefore, the transmission actuation system is modeled such that the shift element torques are directly applied as control inputs.

Engine speed ω_e, rotational speed of the transmission components $\omega_{1,\ldots,8}$, torsion of drive shaft x_{flex} and wheel speed ω_w are the state variables of the model which are described by equations (3), (9)-(16),(21) and (22). Considering actuation torques from all shift elements, $T_{B_{1,2,3,4,5}}$ and $T_{K_{1,2,3}}$ as depicted in Figure 2, there are eight choices for the transmission model control inputs depending on the selected gear. The kinematic analysis is described for `1-2` up-shift only but similar methodology can be followed for other gearshifts. Therefore the control inputs ($u_{\texttt{1-2}}$) for `1-2` up-shift modeling will be:

$$u_{\texttt{1-2}} = \{u_f, T_{B5}, T_{K3}\} \tag{1}$$

where u_f, fuel mass injected per combustion cycle is the control input to the diesel engine model, and T_{B5} and T_{K3} are the torques from brake 5 and clutch 3.

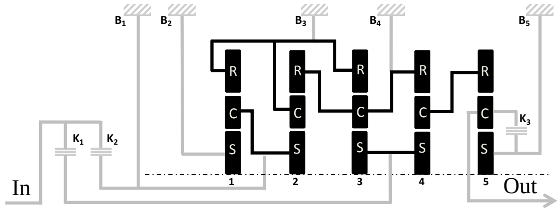

Fig. 3. Schematic view of a nine speed automatic gearbox and mechanical links between planetary gearsets components, Sun(S)-Carrier(C)-Ring(R), with respect to clutches and brakes.

Gear	K1	K2	K3	B1	B2	B3	B4	B5
1	×						×	×
2	×		×				×	
3	×					×		×
4	×		×			×		
5	×		×	×				
6	×		×		×			
7	×	×	×					
8		×			×			×
9		×	×		×			

Table 1. The shift elements (clutches and brakes) that are active at each gear.

2.1 Diesel engine

A simplified version of the mean value engine model (MVEM) in Walström and Eriksson (2011) with properties according to A40G (2015) is used since continuity and differentiability of MVEM makes it more appropriate for optimal control applications. Indicated torque T_i for the six cylinder diesel engine and engine friction torque T_{fr} are calculated as:

$$T_i = \frac{6 \times 10^{-6}}{4\pi} \eta_{ig}\, q_{lhv}\, u_f \tag{2a}$$

$$T_{fr} = c_{f1}\,\omega_e^2 + c_{f2}\,\omega_e + c_{f3} \tag{2b}$$

where η_{ig} is combustion efficiency, q_{lhv} is specific heating value for diesel fuel and c_{fi} are friction model coefficients. The diesel engine model has only one state variable, engine speed ω_e, with the dynamics calculated as follows:

$$\frac{d\omega_e}{dt} = \frac{1}{J_e + J_{tc,p}}(T_i - T_{fr} - T_p) \tag{3}$$

where the inertia of the pumping side of the torque converter $J_{tc,p}$ is added to the engine inertia J_e, and T_p is the required torque on the pumping side of torque converter described in the following section.

2.2 Torque converter

In torque converter modeling, the focus is more on the continuity and differentiability of the model than the dynamics of the torque converter components. Therefore the typical approach as in Haj-Fraj and Pfeiffer (2002) using torque converter characteristic curves, ξ and MP_{1000}, is implemented.

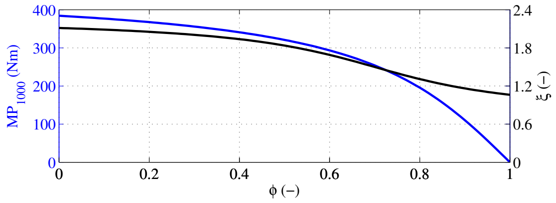

Fig. 4. Torque converter characteristic curves.

Using the characteristic curves, illustrated in Figure 4, the required torque on pumping side and generated torque on turbine side of the torque converter are calculated as follows:

$$T_p = MP_{1000}(\phi)\,(\frac{\omega_e}{1000})^2 \tag{4}$$

$$T_t = MP_{1000}(\phi)\,\xi(\phi) \tag{5}$$

where ϕ=ω_t/ω_e and ω_t is the turbine shaft speed connected to the transmission input and ω_e in equation (4) is in `(rpm)`.

2.3 Transmission

The transmission unit is comprised of five planetary gearsets, three clutches and five brakes illustrated in Figure 3. The brakes are used to fix a component to the casing (zeros speed) while clutches connect two rotating components with each other. Different gear ratios can be obtained by activating shift elements according to Table 1. There are three widely used methods for analysis of automatic transmissions, Shushan and Peng (2013), namely algebraic equation method, lever analogy and matrix methods. While the first two are handy for transmissions with fewer (one or two) number of planetary gearsets, the matrix methods is prioritized for larger transmissions and is also utilized here. In the next two sections it is first described how complying dimensions of sun and ring components, required in matrix method, are calculated and then using the matrix method, dynamics of the transmission components are calculated.

Gearbox model parametrization Dimensions of planetary gearsets are required for calculation of transmission dynamics, however, only the total gear ratio of the transmission at each gear γ_i, individual planetary gearset properties such as ratios and dimensions are in many cases not publicly available. Therefore it is necessary to calculate the complying gearset properties such that correct total gear ratio can be obtained after solving transmission dynamics. The kinematic constraint due to the mechanical structure of each planetary gearset is:

$$\omega_C\,(1+\alpha_i) = \omega_S\,\alpha_i + \omega_R \tag{6}$$

where α_i is the ratio between sun and ring radius in gearset i. Considering equation (6) and the mechanical links between the gearset components illustrated in Figure 3, the total gear ratios can be calculated as the ratio between input and output speeds of the transmission. For example in case of the first gear the gear ratio (γ) calculations is:

$$\omega_{C4} = \omega_{R5}, \omega_{R4} = 0 \Rightarrow \omega_{R5} = \frac{\alpha_4}{1+\alpha_4}\omega_{in} \tag{7a}$$

$$\omega_{S5} = 0 \Rightarrow \omega_{out}(\alpha_5+1) = \omega_{R5} \tag{7b}$$

$$\Rightarrow \frac{\omega_{in}}{\omega_{out}} = \frac{(1+\alpha_4)(1+\alpha_5)}{\alpha_4} = \gamma_1 \tag{7c}$$

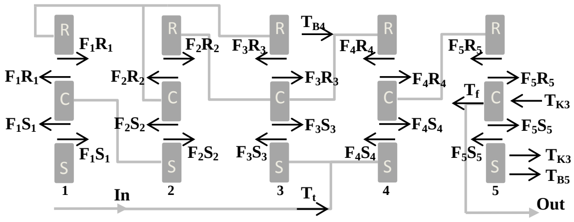

Fig. 5. Graphical representation of exerted torques on transmission components in a `1-2` gearshift.

where $\omega_{in} = \omega_t = \omega_{S3S4}$ and $\omega_{out} = \omega_{C5}$. Performing similar calculations for all nine gears, the gear ratios γ_i as function of α_i are obtained as follows:

$$\gamma_2 = \frac{(1+\alpha_4)}{\alpha_4} \tag{8a}$$

$$\gamma_3 = \frac{(1+\alpha_3)(1+\alpha_4)(1+\alpha_5)}{\alpha_4+\alpha_3\alpha_4+\alpha_3} \tag{8b}$$

$$\gamma_4 = \frac{(1+\alpha_3)(1+\alpha_4)}{\alpha_4+\alpha_3\alpha_4+\alpha_3} \tag{8c}$$

$$\gamma_5 = \frac{\alpha_4+1}{\alpha_4+g_1}, \qquad g_1 = \frac{\alpha_3+\alpha_2\alpha_3}{\alpha_2+\alpha_2\alpha_3+\alpha_3} \tag{8d}$$

$$\gamma_6 = \frac{\alpha_4+1}{\alpha_4+g_2}, \qquad g_2 = \frac{\alpha_3}{1+\alpha_3-\frac{1+\alpha_1}{1+\alpha_2+\alpha_1\alpha_2}} \tag{8e}$$

$$\gamma_7 = 1 \tag{8f}$$

$$\gamma_8 = \frac{\alpha_3(1+\alpha_4)(1+\alpha_5)}{(\alpha_3+\alpha_4+\alpha_3\alpha_4)g_3-\alpha_4(1+\alpha_1)} \tag{8g}$$

$$\gamma_9 = \frac{\gamma_8}{1+\alpha_5}, \qquad g_3 = 1+\alpha_1+\alpha_1\alpha_2 \tag{8h}$$

Using the gear ratios, γ_i and equation (7c)-(8h) the unknown α_i can be calculated. Knowing that the transmission is almost one meter long and using this as a scale, the radii of sun gears (S_i) are read from Figure 1 and then the radii of the ring gears (R_i) are calculated from $R_i = S_i/\alpha_i$.

Applying the matrix method Mechanical links between gearset components are shown in Figure 3. In order to reduce the required number of state variables for describing all component dynamics, the linked components of different gearsets namely `sun(S)-ring(R)-carrier(C)` are considered as a single body resulting in the following inertias:

$$\begin{array}{llll} J_1 : S_1 & J_2 : R_1C_2R_3 & J_3 : C_1S_2 & J_4 : R_2C_3R_4 \\ J_5 : C_4R_5 & J_6 : S_3S_4 + J_{TC,t} & J_7 : C_5 & J_8 : S_5 \end{array}$$

where $J_{tc,t}$ is the inertia of torque converter on the turbine side, the output shaft inertia is lumped into J_7 and the inertia of clutch and brake components are neglected.

A schematic view of the transmission components including applied torques from active shift elements during first and second gear operation together with the input and output torque of the transmission is depicted in Figure 5 where S_i and R_i are the sun and ring radii and F_i represents the internal force in each planetary gearset. Considering Newton's second law for all eight rotating bodies of the transmission, the dynamics are obtained as follows:

$$J_1 \frac{d\omega_1}{dt} - F_1 S_1 = 0 \tag{9}$$

$$J_2 \frac{d\omega_2}{dt} - F_1 R_1 + F_2(S_2 + R_2) + F_3 R_3 = 0 \tag{10}$$

$$J_3 \frac{d\omega_3}{dt} + F_1(S_1 + R_1) - F_2 S_2 = 0 \tag{11}$$

$$J_4 \frac{d\omega_4}{dt} - F_2 R_2 - F_3(S_3 + R_3) + F_4 R_4 = T_{B4} \tag{12}$$

$$J_5 \frac{d\omega_5}{dt} - F_4(S_4 + R_4) + F_5 R_5 = 0 \tag{13}$$

$$J_6 \frac{d\omega_6}{dt} + F_3 S_3 + F_4 S_4 = T_t \tag{14}$$

$$J_7 \frac{d\omega_7}{dt} - F_5(S_5 + R_5) = -T_{K3} - T_f \tag{15}$$

$$J_8 \frac{d\omega_8}{dt} + F_5 S_5 = T_{K3} + T_{B5} \tag{16}$$

The dynamic equations can be written in the following matrix form:

$$\begin{bmatrix} I & K \\ K^T & \mathbf{0} \end{bmatrix} \Omega = \begin{bmatrix} \kappa \\ \mathbf{0} \end{bmatrix} T \tag{17}$$

with I $= \mathtt{diag}(J_1, J_2, \cdots, J_8), \Omega = [\dot{\omega}_1, \ldots, \dot{\omega}_8, F_1, \ldots, F_5]^T$,

$$K = \begin{bmatrix} -S_1 & 0 & 0 & 0 & 0 \\ -R_1 & R_2 + S_2 & R_3 & 0 & 0 \\ R_1 + S_1 & -S_2 & 0 & 0 & 0 \\ 0 & -R_2 & -R_3 - S_3 & R_4 & 0 \\ 0 & 0 & 0 & -R_4 - S_4 & R_5 \\ 0 & 0 & S_3 & S_4 & 0 \\ 0 & 0 & 0 & 0 & -S_5 - R_5 \\ 0 & 0 & 0 & 0 & S_5 \end{bmatrix},$$

$$\kappa = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \text{ and } T = \begin{bmatrix} T_t \\ T_{B4} \\ T_{K3} \\ T_{B5} \\ T_f \end{bmatrix}.$$

where T_f is the transferred torque from the transmission to the final drive and is calculated in the next section. Rearranging equation (17) into (18), the speed dynamics of every rotating component and the internal forces in each planetary gearset can be calculated from the following:

$$\Omega = \begin{bmatrix} I & K \\ K^T & \mathbf{0} \end{bmatrix}^{-1} \begin{bmatrix} \kappa \\ \mathbf{0} \end{bmatrix} T \tag{18}$$

The dynamic equations of the transmission components can be simplified during stationary operations ($\frac{d\omega_i}{dt} = 0$) and also when a component is fixed by means of shift elements to another component or to the transmission housing such that it has same dynamics as another component or is stationary. For example during `1-2` up-shift equation (12) can be removed from dynamic equations since T_{B4} fixes $R_2 C_2 R_1$ to the transmission housing and $\omega_4 = \frac{d\omega_4}{dt} = 0$. The transmission dynamics can then be calculated by removing J_4 from I, $\dot{\omega}_4$ from Ω, fourth row of matrix K, second column of κ and T_{B4} from T vector. The required T_{B4} can then be calculated from equation (12) after obtaining $F_{1,2,3}$ from equation (18) which can be useful for estimation of required torque bandwidth in brake 4. Therefore T_{B4} is not used as a control inputs during the `1-2` up-shift and with same reasoning the control inputs during `4-5` up-shift are reduced to $\{u_f, T_{B5}, T_{B3}\}$ by omitting T_{K3}.

2.4 Drive shaft flexibility and Longitudinal Dynamics

According to Eriksson and Nielsen (2014)-chapter 14, the main flexibility of driveline is located in the drive shaft connecting the final drive to the wheels. Therefore the drive shaft is modeled as a damped flexibility with stiffness k and damping coefficient of c with the output and input torques T_w and T_f as follows:

$$T_w = k\, x_{flex} + c\,(\frac{\omega_7}{\gamma_{fd}} - \omega_w) \tag{19}$$

$$T_f = \frac{T_w}{\gamma_{fd}} \tag{20}$$

where γ_{fd} is the final drive ratio and x_{flex} is the state variable describing the drive shaft torsion which can be obtained as follows:

$$\frac{dx_{flex}}{dt} = \frac{\omega_7}{\gamma_{fd}} - \omega_w \tag{21}$$

The wheel speed ω_w is the last state variable in the complete driveline model calculated from the vehicle longitudinal dynamics for level road condition according to:

$$\frac{d\omega_w}{dt} = \frac{1}{m\, r_w^2 + J_w} \left(T_w - \frac{1}{2}\rho\, c_d\, A\, \omega_w^2 r_w^3 - m\, g\, c_r\, r_w\right) \tag{22}$$

where r_w, m and A are wheel radius, vehicle mass, and vehicle frontal area, c_r and c_d are rolling resistance and aerodynamic drag coefficients and g represent the earth gravity.

3. OPTIMAL CONTROL PROBLEM FORMULATION

Using the developed state space model, optimal control problems are formulated and solved for optimization of gearshift transients. There are several properties of gearshift which can be considered as the optimization objective, see Haj-Fraj and Pfeiffer (2002) and Haj-Fraj and Pfeiffer (2001) for a discussion. Here shift duration (T) corresponding to the operating life of shift elements and changes in the vehicle acceleration (jerk) corresponding to the passenger comfort, are focused. The shift duration as one of the objectives in the optimal control analysis is represented by:

$$T = \int_0^t dt \tag{23}$$

where t denote the duration of the gearshift. The jerk, A, is represented by integrating the squared derivatives of the vehicle accelerations during the gearshift as follows:

$$A = \int_0^t (\frac{da}{dt})^2 dt \tag{24}$$

where $a = \frac{d\omega_w}{dt}$.

To obtain the trade-off between $\min T$ and $\min A$ solutions, the optimal control problem objective is often formulated as the weighted sum of the time and jerk terms

$(v_1 T + v_2 A, v_1 + v_2 = 1)$ where the points on the trade-off are calculated by solving with different weights $(v_{1,2})$. However, considering the large difference in the order of magnitude between the two objectives, when formulating the weighted sum, A and J have to be normalized with respect to their maximums. This makes the solution of the weighted sum sensitive to the normalization and weight values, which makes it difficult to calculate the trade-off with an acceptable spread of points.

Another approach is used here where after solving the $\min T$ and $\min A$ solutions as the start and end points on the trade-off, a time grid is selected on the interval between these two solutions. Then, the gearshift duration t is repeatedly set equal to the grid times and equation (24) is solved to obtain the A values. The first point on the time grid is obtained by solving the $\min T$ problem, but there is no unique solution in time when solving the $\min A$ problem $(v_1 = 0)$ since after a certain gearshift duration, zero jerk can be obtained for various gearshift lengths. Therefore, to obtain the shortest time where zero jerk gearshift is possible, equation (23) is chosen as the objective and the problem is solved including an additional $a = 0$ constraint in the optimal control problem formulation. The calculated gearshift duration is then used as the end point of the time grid.

Considering the 1-2 up-shift, state vector x can be summarized as:

$$x = \{\omega_{1,2,3,5,6,7,8}, \omega_e, \omega_w, x_{flex}\} \tag{25}$$

while the state dynamics $f(x(t), u(t))$ are obtained by the differential equations presented in the previous sections.

The complete optimal control problem formulation for different scenarios stated earlier can be summarized as follows:

$$\begin{aligned}
&\min_{x(t),u(t),\gamma(t)} \quad T \;\text{ or }\; A &(26)\\
&\text{s.t.} \qquad \dot{x}(t) = f(x(t), u(t))\\
&\qquad u_{min} \leq u_i(t) \leq u_{max}\\
&\qquad x_{min} \leq x_i(t) \leq x_{max}\\
&\qquad T_e(\omega_e, u_f(t)) \leq T_{e,max}\\
&\qquad x(0) = x_0\\
&\qquad x(T) = x_T\\
&\qquad \dot{x}(0) = 0\\
&\qquad \dot{x}(T) = 0\\
&\qquad a = 0, \quad \text{when solving for} \min T|_{A=0}
\end{aligned}$$

where the *min* and *max* are the upper and lower limits for states and controls and x_0 and x_T correspond to the state values at the end of the first gear and beginning of the second gear operation, respectively. These are calculated in similar manner as described by equations (7a)-(7c) for $r_w\omega_w$=6 km/h and $r_w\omega_w$=15 km/h in 1-2 and 4-5 up-shifts respectively. The requirements on $\dot{x}$ at the beginning and end of the gearshift are applied to guaranty that the gearshift starts and ends at stationary condition.

It should be noted that the kinematic constraint mentioned in equation (6) does not need to be added to problem constraints as their effect is already present in the dynamic equations of states via internal forces F_i.

Implementing direct multiple shooting method, the formulated optimal control problem is transformed into a nonlinear program (NLP). The NLP is then solved by CasADi, Andersson (2013), which is an open source optimal control framework using Ipopt, Wächter and Biegler (2006), and the powertrain transients are obtained. The results of this optimization are presented and analyzed in the following section.

4. OPTIMAL CONTROL RESULTS

Gearshifting transients are optimized for 1-2 and 4-5 up-shifts where in both cases there are one off-going and one in-coming shift element. The major phenomena observed in the calculated optimal transients are described in the following sections.

4.1 Trade-off between jerk and time

Figure 6 shows the trade-off between $\min T$ and $\min A$ solutions for both gearshift cases where the dashed-lines are obtained by interpolation between the calculated points. The trade-offs show that small increase of gearshift duration from the time optimal solution can largely reduce the jerk. For example, at C_{12} and C_{45} a good compromise is achieved between the two objectives as the gearshift duration in the 1-2 and 4-5 up-shifts has increased only 0.22 % and 4 % while the jerk has reduced as much as 64.7 % and 55.9 % respectively. However, the gain in the jerk reduction reduces when the gearshift time increases further.

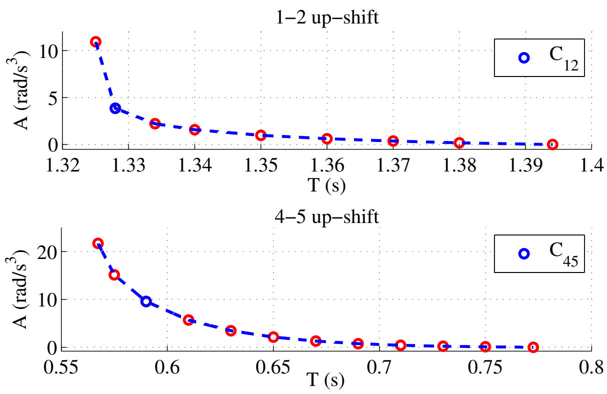

Fig. 6. The trade-off between time and jerk for 1-2 and 4-5 up-shifts. At $C_{12,45}$ there is a compromise between the two objectives.

4.2 Optimal transients for 1-2 and 4-5 up-shifts

The $\min T$, $\min A$ and $C_{12,45}$ state and control input transients for 1-2 and 4-5 up-shifts are illustrated in Figures 7, 8, 9 and 10. According to Figures 7 and 9 it is both time and jerk optimal to release the off-going shift element, u_{b5} and u_{b3} for 1-2 and 4-5 cases respectively, as soon as the gearshifts starts. Also in the $\min A$ case, it is optimal to reach the maximum torque capacity of the in-coming shift element, u_{k3} and u_{b1} for 1-2 and 4-5 up-shifts, immediately after gearshift starts such that the initial torque phase is completely avoided. This requirement for fast actuation of in-coming shift elements

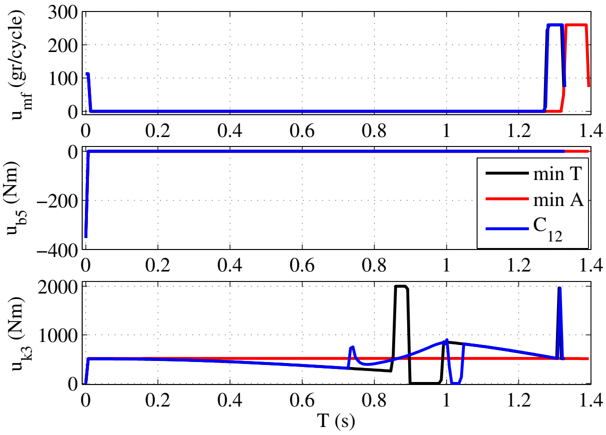

Fig. 7. Optimal control transients for $\min T$, $\min A$ and C_{12} cases in a `1-2` up-shift.

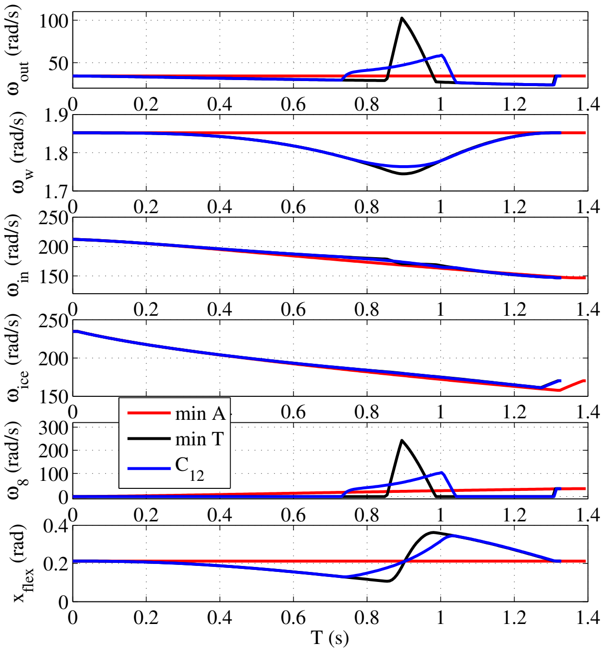

Fig. 8. Optimal state transients for $\min T$, $\min A$ and C_{12} cases in a `1-2` up-shift where $\omega_{out}{=}\omega_7$ and $\omega_{in}{=}\omega_6$.

can be considered when designing the hydraulic actuation system.

Considering $C_{12,45}$ transients, the magnitude of disturbances in the vehicle speed, ω_w, is reduced compared to the $\min T$ solution especially in case of the `1-2` up-shift which can also be verified comparing the $d\omega_w/dt$ values in Figure 11. The required energy in $b1$ and $k3$ shift elements calculated as $\int_0^T u_{b1}\omega_2\, dt$ and $\int_0^T |u_{k3}\omega_8|\, dt$ is also 8.8 % and 12.11 %, respectively, lower than the $\min T$ case.

Figure 11-top shows the input torque to the transmission. The transients are similar for both up-shift cases in the $\min A$ solutions in a sense that the fuel injection u_{mf} is cut off so that the absence of an input torque on the engine side of the torque converter together with the constant torque from the in-coming shift element, smoothly reduce the kinetic energy of the transmission components down to the level required for the engagement of the next gear. However, in the $\min T$ case of both up-shifts, in addition to the fuel cut-off, the bang-bang type dynamics

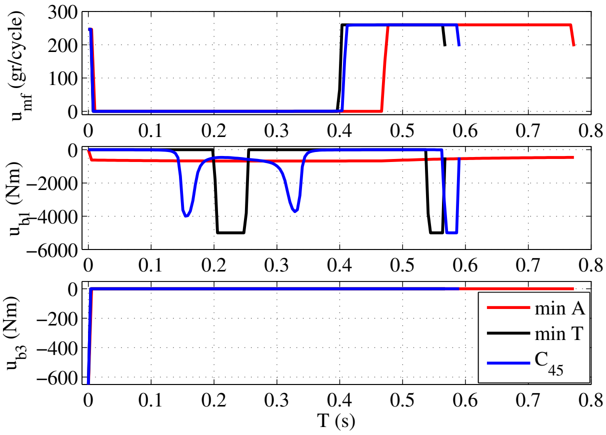

Fig. 9. Optimal control transients for $\min T$, $\min A$ and C_{45} cases in a `4-5` up-shift .

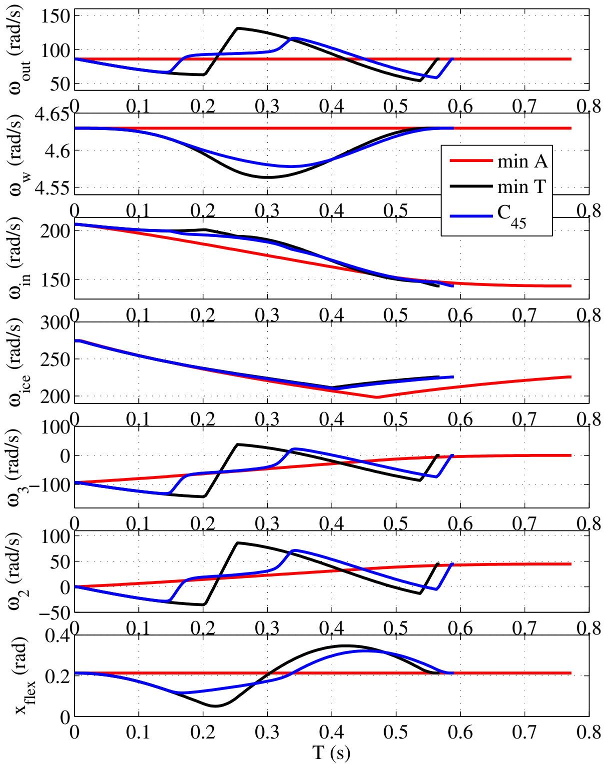

Fig. 10. Optimal state transients for $\min T$, $\min A$ and C_{12} cases in a `4-5` up-shift where $\omega_{out}{=}\omega_7$ and $\omega_{in}{=}\omega_6$.

of the in-coming shift elements introduce disturbances into the driveline such that the induced excitation is transferred to the vehicle mass at wheels via the the drive shaft. These excitations get partially damped by the drive shaft flexibility, vehicle mass and wheel inertia which increases the rate of reduction in the kinetic energy of the transmission components and shortens the time before the next gear can be engaged.

Subtracting the total kinetic energy of all transmission components $(0.5 \sum I_i\omega_i^2)$ at the beginning and end of the gearshifts, the difference is 25.17 % larger in case of the `1-2` up-shift which could be the main reason for the slower transients compared to the `4-5` case as more reduction in the kinetic energy is required before the next gear can be engaged.

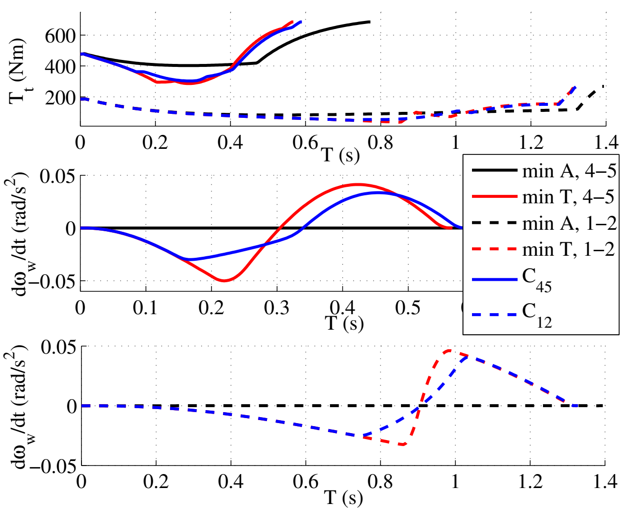

Fig. 11. Transmission input torque and vehicle acceleration/decceleration in min T, min A and C_{45} solutions of `1-2` and `4-5` up-shifts.

5. CONCLUSION

A framework is developed for gearshift transient optimization during inertia phase via state space modeling of a nine speed heavy duty automatic transmission. Using the developed model and in order to analyzed the minimum time/jerk transients and the trade-off between these, optimal control problems are formulated and solved. Two example up-shifts are considered and in order to calculate the trade-off between time and jerk objectives while avoiding objective function normalizations difficulties, the minimum jerk problem is iteratively solved for various preselected gearshift durations.

The results show that the developed framework is applicable for efficient optimization of inertia phase gearshift transients. As future model developments, hydraulic actuation dynamics can be included and sensitivity of the gearshift transients with respect to the parameters such as clutch fill dynamics, components' inertia and driveline flexibilities can be analyzed.

REFERENCES

A40G (2015). Volvo A40G hauler, "`http://www.volvoce.com/SiteCollectionDocuments/VCE/Documents%20Global/articulated%20haulers/Brochure_A35GFS_A40GFS_SV_12_20040745_B_2015.02.pdf`", Accessed July. 2015.

Andersson, J. (2013). *A General-Purpose Software Framework for Dynamic Optimization*. PhD thesis, Arenberg Doctoral School, KU Leuven, Department of Electrical Engineering (ESAT/SCD) and Optimization in Engineering Center, Kasteelpark Arenberg 10, 3001-Heverlee, Belgium.

Eriksson, L. and Nielsen, L. (2014). *Modeling and Control of Engines and Drivelines*. John Wiley & Sons.

Gao, B., Chen, H., Zhao, H., and Sanada, K. (2010). A reduced-order nonlinear clutch pressure observer for automatic transmission. *Control Systems Technology, IEEE Transactions on*, 18(2), 446–453.

Goetz, M., Levesley, M., and Crolla, D. (2005). Dynamics and control of gearshifts on twin-clutch transmissions. *Proceedings of the institution of mechanical engineers, Part D: Journal of Automobile Engineering*, 219(8), 951–963.

Greiner, J., Doerr, C., Nauerz, H., and Graeve, M. (2004). The new 7g-tronic of mercedes-benz: Innovative transmission technology for better driving performance, comfort and fuel economy. *SAE Technical Paper 2004-01-0649*.

Haj-Fraj, A. and Pfeiffer, F. (2001). Optimal control of gear shift operations in automatic transmissions. *Journal of the Franklin Institute*, 338(2), 371–390.

Haj-Fraj, A. and Pfeiffer, F. (2002). A model based approach for the optimisation of gearshifting in automatic transmissions. *International journal of vehicle design*, 28(1), 171–188.

Han, W. and Yi, S.J. (2003). A study of shift control using the clutch pressure pattern in automatic transmission. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 217(4), 289–298.

Horste, K. (1995). Objective measurement of automatic transmission shift feel using vibration dose value. *SAE Technical Paper 951373*.

Huang, Q. and Wang, H. (2004). Fundamental study of jerk: evaluation of shift quality and ride comfort. *SAE Technical Paper 2004-01-2065*.

Kim, D.H., Yang, K.J., Hong, K.S., Hahn, J.O., and Lee, K.I. (2003). Smooth shift control of automatic transmissions using a robust adaptive scheme with intelligent supervision. *International journal of vehicle design*, 32(3), 250–272.

Minowa, T., Ochi, T., Kuroiwa, H., and Liu, K.Z. (1999). Smooth gear shift control technology for clutch-to-clutch shifting. *SAE Technical Paper 1999-01-1054*.

Shushan, Bai, M.J. and Peng, H. (eds.) (2013). *Dynamic Analysis and Control System Design of Automatic Transmissions*. SAE International.

Sun, Z. and Hebbale, K. (2005). Challenges and opportunities in automotive transmission control. In *American Control Conference, 2005. Proceedings of the 2005*, 3284–3289. IEEE.

Thornton, S., Pietron, G.M., Yanakiev, D., McCallum, J., and Annaswamy, A. (2013). Hydraulic clutch modeling for automotive control. In *Decision and Control (CDC), 2013 IEEE 52nd Annual Conference on*, 2828–2833. IEEE.

Wächter, A. and Biegler, L.T. (2006). On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. *Mathematical programming*, 106(1), 25–57.

Wagner, G. (2001). Application of transmission systems for different driveline configurations in passenger cars. *SAE Technical Paper 2001-01-0882*.

Walström, J. and Eriksson, L. (2011). Modeling engines with a variable-geometry turbocharger and exhaust gas recirculation by optimization of model parameters for capturing non-linear system dynamics. *Proceedings of the Institution of Mechanical Engineers, Part D, Journal of Automobile Engineering*, 960–986.