Fuel Efficient Speed Profiles for Finite Time Gear Shift with Multi-Phase Optimization

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Abstract: A method that finds fuel optimal speed profiles for traveling a predefined distance is presented. The vehicle is modeled using a quasistatic formulation and an optimal control problem is defined. In addition, the solving method is based on a multi-phase optimization algorithm based on dynamic programming. This approach results in lower computational time than solving the problem directly with dynamic programming, it also makes the computational time independent of the travel distance. In addition, the simulation generated data can be used to get the solution to several optimal control problems in parallel that have additional constraints. Further a finite time gear shift model is presented to include the gear selection in the optimization problem. The problem also considers speed losses and fuel consumption during the maneuver. The results presented show the optimal speed and gear profiles to cover a distance, making special emphasis at the acceleration phase, where it is optimal to perform a fast acceleration to engage the highest gear as soon as possible. Finally a proposed application is to use the simulation data to provide eco-driving tips to the driver.

Keywords: Dynamic Programming, Simulation, Eco-driving.

1. INTRODUCTION

Driving more efficiently has become an important issue since the fuel cost has increased significantly during the last decades. One way to reduce the fuel consumption, and thereby reduce the CO_2 emissions, is to improve the efficiency of the powertrain by means of technical advances, e.g. hybridization. Another way to reduce the fuel consumption is optimizing the way how the driver operates the vehicle. Giving advice to the driver, or directly controlling the vehicle speed and gear shifting, can be useful to use the powertrain at the most efficient operating points, and thus reduce the fuel consumption.

The problem studied is to find vehicle optimal speed profiles for covering a predefined distance. Previous research has been done in this area. The first analytic approach to solve the problem was done by Schwarzkopf and Leipnik (1977), which was revised later by Chang and Morlok (2005). These two papers solve the problem using the Pontryagin maximum principle.

Optimal speed profiles have also been computed using numerical methods, mostly using Dynamic Programming (DP). Starting with the paper by Hooker (1988), or more recent papers like the ones by Saerens et al. (2009), Gausemeier et al. (2010), Luu et al. (2010), Jorge et al. (2011) and Mensing et al. (2011), where optimal speed profiles for several situations are presented.

The optimal control problem is solved by means of DP, see Bellman and Dreyfus (1962) and Bertsekas (1995). One difference with previous research is that the problem is solved as three separate phases that are joined optimally together. This formulation decreases computational time and is able to compute several speed profiles with the same output data from the optimization. Moreover it also adds the possibility to apply additional constraints once the simulation is computed.

Another contribution consists in presenting a model that optimizes the engaged gear together with the vehicle speed. It also takes into account the gearshift speed losses, due to no traction torque during the maneuver, and fuel consumption in the optimization.

Previous authors that studied the influence of gear shift as an instantaneous maneuver are Hooker (1988) and Mensing et al. (2011). Other studies used an automatic gearbox with a predefined gear shift strategy, see Gausemeier et al. (2010) and Luu et al. (2010). In Saerens et al. (2010), a speed penalty factor is taken into account during gear shift, however no fuel consumption is considered. On the other hand, in Hellström et al. (2010) both speed loss and fuel consumption are considered, however it requires to interpolate between state values to compute the cost.

A criterion for choosing a right discretization to ensure accurate results is also described in the study. Finally several optimal speed profiles are presented and analyzed, making specially emphasis at the acceleration phase.

2. VEHICLE MODEL

A quasistatic approach is used for modeling the vehicle. This approach allows to compute the vehicle fuel consumption as function of vehicle speed and gear. Thus, it is a well suited modeling approach for the DP algorithm, because the gear engaged and the speed at the beginning

Table 1. Vehicle, driveline and engine parameters used in the study.

Parameter	Symbol	Value	Unit
Vehicle mass	m_v	1500	kg
Frontal area	Af	2	m^2
Air drag coeff.	c_d	0.3	-
Rolling res. coeff.	c_r	0.01	-
Wheel radius	r_w	0.3	m
Wheel inertia	J_w	0.6	$kg \cdot m^2$
Gearbox efficiency	η_{gb}	0.98	-
Gear ratio 1	i_1	13.0529	-
Gear ratio 2	i_2	8.1595	-
Gear ratio 3	i_3	5.6651	-
Gear ratio 4	i_4	4.2555	-
Gear ratio 5	i_5	3.2623	-
Displacement	V_d	$2.3 \cdot 10^{-3}$	m^3
Indicated efficiency	e	0.35	-
Idling speed	$\omega_{e,idle}$	95	rad/s
Stroke	S	$79.5 \cdot 10^{-3}$	m
Bore	B	$96 \cdot 10^{-3}$	m
Cylinders	-	4	-
Engine inertia	J_e	0.2	$kg \cdot m^2$

and at the end of the arc are given by the optimization algorithm and the fuel arc cost can be obtained easily. Further information about the quasistatic approach can be found in Guzzella and Sciarretta (2013).

In the quasistatic formulation some assumptions are considered in order to compute the vehicle speed and acceleration for a given arc. The first assumption is that the acceleration remains constant over a step in distance. The second assumption is that the mean speed, computed between the initial and the final speed values of the current arc, is used in order to calculate the arc cost. These are reasonably good approximations for a sufficiently dense speed and distance grids.

2.1 Driveline Model

Following the previous mentioned quasistatic approach, the torque required at the wheels is computed using a longitudinal vehicle model (1), from Guzzella and Sciarretta (2013)

$$T_w = r_w \left(\underbrace{\frac{1}{2} A_f \rho_a c_d v^2}_{\text{air drag}} + \underbrace{c_r m_v g}_{\text{roll resist.}} + \underbrace{\dot{v} \left(m_v + \frac{J_w}{r_w^2} \right)}_{\text{inertia}} \right) \quad (1)$$

and a manual gearbox with a constant efficiency is used. Table 1 contains the main parameters used in this study. More details regarding the model are given in Llamas (2012).

2.2 Engine model

The Willans line approximation is used to compute the fuel consumption. Despite being a simple model, it gives reasonably good fuel consumption values at a low computational cost. The approximation is based on normalized engine variables, that do not depend on the engine size, like the mean effective pressure, p_{me}, and the fuel mean pressure, p_{mf}. The output power of the engine, represented by the mean effective pressure, is computed by an affine function (Guzzella and Sciarretta, 2013)

$$p_{me} = e \cdot p_{mf} - p_{me0} \quad (2)$$

where e is the indicated engine efficiency and p_{me0} represents the friction and pumping losses in the engine. The term p_{me0} is modeled using the ETH friction model from Guzzella and Onder (2004). Table 1 contains the engine parameters used.

The maximum torque available, $T_{e,max}(\omega_e)$, is determined by interpolating the maximum torque data from the QSS toolbox from Guzzella and Amstutz (1999). This data is scaled in order to match the engine dimensions.

3. PROBLEM FORMULATION

The problem goal is to minimize fuel consumption, thus the optimal control problem is formulated as follows:

$$min \int_0^T \dot{m}_f \, dt \qquad \dot{X} = f(X, U) \quad (3)$$

where $X = (x,\ v,\ g)^T$ is the state vector and consists of position, speed and engaged gear and $U = (T_e,\ u_g)^T$ is the control vector that consists of engine torque and gear shift command. This minimization is restricted to the following constraints:

$$T_e \leq T_{e,max} \qquad a \leq a_{max}$$
$$\omega_e \geq \omega_{e,idle} \quad (4)$$
$$X(0) = (0,\ 0,\ 0)^T \qquad X(T) = (x_{final},\ 0,\ 0)^T$$

that are interpreted as functional limits in engine torque and engine speed as well as a limit in vehicle acceleration. Initial and final conditions of the optimal control problem are also defined, as: drive a certain distance, x_{final} and starting and finishing at stand still with neutral gear.

4. MULTI-PHASE DYNAMIC PROGRAMMING ALGORITHM

As DP is used to find the optimal speed profile, a discrete state-space model is required. The variables to be optimized through a certain distance are speed and engaged gear. The gear engaged is already a discrete variable, thus speed has to be discretized as well as distance, which is used as a tracking variable.

A three dimensional DP algorithm is required to compute the optimal speed profile. More dimensions in the DP algorithm means that the computational time becomes much more influenced by the discretization size, due to the "curse of dimensionality" (Bellman and Dreyfus, 1962). Hence, it is interesting to adapt the DP algorithm to the particularities of this specific problem to save computational time. It is known from previous research that keeping constant speed is a fuel optimal policy (Chang and Morlok, 2005; Schwarzkopf and Leipnik, 1977). It is also pointed out in Llamas (2012) that the optimal policy is to accelerate fast to reach a constant cruising speed and then do coasting until stand still.

Thus, there is a long phase of the optimization that consists of keeping constant speed and therefore having a fine grid in this phase is a waste of resources. The key idea is to split the optimization algorithm into three phases that later on are optimally joined together. These three phases are: acceleration, constant cruising and deceleration, see Figure 1.

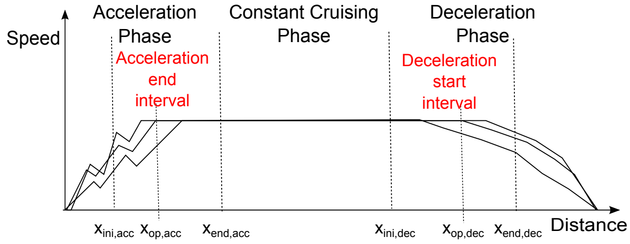

Fig. 1. Multi-Phase DP algorithm diagram.

With this approach the acceleration and deceleration phases are still optimized using DP with a dense grid to obtain accurate results. Note that the deceleration phase consists in doing coasting and thereby the speed profile could be computed analytically. However, this is not possible because the gearshift profile needs to be optimized. The two phases do not require from data of each other, and they can thus be solved in parallel to save computational time. Using the results from both DP phases, optimal profiles for different constant cruising speeds can be computed.

The algorithm that optimally merges the three phases requires a predefined constant cruising speed value and gear. Then, as illustrated in Figure 1, it generates the cost to go matrices over the distance intervals at the acceleration $(x_{ini,acc}, x_{end,acc})$ and deceleration phases $(x_{ini,dec}, x_{end,dec})$ to find the joining points $(x_{op,acc}, x_{op,dec})$ that result in the lowest fuel consumption. A reference value for setting suitable intervals can be found by looking at the optimal profiles in Llamas (2012). Note that the maximum length of these intervals is given by the distance grid used in the DP simulations and also that the required distance grid is proportional to the desired cruising speed.

In addition, obtaining longer distance optimal profiles do not affect the computational time because it only requires a longer constant cruising speed phase. It has also the possibility to add other constraints to the problem, e.g. maximum distance to accelerate or decelerate the vehicle.

4.1 Acceleration Phase

The acceleration profile is optimized using Forward DP. The difference with the well known Backward DP is that the arc costs are computed forwards, from the initial state to the end state, while Backward DP computes them from the final state to the initial state. Once the arc costs are known, the Forward algorithm computes the optimal profile from a given final speed, gear and distance to the initial condition point $X(0)$. Thus with one simulation all optimal acceleration profiles from the initial condition point $X(0)$, to every speed, gear and distance grid points are known. In this phase the vehicle is only allowed to accelerate if no gear shift is performed.

4.2 Deceleration Phase

The deceleration profile is computed using two strategies that are later compared in Section 7. The first one is to compute the optimal profiles using a Backward DP algorithm. The Backward formulation is used here because it gives all optimal profiles from a certain grid point to the final condition point $X(T)$. For this strategy the engine is either running or in fuel cut.

The second strategy is coasting with the engine shut down. That means that the engine is disengaged from the powertrain and thus the vehicle speed loss is only due to air drag and rolling resistance. The speed profiles are computed analytically from an initial distance and speed state.

4.3 Constant Cruising Phase

This phase joins the acceleration and deceleration phases with a constant speed and gear profile. Given the cruising speed and gear, the algorithm finds the optimal joining points, $x_{op,acc}$ and $x_{op,dec}$, that result in the global lowest fuel consumption, see Figure 1.

5. FINITE TIME GEAR SHIFT MODEL

A new way to approach a finite time gear shift maneuver is developed. The model computes the fuel consumption and the speed loss during the maneuver time and then these values are taken into account by the optimization algorithm. This approach has been inspired by the gear shift model from Hellström et al. (2010), however when it is implemented for the DP algorithm it does not require to interpolate between state values if there is a gearshift.

The maneuver time is defined as the time elapsed since the clutch is pressed until the engine is engaged again. In this study it is set to be 1 s. The assumption made to compute the fuel consumption and the speed loss is that during the maneuver time the engine cannot provide traction torque. Thus the vehicle speed is computed by

$$\frac{dv}{dt} = \frac{-1}{m_v + \frac{J_w}{r_w^2}} \left(\frac{1}{2} \rho_a A f c_d v^2 + m_v g c_r \right) \tag{5}$$

since T_ω in (1) is set to zero. The cost of the maneuver is assumed to be the fuel consumed during the maneuver and that the engine is idling.

The distance traveled during the gear shift maneuver can be computed using the maneuver time and the final speed computed with (5). Hence, with that information the point in the state-space grid can be set (x_l, v_l). If $x_l \ll x_{k+1}$ the cost to go from that point to every next grid point, in x_{k+1}, can be computed. A gear shift computation is illustrated in Figure 2.

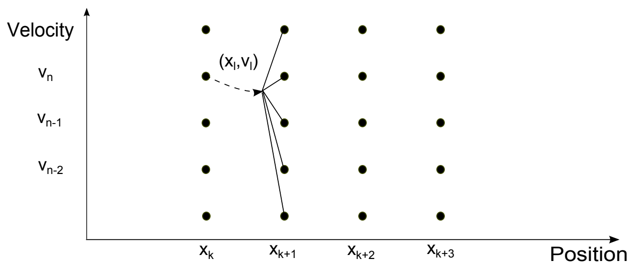

Fig. 2. Computation of the cost to go from (x_k, v_n) to each state of x_{k+1} with a gear shift maneuver.

However, some problems arise when the distance traveled during the gear shift is bigger, or nearly equal, to the step length used in the algorithm. These two situations have two related problems. The first one is that if the maneuver finishes close to the next distance step, the next speed points might not be reachable due to a high acceleration or braking torque is requested in the small distance that exists from the maneuver end point to the next grid step. The second issue is that if the gear shift maneuver ends after the next distance grid step, the cost to go cannot be computed following the standard DP algorithm.

In order to solve these problems, the standard DP algorithm is modified. If the gear shift maneuver ends after or too close to the next distance grid step, the algorithm computes the costs to the next available grid point and saves to which distance step the costs are computed. The model makes a difference if the next step has to be reached by accelerating or braking because if braking is needed, the model allows to do a gradual braking through the distance step even while performing the gearshift maneuver. The strategy is illustrated in Figure 3.

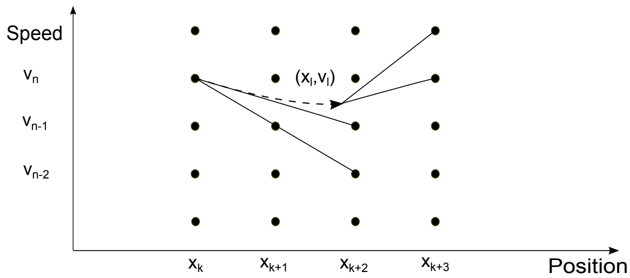

Fig. 3. Skipping grid points strategy from (x_k, v_n) during a gear shift maneuver.

6. DISCRETIZATION

The discretization of the continuous problem plays an important role in the accuracy of the results. Hence, how the discretization is defined on the acceleration phase is investigated because it is the phase with highest fuel consumption per distance. Then the discretization used in the deceleration phase is the same that is considered sufficiently accurate for the acceleration phase.

The acceleration profile to reach $8.75\,m/s$ with fifth gear in $50\,m$ is optimized with several grid choices and the results are presented in Figure 4. It can be observed that speed and gear profiles change especially at the beginning if a smaller distance step is selected. First gear is engaged instead of second for a certain step distance smaller than $5\,m$, in Figure 4 for values of $1\,m$ and $0.5\,m$.

In addition, one also must ensure that the assumptions made to compute the arc costs are reasonably fulfilled with the grid choice. If the grid is too sparse, there will be less distance points where the profile is computed, making the computations less accurate. E.g. for $\Delta x = 5\,m$, with only one distance step and thus one computation, the speed profile reaches $6\,m/s$ which is more than the half of the final speed value.

A grid is sufficiently accurate if further decrease of the grid size do not change the optimal profile, thus the optimal profile converges. Looking again at Figure 4, once first gear is selected (for $\Delta x \leq 1\,m$), decreasing the distance step more does not change the optimal profile and thereby the distance step is sufficiently small.

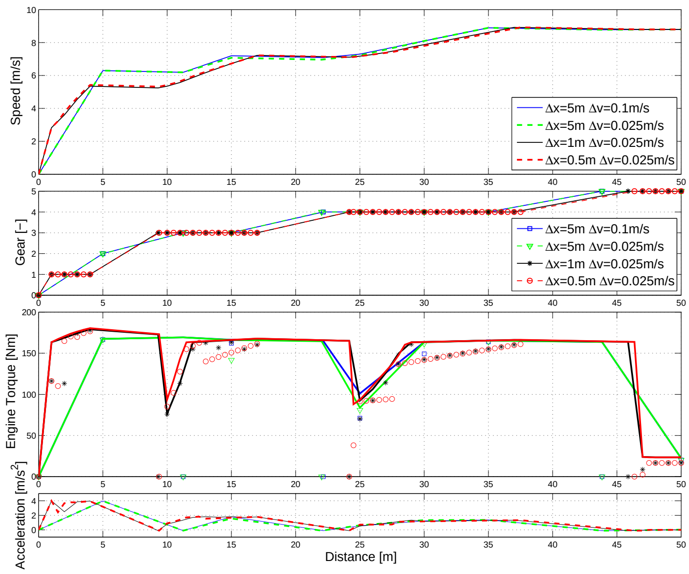

Fig. 4. Acceleration Phase with several grid step sizes. The thick lines in the Engine Torque plot represent the maximum engine torque.

Moreover, looking at the acceleration values for the different grid choices, it can be seen that for the first step the algorithm choice is always at the maximum acceleration boundary (which for this test is set to $a_{max} = 4\,m/s^2$). Hence the maximum acceleration value defines the profile shape at the very beginning. In this case, in order to ensure convergence, one must select a distance step that engages first gear at the beginning. This means that the optimal speed value at the first distance step must be lower than the minimum speed needed to engage second gear.

The speed at one step is computed as the mean value between the start and end points

$$v_m = \frac{v_{k+1} + v_k}{2} \tag{6}$$

thereby for the first step, using (6) with $v_1 = 0$ and kinetic energy, the speed after the first distance step is

$$v_2 = \sqrt{2 a_{max} \Delta x} \tag{7}$$

thus the relation that must hold to ensure that the first gear can be selected at the first distance step is

$$v_2 < \frac{\omega_{idle} r_w}{i_2} \tag{8}$$

that can be satisfied with a small enough value of Δx. With the parameters used, the relation is fulfilled with $\Delta x < 1.5\,m$, and the selected value is $\Delta x = 1\,m$. In addition, the selected speed step value is $\Delta v = 0.025\,m/s$.

7. RESULTS

First of all, an optimization of a speed profile simulation is carried out without additional constraints. Figure 5 presents the optimal speed profiles for several cruising speed values. It can be seen that the acceleration and deceleration phases are really similar for the each cruising speed.

The cruising speed of $8.75\,m/s$ is selected because this is the one that leads to the lowest fuel consumption, see

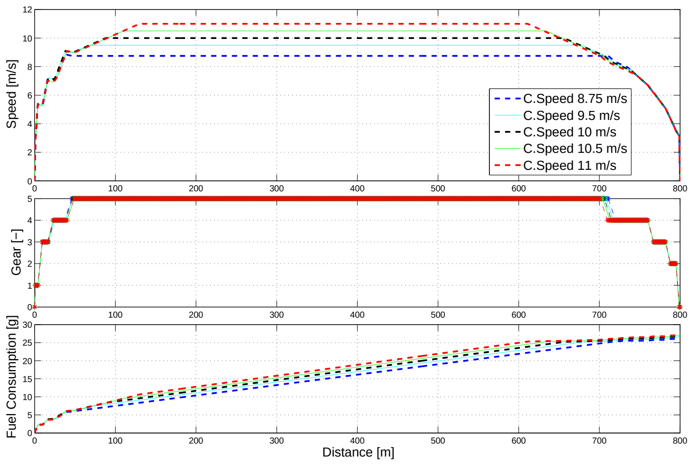

Fig. 5. Optimal speed profiles for several cruising speeds without constraints.

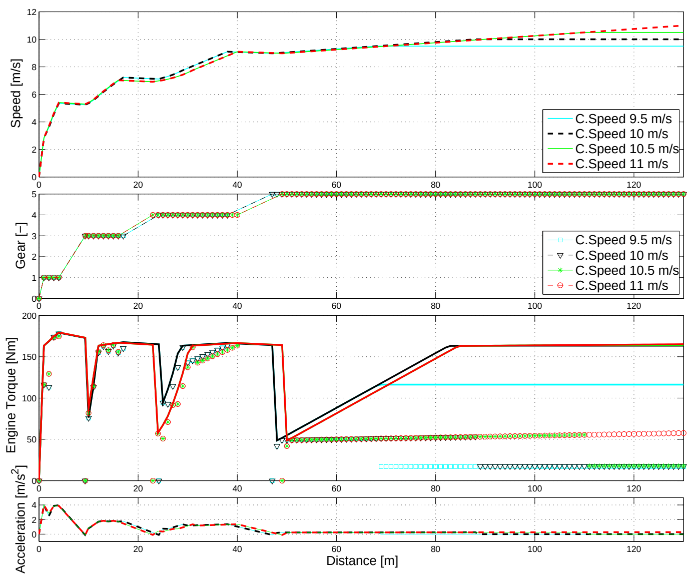

Fig. 6. Optimal acceleration profiles extracted from Figure 5. The thick lines in the Engine Torque plot represent the maximum torque available.

Table 2. In Llamas (2012) the whole drive mission is optimized by a one phase DP algorithm, and the results show that the optimal policy is to cruise at the lowest speed possible with the highest gear engaged. This optimal cruising speed is defined by the engine idling speed, the wheel radius and the gear ratio at the highest gear. The other cruising speeds are selected near that value to provide a comparison.

Figure 6 presents a closer look at the acceleration phase. It can be seen that the torque values are always close to the maximum, thus the optimal policy is to have high acceleration and engage higher gears as soon as they are available in order to reach fifth gear as soon as possible (see also Figure 4). As there is no time constraint, once the acceleration profile reaches the optimal cruising speed, $8.75\,m/s$, it continues accelerating as slow as possible until the fixed cruising speed. This slow acceleration depends on the grid choice (for each Δx, the speed increases Δv), due to that the model is not allowed to keep constant speed.

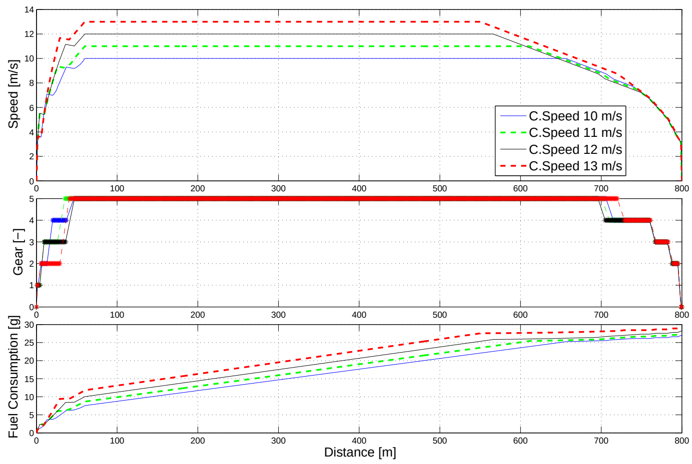

Fig. 7. Optimal speed profiles for several cruising speeds a maximum acceleration distance of 60 m

Table 2. Fuel consumption and trip time for both strategies and several cruising speeds.

C. Speed	Fuel DP	Fuel E. off	Time DP	Time E. off
$8.75m/s$	$4.48l/100km$	$2.95l/100km$	$104.65s$	$145.68s$
$9.5m/s$	$4.54l/100km$	$2.82l/100km$	$98.96s$	$140.83s$
$10m/s$	$4.58l/100km$	$2.72l/100km$	$95.88s$	$141.18s$
$10.5m/s$	$4.61l/100km$	$2.61l/100km$	$93.43s$	$140.93s$
$11m/s$	$4.65l/100km$	$2.50l/100km$	$91.37s$	$140.03s$

If constant speed is allowed, then the acceleration profile does cruising at $8.75\,m/s$ during a certain distance before accelerating to reach the imposed cruising speed.

In order to see the effects of applying additional constraints into the original problem, a limit is set into the acceleration distance, e.g. the cruising speed must be reached within $60\,m$. Figure 7 displays the results with the new constraint. Note that these new profiles can be obtained with the same DP simulation data.

Looking again at the acceleration phase, presented in Figure 8. With the applied constraint, the acceleration profiles now differ depending on the cruising speed. The gearshift strategy depends on this as well. However, the acceleration limit is reached at the very beginning by all speed profiles, the same acceleration is reached in Figures 4 and 6. Moreover the engine torque values are again close to its maximum values. Hence, in general the optimal speed and gear choice is the one that places the engine torque as close to the maximum as possible, which is due to that these are the zones of the engine map with high efficiency and thus lead to the lowest fuel consumption.

The results for the second deceleration strategy are presented in Figure 9. The optimal profiles have much longer coasting profiles because the speed losses are lower without the engine engaged to the driveline. In addition, the acceleration to the cruising speed is exactly the same as for the optimized deceleration using DP.

Table 2 presents the total fuel consumption as well as the trip time for the different cruising speed values with the two deceleration strategies. As one might think beforehand, it is true that shutting down the engine leads to a lower fuel consumption.

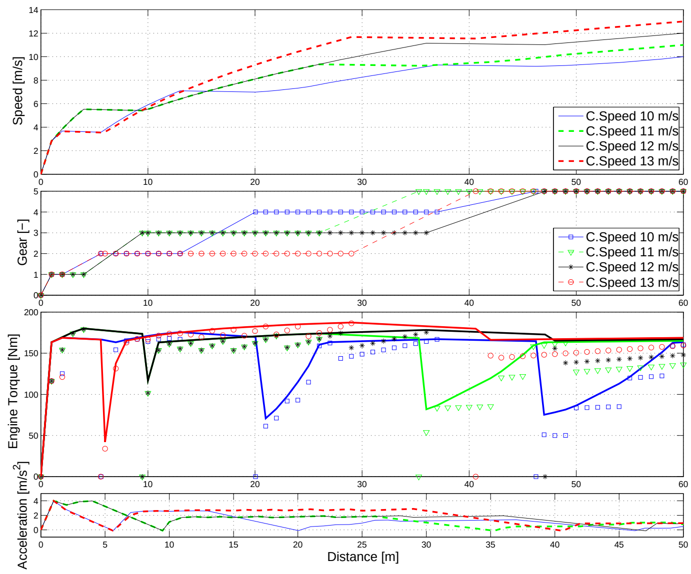

Fig. 8. Optimal acceleration profiles extracted from Figure 7. The thick lines in the Engine Torque plot represent the maximum torque available.

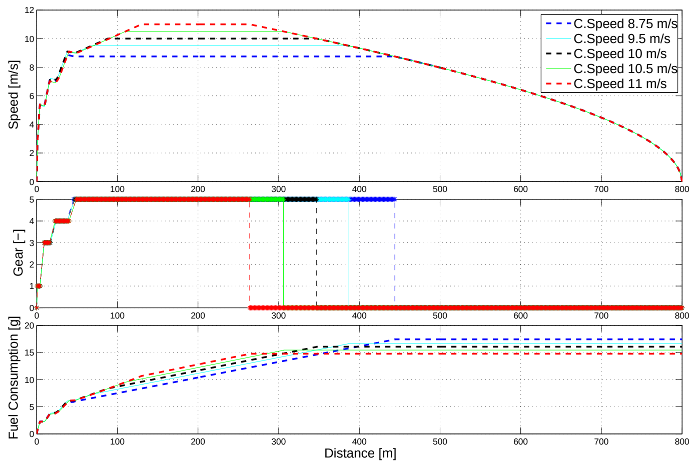

Fig. 9. Optimal speed profiles for several cruising speeds with engine shut down strategy.

8. CONCLUSION

A new approach to obtain speed profiles has been proposed. The algorithm is able to provide results applying different constraints with one single simulation data, while a normal DP algorithm would need one simulation for each optimal profile. This new algorithm has been built by taking assumptions about the shape of the optimal speed profile, i.e. constant cruising speed and coasting.

An advantage to use this method is to store simulation data for acceleration and deceleration conditions and consult them to obtain multiple optimal profiles without much more computational effort. It can be easily implemented into a vehicle computer to provide eco-driving tips to the driver.

Also a new way to handle gearshifts has been proposed, that takes into consideration the speed losses and the fuel consumption during the time that the gearshift maneuver takes place. This model obtains more realistic results than without consider gearshift losses, and takes into consideration the vehicle engaged gear.

Results regarding optimal speed policies have been presented. Performing fast gearshifts until the highest gear is engaged appoints as optimal, as well as keeping constant cruise speed with the highest gear engaged until decelerate using the fuel cut-off feature.

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