Generation of Equivalent Driving Cycles Using Markov Chains and Mean Tractive Force Components ⋆

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Abstract: In the automotive industry driving cycles have been used to evaluate vehicles in different perspectives. If a vehicle manufacturer focuses only on a fixed driving cycle there is a risk that controllers of the vehicle are optimized for a certain driving cycle and hence are sub-optimal solutions to real-world driving. To deal with this issue, it is beneficial to have a method for generating more driving cycles that in some sense are equivalent but not identical. The idea here is that these generated driving cycles have the same vehicle excitation in the mean tractive force, MTF. Using the individual force components of the MTF in the generation of driving cycles with Markov chains makes it possible to generate equivalent driving cycles that have the same vehicle excitation from real-world driving data. This is motivated since the fuel consumption estimation is more accurate when the MTF components are considered. The result is a new method that combines the generation of driving cycles using real-world driving cycles with the concept of equivalent driving cycles, and the results are promising.

Keywords: Drive cycle, markov process, vehicle excitation, driver behavior, vehicle propulsion, automotive emissions, test procedures, specific energy, equivalence measures.

1. INTRODUCTION

A driving cycle, also called driving schedule or speed profile, is a representation of vehicle speed versus time. In the automotive industry driving cycles have been used to evaluate vehicles in different perspectives. To name a few, driving cycles have been used in exhaust gas emissions tests, in vehicle traffic control, or as an engineering tool for comparison and design. [André, 1996], [Tong et al., 1999], or [Stockar et al., 2010]. An example of a common driving cycle is the New European Driving Cycle, NEDC, which is seen in Figure 1. The NEDC is the certification driving cycle for light-duty trucks in Europe. If a vehicle manufacturer focuses only on such a fixed driving cycle there is a risk that controllers of the vehicle are optimized for a certain driving cycle, and if the driving cycle is not representative for real-world driving there is a considerable risk that the controls will be sub-optimal solutions for real-world driving [Schwarzer and Ghorbani, 2013, Kågeson, 1998]. Usually a representative driving cycle means that some criteria of interest is sufficiently close to data from real-world driving. The general consensus is that NEDC is not a representative of real-world driving [Fontaras and Dilara, 2012, Zaccardi and Le Berr, 2012].

Zaccardi and Le Berr [2012] studies how several different driving cycles can be used together in order to represent real-world driving. However, the results of one driving cycle is difficult to compare to the results of another driving cycle since two different driving cycles can have very different vehicle excitation.

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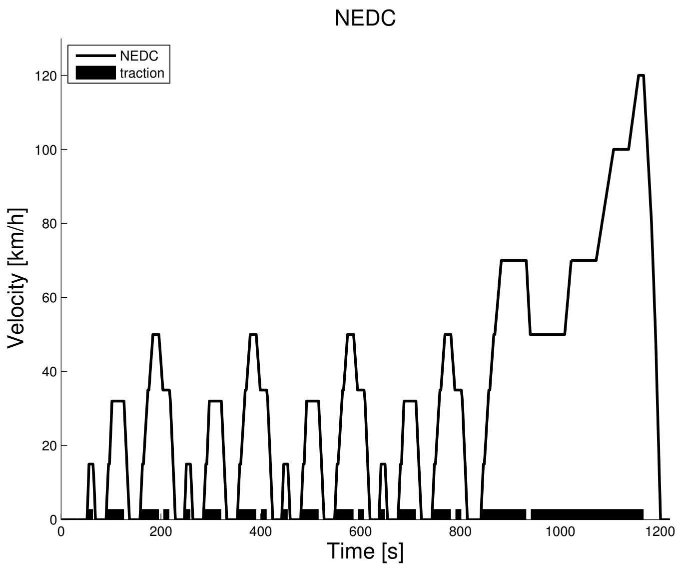

Fig. 1. The NEDC with marked traction regions which indicates the instants where the powertrain needs to deliver positive power to the wheels so that the vehicle is able to track the driving cycle.

There are different techniques to construct a driving cycle and one approach is to randomly choose micro-trips (speed profile between two successive stops) in the assemble of the driving cycle [Tong et al., 1999], and another is the mode-based approach [Lin and Niemeier, 2002] where the speed-acceleration frequency distribution are made similar to the real-world driving data. More recently a Markov chain approach have been used for generating representative driving cycles from real-world driving data in a compact

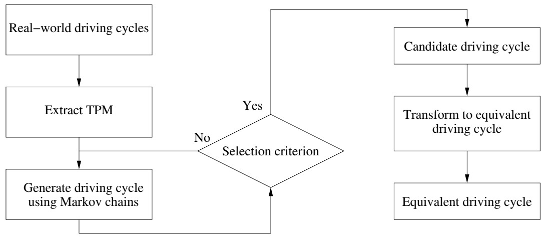

Fig. 2. Overview of the algorithm for generation of equivalent driving cycles from real-world driving cycles using Markov chains.

way, see Lee and Filipi [2011], Gong et al. [2011] and the references therein.

1.1 Overall Idea with this Paper

To deal with the sub-optimization problem it would be advantageous to have mechanisms that generate similar driving cycles which excites the vehicle in a similar way so that any performance comparison made, between the generated cycles, is more fair. The approach in this paper is to merge the Markov chain approach with the concept of equivalent driving cycles introduced in Nyberg et al. [2013], so the idea is to have a method that given real-world driving data generates equivalent driving cycles that have the same excitation in the mean tractive force meaning.

2. DRIVING CYCLE GENERATION

As a first part of the proposed algorithm, driving cycles will be generated from real-world driving data using a Markov chain approach. This corresponds to the left part of Figure 2. As a second part, these driving cycles will be determined if they are candidate driving cycles that pass the selection criterion, and finally are transformed to equivalent driving cycles which corresponds to the middle and right part of Figure 2. The second part will be explained in Section 4.

Now we focus on the first part, and in this work the driving cycle generator of Torp and Önnegren [2013], similar to the one presented in Lee and Filipi [2011], has been used to generate driving cycles.

2.1 Real-world Driving

Input data to the driving cycle generation are real-world driving cycles that have been measured on instrumented vehicles driving in real traffic. An example of a real-world driving cycle is seen in Figure 3, and the data used here is a set of 466 drives from a test in the western parts of Sweden. A categorization of driving cycles have been performed by either mean positive velocity, $\bar{v}_{\text{pos}} = \bar{v}(t) : v(t) > 0$, or based on distance traveled in the driving cycle, $x_{\text{tot}} = \int v(t)dt$, which are similar to the categorization in Lee and Filipi [2011]. This yields a possibility to generate driving cycles that for example specifically show urban tendencies or highway driving with high speeds.

The chosen limits in the categorization and the number of real-world driving cycles that fit into each category can be seen in Table 1 and in Table 2. For more details of the real-world driving data see Torp and Önnegren [2013].

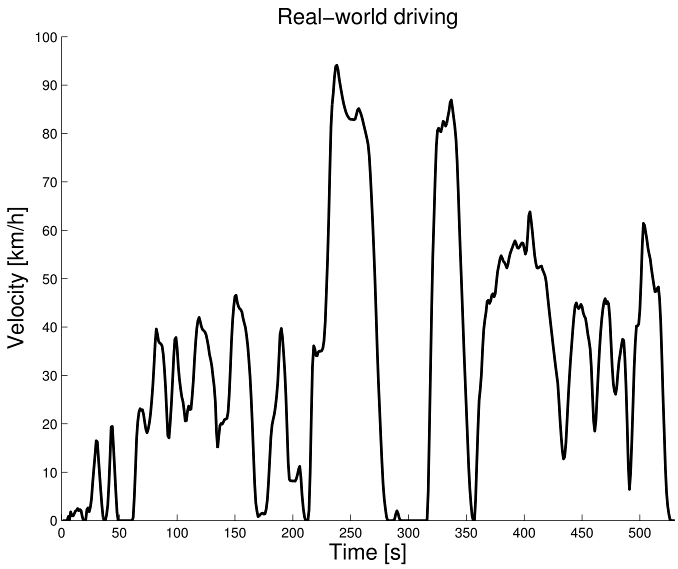

Fig. 3. Example of real-world driving cycle.

Table 1. Categorization based on mean positive velocity $\bar{v}_{\text{pos}}$.

Category	Limits [km/h]	#Cycles
Urban	$0 < \bar{v}_{\text{pos}} \leq 40$	328
Mixed	$40 < \bar{v}_{\text{pos}} \leq 72$	133
Highway	$72 < \bar{v}_{\text{pos}} < \infty$	5

Table 2. Categorization based on driving distance x_{tot}.

Category	Limits [km]	#Cycles
Short	$0 < x_{\text{tot}} \leq 14$	409
Medium	$14 < x_{\text{tot}} \leq 32$	42
Long	$32 < x_{\text{tot}} < \infty$	15

2.2 Markov Chains using Transition Probability Matrix

In order to extract information from the real-world driving cycles and use it in a compact way, transition probability matrix, TPM, is used. The transition probabilities represents the Markov chain and are used to generate driving cycles.

Let the state $x_n = (v_n, a_n)$ where v_n and a_n are the current velocity and acceleration, respectively. The Markov property is that the present state contains all information that conditions the future state

$$P(X_{n+1} = x_{n+1}|X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n) = P(X_{n+1} = x_{n+1}|X_n = x_n), \quad (1)$$

which means that the current velocity and acceleration contain all relevant information in order to predict the next state $x_{n+1} = (v_{n+1}, a_{n+1})$. The probability of such a transition from state i to state j is

$$p_{ij} = P(X_{n+1} = j|X_n = i), \quad (2)$$

and p_{ij} is predicted by all such transitions that have occurred in the real-world driving data. All the transition probabilities are stored as a TPM. The data have been for practical reasons been discretized in steps of 1 km/h and 0.2 m/s^2 in velocity and acceleration, respectively. For more information about implementation aspects, see Lee and Filipi [2011], Torp and Önnegren [2013].

Generation of a driving cycle, v, using Markov chains in this paper is the process of initiating the driving cycle with velocity and acceleration of zero ($x_0 = (v_0, a_0) = (0,0)$) and then randomly choose the next state based on the probabilities, see (2), in the previously generated TPM. This will continue until a certain duration in time is achieved and the velocity has reached zero, $v_n = 0$. For validation of the Markov property when generating driving cycles and the validation of the used driving cycle generation software, see Shuming et al. [2013] and Torp and Önnegren [2013].

The generated driving cycle, $v = \{v_1, v_2, \ldots, v_n\}$ are then checked if it fulfills the selection criterion which will be explained in more detail in Section 4. If the selection criterion is not fulfilled the generation of the driving cycles will continue until it does.

3. MEAN TRACTIVE FORCE AND EQUIVALENT DRIVING CYCLES

The concept of equivalent driving cycles based on mean tractive force, MTF, was introduced in Nyberg et al. [2013] and it focuses on the vehicle's tractive energy per distance at the wheels in a driving cycle. The main points are now recapitulated.

The total tractive force, $F(t)$, at the wheels for flat roads consists of aerodynamic resistance, F_{air}, rolling resistance, F_{roll}, and inertia resistance, F_{m}, and the three components are here modeled as

$$F(t) = F_{\text{air}} + F_{\text{roll}} + F_{\text{m}} \tag{3}$$
$$F_{\text{air}} = \frac{1}{2}\rho_a c_d A_f v^2(t) \tag{4}$$
$$F_{\text{roll}} = mgc_r \tag{5}$$
$$F_{\text{m}} = ma(t), \tag{6}$$

where ρ_a is the air density, c_d the drag coefficient, and the frontal area of the vehicle is denoted A_f. Moreover, the vehicle mass is m, c_r is the rolling friction coefficient, and the gravitational constant is g.

When comparing driving cycles the measures that have been used previously have usually been defined on the whole time interval $\tau = [0, t_{\text{final}}]$. Example of such is the mean speed, standard deviation of acceleration, percentage of time spent in cruising etc. However, the characterization used here is based on the MTF [Guzzella and Sciarretta, 2007]. It is a measure on what mean tractive force the powertrain needs to provide during a driving cycle. Since the powertrain does not need to provide any forces to the wheels during coasting or braking regions ($F(t) \leq 0$) the integration intervals are those when the powertrain need to provide positive power to the wheels ($F(t) > 0$). For the MTF, $\bar{F}_{\text{trac}}$ is defined on a subset $\tau_{\text{trac}} = \{t \in \tau : F(t) > 0\}$ (see the marked regions near the x-axis in Figure 1), and is written as

$$\bar{F}_{\text{trac}} = \frac{1}{x_{\text{tot}}} \int_{t\in\tau_{\text{trac}}} F(t) \cdot v(t)\, dt, \tag{7}$$

where x_{tot} is the total distance traveled in the driving cycle.

From (3) and (7) the MTF can be partitioned into its components according to

$$\bar{F}_{\text{trac}} = \bar{F}_{\text{air}} + \bar{F}_{\text{roll}} + \bar{F}_{\text{m}} \tag{8}$$
$$\bar{F}_{\text{air}} = \frac{1}{x_{\text{tot}}} \int_{t\in\tau_{\text{trac}}} \frac{1}{2}\rho_a c_d A_f v^3(t)\, dt = \frac{1}{2}\rho_a c_d A_f \alpha \tag{9}$$
$$\bar{F}_{\text{roll}} = \frac{1}{x_{\text{tot}}} \int_{t\in\tau_{\text{trac}}} mgc_r v(t)\, dt = mgc_r\beta \tag{10}$$
$$\bar{F}_{\text{m}} = \frac{1}{x_{\text{tot}}} \int_{t\in\tau_{\text{trac}}} ma(t)v(t)\, dt = m\gamma. \tag{11}$$

For a given vehicle the vehicle parameters are fixed and if the air density is assumed to be constant then two driving cycles have the same values on the MTF components, $\bar{F}_{\text{air}}$, $\bar{F}_{\text{roll}}$, and $\bar{F}_{\text{m}}$, if they have the same values on α, β, and γ which are defined as

$$\alpha(v) = \frac{1}{x_{\text{tot}}} \int_{t\in\tau_{\text{trac}}} v^3(t)\, dt \tag{12}$$
$$\beta(v) = \frac{1}{x_{\text{tot}}} \int_{t\in\tau_{\text{trac}}} v(t)\, dt = \frac{x_{\text{trac}}}{x_{\text{tot}}} \tag{13}$$
$$\gamma(v) = \frac{1}{x_{\text{tot}}} \int_{t\in\tau_{\text{trac}}} a(t)v(t)\, dt, \tag{14}$$

where x_{trac} is the distance traveled in the traction regions.

The definition of equivalent driving cycles in Nyberg et al. [2013] is

Definition 1. For a given vehicle, two driving cycles, $v_1(t)$ and $v_2(t)$, are said to be *equivalent*, denoted $v_1(t) \sim v_2(t)$, if the following are fulfilled

$$\alpha(v_1(t)) = \alpha(v_2(t))$$
$$\beta(v_1(t)) = \beta(v_2(t))$$
$$\gamma(v_1(t)) = \gamma(v_2(t)).$$

In order to change the MTF by doing changes in the driving cycle the following is important to note

$$\begin{aligned}\bar{F}_{\text{m}} = m\gamma &= \frac{1}{x_{\text{tot}}} \int_{t\in\tau_{\text{trac}}} m\dot{v}(t) \cdot v(t)\, dt \\ &= \frac{1}{x_{\text{tot}}} \int_{t\in\tau_{\text{trac}}} \frac{m}{2} \cdot \frac{dv^2(t)}{dt}\, dt \\ &= \frac{1}{x_{\text{tot}}} \sum_{i}^{\#trac} [\frac{m \cdot v^2(t)}{2}]_{t_{i,\text{start}}}^{t_{i,\text{end}}}. \end{aligned} \tag{15}$$

Hence the MTF components related to vehicle inertia is the sum of the difference in kinetic energy over all traction intervals. The number of traction intervals is denoted *#trac* and each traction interval has its start, $t_{i,\text{start}}$, and end point $t_{i,\text{end}}$.

3.1 Motivation for using the MTF components and not only the total MTF

In the definition above the individual components of MTF are used, and a discussion of this is given now. In Lee and Filipi [2011] the response variable in their regression model is the specific energy which is the same as the total MTF, $\bar{F}_{\text{trac}}$. In their work the minimum number of explanatory variables that still can explain $\bar{F}_{\text{trac}}$ sufficiently good ($r^2 > 0.9$) are chosen. The MTF is a tentative value of the fuel consumption [Guzzella and Sciarretta, 2007] and to investigate this, a simulation study of the fuel consumption has been performed.

In order to perform the fuel consumption study a model of the powertrain is used. For simplification the driveline

Table 3. Parameters used in the simulation.

Vehicle		Engine		Gearbox
$A_f = 2$	[m^2]	$H_l = 44.5 \cdot 10^6$	[J/kg]	$gr_1 = 9.97$
$c_d = 0.4$	[-]	$\rho_f = 737.2$	[kg/m^3]	$gr_2 = 5.86$
$c_r = 0.013$	[-]	$J_e = 0.2$	[kg m^2]	$gr_3 = 3.84$
$m_v = 1600$	[kg]	$e_g = 0.4$	[-]	$gr_4 = 3.68$
$\rho_a = 1.29$	[kg/m^3]	$p_{me0} = 10^5$	[Pa]	$gr_5 = 2.14$
$g = 9.81$	[m/s^2]	$V_d = 0.03$	[m^3]	

is assumed to be ideal, hence the efficiency is 1. The gear-shifting strategy is extracted from the NEDC. The fuel consumption study is based on a vehicle with parameters according to Table 3 and where the engine is modeled as a Willan's line. The model is (16) [Guzzella and Sciarretta, 2007] and it is based on a constant friction loss inside the engine, $T_{\text{fric}} = p_{\text{me0}}\frac{V_d}{4\pi}$, and the torque originating from acceleration of the engine, $J_e\dot{\omega}_{\text{ICE}}$, is also considered. The fuel power P_{fuel} is

$$P_{\text{fuel}} = \frac{1}{e_g}(T_{\text{ICE}}\omega_{\text{ICE}} + p_{\text{me0}}\frac{V_d}{4\pi}\omega_{\text{ICE}} + J_e\dot{\omega}_{\text{ICE}}\omega_{\text{ICE}}), \quad (16)$$

where the parameters are the indicated engine efficiency, e_g, engine torque, T_{ICE}, engine speed, ω_{ICE}, engine acceleration, $\dot{\omega}_{\text{ICE}}$, loss mean effective pressure, p_{me0}, volume of the engine, V_d, and finally the engine inertia, J_e.

The fuel mass, m_{fuel}, is calculated as

$$m_{\text{fuel}} = \int_0^{t_{\text{final}}} \frac{\max(P_{\text{fuel}}, 0)}{H_l} dt, \quad (17)$$

where t_{final} is the final time of the driving cycle, and H_l is the lower heating value for gasoline. The efficiency of the engine can be written as

$$\eta_{\text{ICE}} = \frac{P_{\text{ICE}}}{P_{\text{fuel}}} = \text{e}_g \frac{T_{\text{ICE}}}{T_{\text{ICE}} + T_{\text{fric}} + J_e\dot{\omega}_{\text{ICE}}}, \quad (18)$$

and is affected by the indicated engine efficiency, the engine torque, the friction torque, and also the torque for acceleration or deceleration of the engine. For the steady-state case ($\dot{\omega}_{\text{ICE}} = 0$) the only variable is the engine torque since e_g and T_{fric} are constant parameters and the engine efficiency in (18) is a function of engine torque. The steady-state case can be seen in Figure 4 as the solid line. The dots in the same figure are the efficiency of the engine operating in a driving cycle where acceleration and deceleration of the engine is considered. For example, the dots below the steady-state (solid) line corresponds to points where the extra fuel mass injected, hence lower efficiency, are due to acceleration of the engine's inertia.

A comparison between the ability to predict the simulated fuel consumption with linear regression is shown in Figure 5. Using the driving cycle generation tool 100 driving cycles have been generated and split into two halves. The first half consists of 50 driving cycles that is used as estimation data for the linear regression where the response variable is the simulated fuel consumption and the regressors are either the MTF components, $\bar{F}_{\text{air}}$, $\bar{F}_{\text{roll}}$, and $\bar{F}_{\text{m}}$, or the total MTF, $\bar{F}_{\text{trac}}$. The other 50 driving cycles are used for validation data and in the figure the circles correspond to the MTF components as regressors and the triangles correspond to the regressor $\bar{F}_{\text{trac}}$. The r^2-fit are 0.95 and 0.89, respectively. The mean relative error are 1.3 and 2.0, respectively. Splitting up the MTF into its components clearly gives a better estimation of the fuel consumption. This shows that using the MTF components as an equivalence measure between driving cycles are a better choice than using the total MTF.

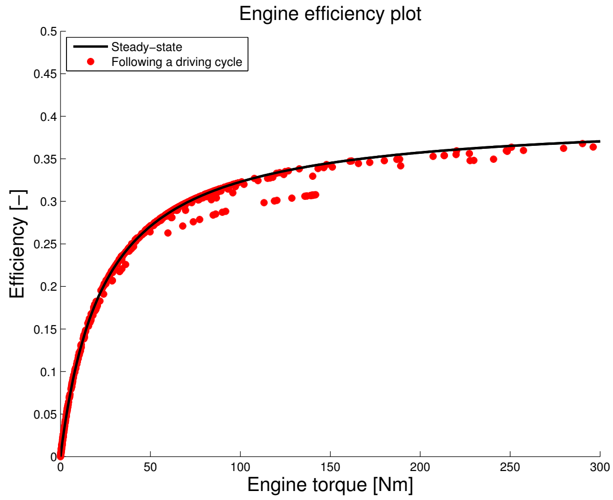

Fig. 4. Steady-state ($\dot{\omega}_{\text{ICE}} = 0$) efficiency from (18) (solid line) and engine efficiency operating in a driving cycle (see solid line in Figure 6) (dots).

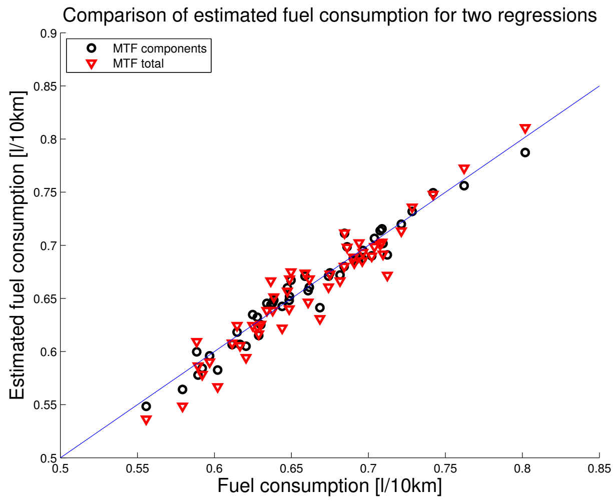

Fig. 5. A comparison between the two regressions that estimates the fuel consumption based on the MTF components, $\hat{y} = b_0 + b_1x_1 + b_2x_2 + b_3x_3$, (circles) or the total MTF, $\hat{y}' = b_0' + b_1'x_1'$, (triangles).

4. EQUIVALENT DRIVING CYCLE GENERATION

The concept of equivalent driving cycle from the previous section will be used as the final steps to generate equivalent driving cycles from real-world driving data. The process can be viewed in Figure 2 where the middle and right part of the figure shows the transformation from candidate driving cycles, which fulfill the selection criterion, to equivalent driving cycles. These steps will now be explained in more detail.

4.1 Selection Criterion

If a generated driving cycle, $v(t)$, from the steps in Section 2 fulfills the selection criterion it has the required characteristics, and it is called a candidate driving cycle. There are different selection criteria that can be used and some examples are statistical measures on key parameters such as velocity, acceleration, time spent in certain modes, number of steps etc. need to be close enough to some values. Lee and Filipi [2011] use linear regression in order to determine which parameters that are significant in order to describe the specific energy, also called mean tractive force, MTF. Principal components analysis, PCA, is used in Gong et al. [2011] in order to decrease the number of correlated parameters, and in Torp and Önnegren [2013] the selection criterion can be changed from different options, from linear regression of the MTF as in Lee and Filipi [2011] to PCA as in Gong et al. [2011].

The selection criterion used in this work is that the MTF components shall be similar, that is the selection criterion is fulfilled if the following holds

$$|1 - \alpha(v)/\alpha'| \leq \psi_\alpha \tag{19}$$

$$|1 - \beta(v)/\beta'| \leq \psi_\beta \tag{20}$$

$$|1 - \gamma(v)/\gamma'| \leq \psi_\gamma \tag{21}$$

where α', β', and γ' are the desired predetermined values, possibly from an existing driving cycle, and the thresholds for respectively value are ψ_α, ψ_β, and ψ_γ. The thresholds are currently set to some percent in deviation compared to the predetermined values since the difference in MTF should not be too large between the candidate driving cycle, and the predetermined values in order for the next step below to work well.

There is a trade-off between the size of the thresholds and the computational time. With more conservative thresholds, the probability to generate a candidate driving cycle within a certain time will go down and vice verse. The underlying real-world driving cycles also affect the time to generate a candidate driving cycle. However, a detailed analysis of this is out of scope of this paper.

4.2 Transformation to Equivalent Driving Cycles

When a candidate driving cycle has been generated it will then be transformed according to Algorithm 2 in Nyberg et al. [2013] to produce an equivalent driving cycle. The equivalent driving cycle, which were explained in Section 3, has the same vehicle excitation regarding the components of the mean tractive force.

Let $\tilde{v}$ be the transformed driving cycle, and ϵ_β, ϵ_γ, and ϵ_α be thresholds for the absolute allowed differences. The final transformation is performed in three steps as

1: The quantity β in (13) is the ratio between traveled distance during traction regions and the total driven distance in the driving cycle. To change the driving cycle such that $|\beta(\tilde{v}) - \beta'| \leq \epsilon_\beta$ the speed points within the traction regions will be altered iteratively until the absolute difference between $\beta(\tilde{v})$ and β' is sufficiently small.

2: To change F_m and thus γ when x_{tot} is constant and traction regions are intact the only solution is to change the speed points at the start or end of each traction region according to (15). By iteratively changing the end points it is possible to achieve $|\gamma(\tilde{v}) - \gamma'| \leq \epsilon_\gamma$.

3: Finally, to change the driving cycle to get $|\alpha(\tilde{v}) - \alpha'| \leq \epsilon_\alpha$ while maintaining both $\beta(\tilde{v})$ and $\gamma(\tilde{v})$, is achieved by expanding or contracting of the speed points (keeping the average speed) in the driving cycle.

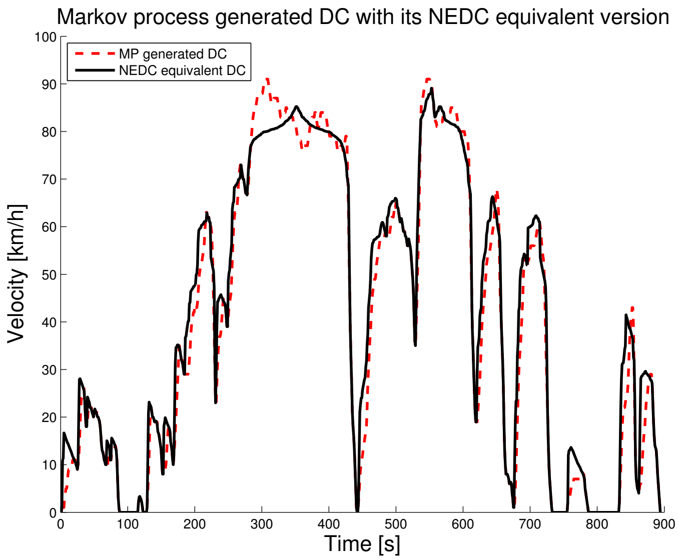

Fig. 6. A Markov process generated driving cycle (dashed) that have similar MTF components as the NEDC (4%, 1%, and 17% difference) and a transformed equivalent driving cycle (solid) to NEDC.

4.3 Method Summary

To summarize, relating to Figure 2, the method creates a TPM from real-world driving data and generates driving cycles with a certain duration. These pre-candidate driving cycles are regarded as candidate driving cycles, by the selection criterion, if the relative difference between the desired values is sufficiently small enough according to (19 - 21). As a final step the candidate driving cycles are transformed to equivalent driving cycles using the three-step algorithm that was described in the previous section.

5. RESULTS

Equivalent versions to the NEDC will be generated for the illustrative purpose of using the whole process from real-world driving cycles to equivalent driving cycles using Markov chains and MTF. Even if the NEDC is not representative for real-world driving it is still interesting to see what an equivalent version of it may look like.

Three driving cycles that are equivalent to NEDC have been generated from the database using the categories *Short*, *Urban*, and *Mixed*. Each of the equivalent driving cycles have its corresponding candidate driving cycle. From the category 'Mixed', see Table 1, an equivalent driving cycle has been generated that can be seen in Figure 6. The candidate driving cycle (dashed line) has the relative difference compared to the NEDC of 4%, 1%, and 17% for the MTF components ($\bar{F}_{air}$, $\bar{F}_{roll}$, and $\bar{F}_m$) and the equivalent driving cycle (solid line) has the maximum relative difference of 0.07% in $\bar{F}_m$.

Two more equivalent driving cycles have also been generated. The real-world driving cycles that have been used are from the categories 'Urban' and 'Short', see Table 1 and Table 2. For the first category an equivalent driving cycle to NEDC and its candidate driving cycle are shown in Figure 7 where the candidate driving cycle (dashed line) has the relative difference compared to the NEDC of 1%, 1%, and 11% for the MTF components and the equivalent

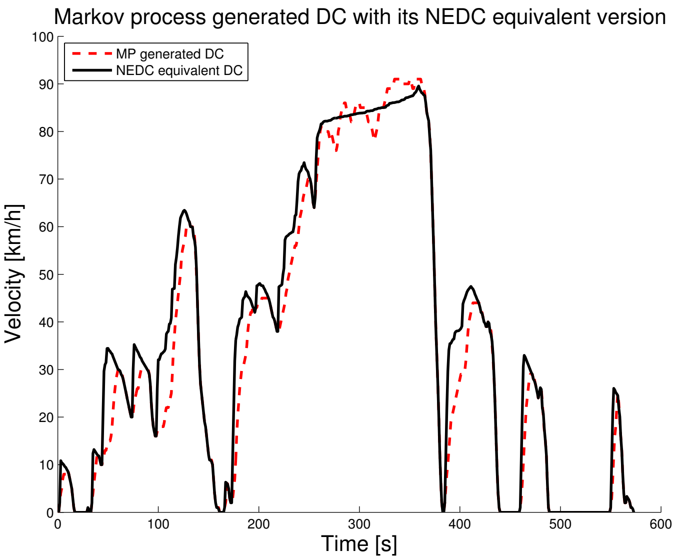

Fig. 7. A Markov process generated driving cycle (dashed) that have similar MTF as the NEDC (1%, 1%, and 11% difference) and a transformed equivalent driving cycle (solid) to NEDC.

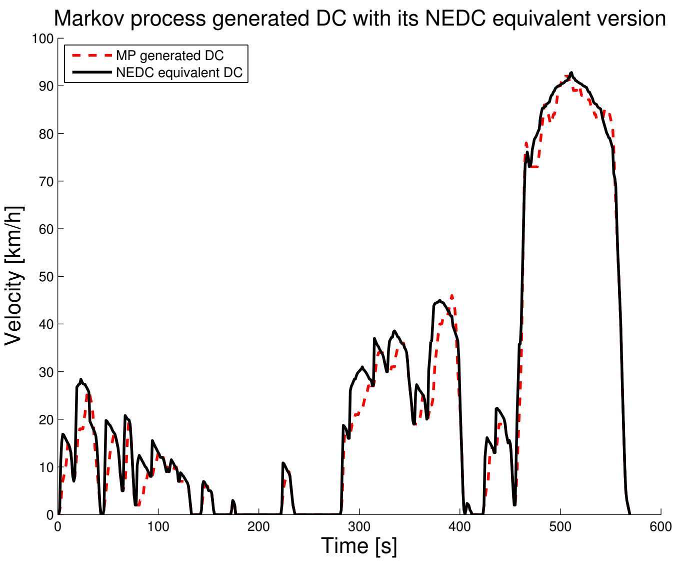

Fig. 8. A Markov process generated driving cycle (dashed) that have similar MTF as the NEDC (1%, 1%, and 20% difference) and a transformed equivalent driving cycle (solid) to NEDC.

driving cycle (solid line) has the maximum relative difference of 0.03% in $\bar{F}_{\mathrm{m}}$. For the latter category an equivalent driving cycle to NEDC and its candidate driving cycle are shown in Figure 8 where the candidate driving cycle (dashed line) have the relative difference compared to the NEDC of 1%, 1%, and 20% for the MTF components and the equivalent driving cycle (solid line) has the maximum relative difference of 0.008% in $\bar{F}_{\mathrm{m}}$.

6. CONCLUSIONS

The problem of generating representative driving cycles has been addressed, with the objective of being able to generate several equivalent driving cycles that are similar but not the same. A novel part is to use individual force components of the mean tractive force, instead of only using the total MTF, in the generation of driving cycles. This is motivated since the fuel consumption estimation is more accurate when the MTF components are considered as seen in Figure 5. Further, another novelty is to combine the generation of driving cycles from real-world driving data with the new concept of equivalence. One may note that the equivalence concept is used both in the selection criterion and in the final step. Hence, the methods are interwoven in the resulting overall algorithm, and the feasibility of the approach has been demonstrated in a number of examples. Thus, a new method has been presented and the results are promising.

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