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Fuel Optimal Powertrain Control for Heavy Trucks Utilizing Look Ahead

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Abstract

The road topography in highways affects the powertrain control of a heavy truck substantially since the engine power is low in relation to the vehicle weight. In large road gradients constant speed is not possible to keep, which would have been beneficial otherwise, and in some uphills shifting gears becomes inevitable. If information about the road ahead, i.e. look ahead information, is available, then the powertrain can be controlled in a more fuel efficient way. Trial runs are performed, where the velocity trajectory that minimizes energy consumption, is calculated and communicated in real time as set points to the conventional cruise control. This look ahead control gives significant fuel consumption reductions compared to a standard cruise control, while keeping to the same mean speed. The results are the inspiration to further studies in how powertrain control can benefit from look ahead information. An engine with a non-linear fuel map is studied to understand its impact on fuel optimal speed. It is shown that for a significant fuel map non-linearity, quantified by a threshold value, constant speed in small road gradients is no longer optimal. Further, an automated manual transmission (AMT) optimal gear control is studied. It is shown that the reduced propulsion of a typical AMT gear-shifting process must be considered when choosing when to shift gears. Thus, additional reductions of fuel consumption are obtained with a look ahead control based on knowledge of engine and transmission characteristics.

Acknowledgments

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Finally, and most importantly, Per and Wilmer, you are my meaning of life!

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Contents

Part I

Introduction

1

Truck driving

Heavy trucks, defined as trucks with a gross vehicle weight of more than 16 tonnes, are built according to widely varying specifications. They are used in mining or construction applications as well as to distribute goods in cities or in long haulage driving. These applications all imply different requirements on functionality, robustness and driveability of the truck.

In long haulage for example, long distance highway driving is the most frequent driving scenario. In such stationary driving conditions, some automatic systems developed for heavy trucks are particularly suitable. A requested and commonly used automatic system is the cruise control, another is the automated transmission. Besides being a convenient help system in long haulage driving, the cruise control is sometimes also the most fuel efficient way of driving, depending on the road topography of the driving scenario and the efficiency of the engine.

In a market which is becoming more and more global, there is an increasing demand of transporting goods. This indicates an increasing need of long haulage heavy trucks in the long term. Still, a concern is the global warming of today, an effect partly due to combustion of fossil fuels, since the greenhouse gas carbon dioxide, CO_2, is a residual product when e.g. truck diesel is combusted. Fuel optimal powertrain control, i.e. control of engine and transmission, is thus of great interest and will be developed and evaluated. The take-off point is that fuel optimal control can benefit from look ahead information. In the following two sections, characteristics of long haulage operation is presented as well as the definition of look ahead control. Subsequently, the objective is stated and the potentials and difficulties in finding the optimal solution are discussed. Thereafter, the reader is briefly guided through some fundamentals of powertrain control and previous research in this area.

1.1 Implications of the long haulage business

The objective of all heavy truck contractors is of course to fulfill their commitments whilst associated costs are minimized. However, the distribution of costs varies between different businesses. The two largest costs for a long haulage contractor are the truck driver salary and the fuel consumption of the truck. Fuel consumption accounts for as much as 25-30% of total life cycle cost. As all the costs are added there is most often not a wide margin to make a profit. Since fuel consumption is a big part of the haulier's total cost, a reduction of fuel consumption as small as one percent is significant to reduce the total cost and thus make long haulage more profitable. Moreover, the goods must still be delivered in time which means that the required final time of the driving scenario must not be violated while trying to minimize fuel consumption.

There is a wide range of skills of truck drivers. A skilled driver can reduce the fuel consumption with more than 10% compared to an unexperienced driver, by reducing the need of braking and by adapting speed and gear to e.g. the surrounding traffic and according to the topography of the road ahead, and still deliver the goods in time. However, automatic systems are commonly used in highway driving and consequently the impact of the driver's experience is decreased. A conclusion to be drawn is that there is potential of reducing fuel consumption, and hence diminishing costs of the haulier, if information about the road ahead is available to the control systems.

1.2 Look ahead control

Control strategies that use information about the road ahead, i.e. look ahead control, could improve functionality of the vehicle in many senses. This is particularly true for a heavy truck since it often has a large vehicle mass in relation to its engine power, generating a slow longitudinal dynamics of the vehicle. Input data to a look ahead system could include various attributes such as topography, road curvature, stop lights, speed limits, road restrictions and surrounding traffic. The vehicle continuously evaluates its position, by e.g. GPS, and sets up a look ahead horizon of a predicted future route, see Figure 1.1. The desired attributes are collected from a database for the distance ahead of the truck, the look ahead horizon. There are many systems that could benefit from look ahead information and the examples are diverse. Example applications go from powertrain control such as engine and transmission control, to control of brakes, state of charge in hybrid vehicles or control of auxiliary units such as coolant pump, cooling fan or air compressor. The benefits could be reductions of fuel consumption or enhancements in comfort or active safety.

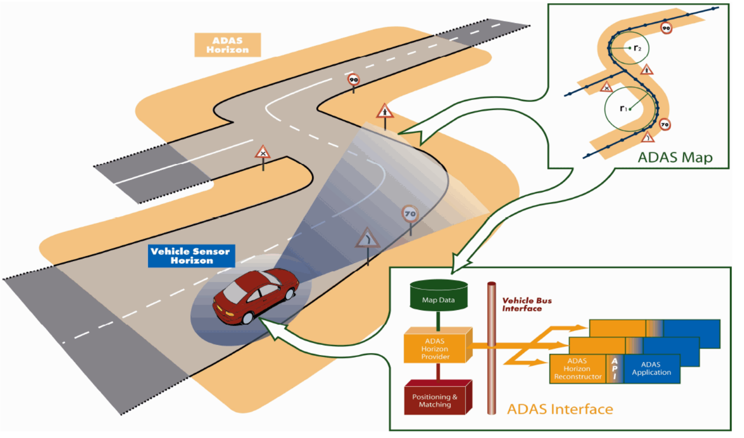

Figure 1.1: Look ahead horizon of a predicted future route with different attributes such as speed limits and road curvature. (Illustration courtesy of the PReVENT project).

1.3 Objective – difficulties and potentials

By respecting the scenario of global warming and how combustion of diesel relates to that, in combination with the financial difficulties that hauliers are facing every day to make a profit out of their business, a strong demand on all sides is to decrease fuel consumption for heavy trucks. The objective is thus to find the fuel optimal way of driving a heavy truck, while trip time is respected. Optimal control of engine fueling and gear shifting, which generates an optimal vehicle speed profile and an optimal engine speed profile, is studied for long haulage highway driving missions. For this application and purpose, only the effects of topography are assumed to affect the objective and thereby only this look ahead attribute is considered. Topography maps are expected to be available for commercial purposes in a near future. However, since a long haulage truck often completes the same route many times, the gradient of the road traveled could alternatively be estimated off-line, see (Sahlholm, 2008), and used for control purposes in the coming drives.

The difficulties that arise with this problem formulation are various, e.g. the system models may be non-linear, and in order to find the optimal gear of a discrete transmission in combination with the optimal control of fueling, a hybrid optimization is required. Further on, this is very much a problem close to the real application, where the effects of a real enviroment must be evaluated in terms of unknown disturbances.

To conclude, an automatic powertrain control system, using look ahead information, has the potential to reduce fuel consumption in long haulage driving missions, independent of the experience of any specific driver. This is beneficial from many perspectives, both for environmental and economical reasons. Legislation will eventually force reductions of CO_2 and consequently reductions of fuel consumption but, again, the greatest incentive of reducing fuel consumption is already present as the haulier significantly increases his profit with a low fuel consumption.

1.4 Powertrain control and road topography

The topography affects powertrain control of heavy trucks strongly, since uphill driving requires a much higher torque compared to going downhill and some hills are not possible to climb without shifting gears. Since the considered driving scenarios are restricted to highways, also the considered road gradients, α, are restricted. According to the Swedish Road Administration (2004), a highway classified as high standard has road gradients according to $-6\% \leq \alpha \leq 6\%$, where uphill gradients are positive and downhill gradients are negative. The road gradient unit [%] corresponds to a 1 meter vertical elevation/hollow in a horizontal move of 100 meters. There are also restrictions in speed, and the maximum speed allowed for heavy trucks is 89 km/h.

Large road gradients, also denoted as significant hills, are defined in (Fröberg et al., 2006), as gradients where the vehicle looses speed when going uphill, even with maximum fueling, and gains speed when going downhill, even with no injected fuel. Large highway gradients are interesting in order to find slopes that enforce gear shifting, whereas small gradients are interesting when studying the characteristics of the fuel map. In the following subsections, general results from previous automotive research and common know-how in powertrain control are presented in terms of optimal speed, engine efficiency and gear shifting.

1.4.1 Optimal speed

In a driving mission on small gradients with a constraint on trip time, constant speed has been found to be fuel optimal by several publications, see (Chang and Morlok, 2005; Fröberg et al., 2006; Hellström, 2007), when assuming an affine relationship between injected fuel and engine torque. In roads with steeper slopes the propulsive control signal, i.e. injected fuel or engine torque, runs into its upper or lower limit and constant speed is no longer possible to keep only by controling the propulsion. This is more often the case when driving a heavy truck, in contrast to passenger cars, due to its high vehicle weight in relation to engine power.

In a driving mission with significant hills trying to keep constant speed, as e.g. a standard cruise control, is not beneficial. Figure 1.2 shows a large uphill gradient followed by a large downhill gradient, classified for a 18000 kg truck with a 13 liter engine. The optimal speed is plotted for a required final time corresponding to a mean speed of 80 km/h. Prior to a large downhill gradient it is beneficial to decelerate, to let the potential energy gained when climbing up the hill accelerate the vehicle without violating the constraint of maximum

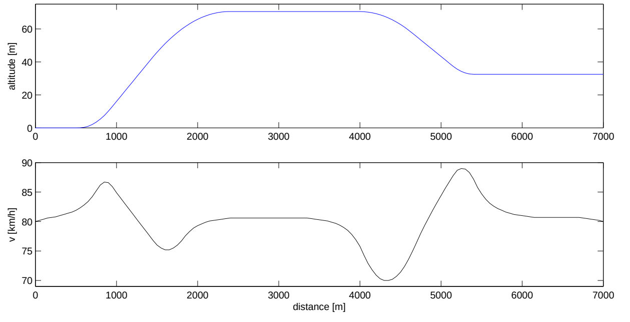

Figure 1.2: Optimal speed profile in significant hills.

velocity. To fulfill the constraint on final time it is optimal to accelerate prior to large uphill gradients.

Compared to the optimal speed profile, a vehicle with a standard cruise control would have a lower speed when starting to climb the uphill, which might lead to the need of shifting gears. Moreover, the cruise control would have a higher speed when the downhill gradient starts, compared to the optimal speed profile, which leads to braking at the end of the slope, i.e. a waste of energy. The optimal acceleration before significant uphills is acheived by using maximum engine fueling and the optimal deceleration prior to significant downhills is acheived by using minimum engine fueling (Fröberg et al., 2006), assuming an affine relation between engine torque and fueling.

1.4.2 Fuel map – engine efficiency

When driving in small road gradients, fueling is not constrained by its upper or lower limit. Hence, for such a driving mission, the characteristics of the engine in partial loads is interesting. A fuel map contains data from measurements in an engine test cell for various stationary operating points, in a range from no load to maximum load and with various engine speeds. The fuel map studied describes the efficiency of the engine in terms of specific fuel consumption [g/kWh], sfc, which is proportional to engine fueling over engine torque on the output shaft. In a traditional combustion engine, without any electric control system, the sfc map typically has a concave shape, see Figure 1.3.

For this type of fuel map the common assumption of affine relation between torque and fueling is most often a good approximation. However, the best efficency of a combustion engine is traditionally at about 80% of maximum torque, and not at maximum torque as is the result of an affine relation. This concave phenomena is captured in one way in (Schwarzkopf and Leipnik, 1977) where the sfc model is a product of a second order polynomial in engine torque and a second order polynomial in engine speed. Another way of modeling the

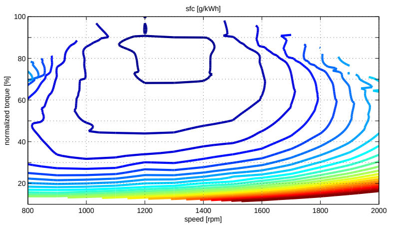

Figure 1.3: Conventional concave fuel map.

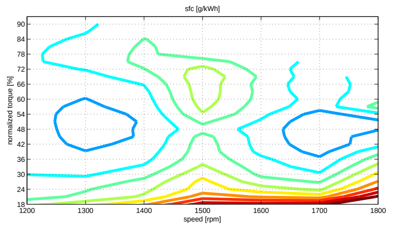

Figure 1.4: Fuel map with peaks and valleys.

concave fuel map is as a piece-wise affine relation between fueling, engine speed and torque, as in (Fröberg and Nielsen, 2007). The operating point, in terms of engine speed and torque, with the best engine efficiency is not always the best operating point from a complete vehicle perspective, since an optimal velocity profile on a non-constant road gradient requires a varying engine torque, and for large road gradients also a varying engine speed. The optimal torque/speed profile is a balance between a high engine efficiency and a low total energy consumption.

Modern engines may have other characteristics compared to the traditional concave fuel map, like peaks and valleys in sfc (see Figure 1.4), because of control strategies that today are possible to implement as for example after treatment of exhaust gases, optimization of gas flows and engine cooling control that are now common for combustion engines of heavy trucks. Modern fuel maps often contain non-linearities such that neither the assumption of affine relation between engine torque and fueling is valid nor are models fitted to concave fuel maps.

1.4.3 Shifting gears

Gear shifting is necessary in large road gradients if the maximum engine torque on the direct gear is not high enough to manage propeling the vehicle within an acceptable speed range. However, when cruising with fuel injection not in the limit, then as stated earlier the optimal speed, v, is constant. In this driving condition, when the power demand at the wheels, P_w, is low, it is always optimal to choose a low gear ratio and the reason to this is due to how the engine efficiency depends on engine speed for a certain power level. The demanded power is generated by the engine, P_e, and in the ideal case with no losses

$$P_e = T_e \omega_e = T_w \omega_w = T_w \frac{v}{r} = P_w \tag{1.1}$$

where T_e is engine torque and ω_e is engine speed. The rotational speed of the wheels, ω_w, is a function of wheel radius, r. As $T_w = T_e i$ and $\omega_w = \frac{\omega_e}{i}$ it can be seen that delivered power is not affected by gear ratio, i. A certain power can accordingly be generated by a low engine speed and a high engine torque or vice versa. If the fuel map is assumed to be affine in fueling, δ, and engine speed, according to

$$T_e = c_\delta \delta - T_{e,loss}(\omega_e) = c_\delta \delta - (c_\omega \omega_e + c_c) \tag{1.2}$$

then if engine speed is low and engine torque is high, the engine has a high efficiency as the engine losses are relatively small. For a realistic concave fuel map (as in Figure 1.3) and realistic gear ratios of a discrete transmission, the gear ratio that gives the best efficiency is still the lowest gear ratio. This is true for the relatively low power demand in small road gradients. In large road gradients, the demanded power is higher and in this case maximum torque is required in combination with a high engine speed, which leads to the need of a higher gear ratio. If gearing down is necessary, gear shifting control determines when to switch gears.

Heavy trucks in long haulage are often equipped with a manual gear box which is automatically controled, i.e. an automated manual transmission (AMT). In Figure 1.5 a typical AMT gear shifting process is illustrated, in terms of en-

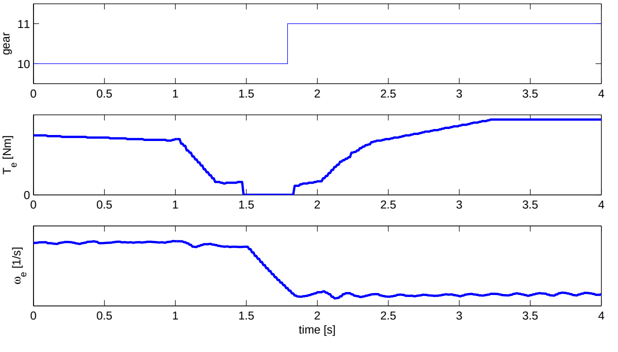

Figure 1.5: Typical AMT gear shifting.

gine torque and speed. An AMT system does not contain a clutch or torque converter. Instead engaging/disengaging of gears is enabled by engine torque control (as shown in Figure 1.5). This leads to a lower propulsive force for a couple of seconds. For tenths of a second, while shifting gears, there is no propulsive work produced at all. Of course, the traction of the vehicle is affected by the AMT gear shifting process. The low propulsive work during a gear shift leads to that vehicle speed is, for a short period of time, only determined by the driving resistance. The driving resistance varies along the road profile and consequently vehicle speed is differently affected dependent on when the gear shift is executed.

A difficult problem for a truck driver is to choose the optimal gear prior to and whilst in a steep slope. To ensure driveability, comfort and performance an experienced driver plans the necessary gear shifts when approaching steep slopes, according to his/her knowledge of the behaviour of the truck while shifting gears in a certain driving resistance and for a given speed. However, finding the fuel optimal gear shift control is not as intuitive. The AMT aims at optimizing all criteria, but since the future road gradient is not known, the system chooses gears with a safety margin necessary if the driving conditions would change suddenly, e.g. if the slope would become steeper. However, the safety margin leads to that gear shifts are most often not performed in an optimal manner. With look ahead information the need for safety margins is decreased and the system becomes more reliable in all driving scenarios.

1.5 Outline and contributions

The contributions are hereby stated, also giving an outline of the thesis. This thesis is based on three papers with the same objective, reducing fuel consumption while keeping to a set trip time, by using look ahead information.

Paper 1 is an article published in **Control Engineering Practice** Volume 17 no. 2, pp. 245-254, 2009;

Look-ahead Control for Heavy Trucks to Minimize Trip Time and Fuel Consumption, Erik Hellström, Maria Ivarsson, Jan Åslund and Lars Nielsen.

It is an extended version of a conference paper with the same title and authors presented on the Fifth IFAC symposium on advances in automotive control, California, 2007, (Hellström et al., 2007). A control algorithm previously developed, (Hellström et al., 2006), is in this paper adapted and evaluated in real trial test runs. In (Hellström et al., 2006) it was shown in simulations that for heavy trucks it is possible to reduce fuel consumption by controling vehicle speed in an optimal way. Another result from the simulations is that gear shifting is less likely to occur when driving with an optimal speed profile compared to driving with a standard cruise control. The model used in the optimization assumed an affine relation between injected fuel and engine torque. In Paper 1 the same optimization is performed in real time in a heavy truck, where the optimal solution is executed by adjusting the set speed of the standard cruise control. It is shown that the predicted fuel reductions are possible to obtain also in a real environment with its disturbances, model errors and time lags. Paper 2 and Paper 3 present studies that are inspired from the results of Paper 1.

Paper 2 is an extended version of the conference paper, *Optimal Speed on Small Gradients – Consequences of a Non-Linear Fuel Map*, Maria Ivarsson, Jan Åslund and Lars Nielsen, presented at the **IFAC World Congress** in Korea 2008 (Ivarsson et al., 2008);

Look Ahead Control – Consequences of a Non-Linear Fuel Map on Truck Fuel Consumption, Maria Ivarsson, Jan Åslund and Lars Nielsen.

Engines with non-linear dependencies between injected fuel and engine torque are studied. The commonly used efficiency measure sfc, specific fuel consumption, is studied in a realistic fuel map. Peaks and valleys in sfc indicate that some operating points of the engine are more efficient than others. To investigate how the characteristics of the fuel map non-linearities affect the fuel efficiency of the vehicle, driving missions with small road gradients are considered. It is shown that for some significant sfc non-linearities, constant speed is not optimal, as is the case when engine torque is affine in fueling. The critical threshold value of a significant non-linearity is defined, and the resulting behaviour is investigated.

Paper 3 is a technical report at the Departement of Electrical Engineering, Linköpings Universitet, LiTH-ISY-R-2883;

Impacts of AMT Gear-Shifting on Fuel Optimal Look Ahead Control, Maria Ivarsson, Jan Åslund and Lars Nielsen.

A model is set up of a standard AMT gear-shifting process reflecting the characteristics that influence final time and fuel efficiency. In order to minimize fuel and time, optimal engine fueling and gear control is found for realistic uphill slopes. The optimal solutions show that gearing down unnessecarily is always unbeneficial, but still not an uncommon scenario for a standard AMT. Further on, it is shown that the reduced propulsion of a typical AMT gear-shifting process, and the resulting vehicle retardation, must be considered when choosing when to shift gears, in order to ensure an adequate engine speed and a sufficient engine power after the gear shift and consequently to acheive a reduction of fuel consumption and final time.

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Part II

Papers

Paper 1

Look-ahead Control for Heavy Trucks to Minimize Trip Time and Fuel Consumption[1]

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Abstract

The scenario studied is a drive mission for a heavy diesel truck. With aid of an on board road slope database in combination with a GPS unit, information about the road geometry ahead is extracted. This look-ahead information is used in an optimization of the velocity trajectory with respect to a criterion formulation that weighs trip time and fuel consumption. A dynamic programming algorithm is devised and used in a predictive control scheme by constantly feeding the conventional cruise controller with new set points. The algorithm is evaluated with a real truck on a highway, and the experimental results show that the fuel consumption is significantly reduced.

[1]This article was published in *Control Engineering Practice*, Vol. 17, No. 2, pp. 245-254, 2009.

1 Introduction

As much as about 30% of the life cycle cost of a heavy truck comes from the cost of fuel. Further, the average mileage for a (European class 8) truck is 150,000 km per year and the average fuel consumption is 32.5 L/100km (Schittler, 2003). Lowering the fuel consumption with only a few percent will thus translate into significant cost reductions. These facts make a system which can reduce the fuel consumption appealing to owners and manufacturers of heavy trucks. The problem scenario in the present work is a drive mission for a truck where the route is considered to be known. It is, however, not assumed that the vehicle constantly operates on the same route. Instead, it is envisioned that there is road information on board and that the current heading is predicted or supplied by the driver. In the current work, information about the road slope is exploited aiming at a fuel consumption reduction.

One early work (Schwarzkopf and Leipnik, 1977) formulates an optimal control problem for a nonlinear vehicle model with the aim at minimizing fuel consumption and explicit solutions are obtained for constant road slopes. A dynamic programming (DP) approach is taken from Monastyrsky and Golownykh (1993) to obtain solutions for a number of driving scenarios on shorter road sections. Inspired of some of the results indicated in these and other works it was shown in Chang and Morlok (2005); Fröberg et al. (2006) with varying vehicle model complexity that constant speed is optimal on a constant road slope within certain bounds on the slope. The result relies on that there is an affine relation between the fuel consumption and the produced work. Analytical studies of the situation when this relation is nonlinear were conducted in Fröberg and Nielsen (2007). Predictive cruise control is investigated through computer simulations in, e.g., Lattemann et al. (2004); Terwen et al. (2004), but constructing an optimizing controller that works on board in a real environment puts additional demands on the system in terms of robustness and complexity.

In Hellström et al. (2006) a predictive cruise controller (CC) is developed where discrete DP is used to numerically solve the optimal control problem. The current paper is a continuation where an improved approach is realized and evaluated in actual experiments. One objective is to evaluate the order of fuel reduction that can be obtained in practical driving. The strategies to achieve fuel reductions may be intuitive, but only in a qualitatively manner. Another objective is therefore to find the optimal solution and thereby quantify the characteristics of the best possible strategy. The purpose is also to analyze controller behavior in real trial runs and evaluate potential benefits.

To perform this study a chain of elements is needed. Section 2 presents a vehicle model of standard type. Section 3 deals with the DP algorithm, and it is described how the problem characteristics are utilized to reduce the complexity, to determine penalty parameters, and efficiently compute the criterion by an inverse technique. In Section 4 the experimental setup is presented, and finally the quantitative evaluation concludes the study.

2 Truck model

The modeling follows (Kiencke and Nielsen, 2005; Sandberg, 2001), and the resulting model is then reformulated and adapted for the numerical optimization that is performed.

The engine torque T_e is modeled as

$$T_e(\omega_e, u_f) = a_e\omega_e + b_e u_f + c_e \tag{1}$$

where ω_e is the engine speed and u_f is the control signal which determines the fueling level.

The control u_f is assumed to be bounded by

$$0 \leq u_f \leq u_{f,max}(\omega_e) \tag{2}$$

where the upper limit $u_{f,max}(\omega_e)$ is modeled by a second-order polynomial in engine speed ω_e,

$$u_{f,max}(\omega_e) = a_f\omega_e^2 + b_f\omega_e + c_f.$$

When a gear is engaged, the engine transmits a torque T_c to the clutch and

$$J_e\dot{\omega}_e = T_e - T_c \tag{3}$$

where J_e is the engine inertia and ω_e is the engine speed. The clutch, propeller shafts and drive shafts are assumed stiff and their inertia are lumped into one together with the wheel inertia, denoted by J_l. The resulting conversion ratio of the transmission and final drive is denoted by i and energy losses are modeled by an efficiency η. When a gear is engaged, this gives

$$\begin{aligned} \omega_e = i\omega_w,\ T_w = i\eta T_c \\ J_l\dot{\omega}_w = T_w - T_b - r_w F_w \end{aligned} \tag{4}$$

where T_w is the torque transmitted to the wheel, T_b is the braking torque and r_w is the wheel radius. F_w is the resulting friction force.

When neutral gear is engaged, the engine transmits zero torque to the driveline. The driveline equations (3) and (4) then become

$$\begin{aligned} J_e\dot{\omega}_e = T_e,\ T_c = T_w = 0 \\ J_l\dot{\omega}_w = -T_b - r_w F_w. \end{aligned} \tag{5}$$

The motion of the truck is governed by

$$m\frac{dv}{dt} = F_w - F_a(v) - F_r(\alpha) - F_N(\alpha) \tag{6}$$

where α is the road slope. The models of the longitudinal forces are explained in Table 1.

Table 1: Longitudinal forces.

Force	Explanation	Expression
$F_a(v)$	Air drag	$\frac{1}{2}c_w A_a \rho_a v^2$
$F_r(\alpha)$	Rolling resistance	$mgc_r \cos\alpha$
$F_N(\alpha)$	Gravitational force	$mg \sin\alpha$

It is assumed that the transmission is of the automated manual type and that gear shifts are accomplished through engine control, see (Pettersson and Nielsen, 2000). A shift is modeled by a constant period of time τ_{shift} where the neutral gear is engaged before the new gear is engaged. The number of the currently engaged gear is denoted by g. The ratio i and efficiency η then becomes functions of the integer g. The control signal that selects gear is denoted by u_g. Neutral gear corresponds to gear zero, equivalent with a ratio and efficiency of zero.

The vehicle velocity v is

$$v = r_w \omega_w \tag{7}$$

where ω_w is the wheel speed of revolution and r_w is the effective wheel radius. Equations (3)-(7) are combined into

$$\begin{aligned}\frac{dv}{dt}(x,u,\alpha) = \frac{r_w}{J_l + mr_w^2 + \eta(g)i(g)^2 J_e}\Big(&i(g)\eta(g)T_e(v,u_f) \\ &- T_b(u_b) - r_w\left(F_a(v) + F_r(\alpha) + F_l(\alpha)\right)\Big)\end{aligned} \tag{8}$$

where

$$x = [v, g]^T \qquad u = [u_f, u_b, u_g]^T \tag{9}$$

denote the state and control vector, respectively. The states are the velocity v and currently engaged gear g and the controls are fueling u_f, braking u_b and gear selector u_g.

The mass flow of fuel is determined by the fueling level u_f (g/cycle) and the engine speed ω_e (rad/s). The flow in (g/s) is then

$$\frac{n_{cyl}}{2\pi n_r}\omega_e u_f \tag{10}$$

where n_{cyl} is the number of cylinders and n_r is the number of crankshaft revolutions per cycle. Using (4) and (7) in (10) gives

$$\dot{m}(x,u) = \frac{n_{cyl}}{2\pi n_r}\frac{i}{r_w} v u_f, \; g \neq 0 \tag{11}$$

whereas in the case of neutral gear, $g = 0$, the fuel flow is assumed constant and equal to an idle fuel flow $\dot{m}_{idle}$.

2.1 Reformulation

Models (8) and (11) are transformed to be dependent on position rather than time. Denoting traveled distance by s and the trip time by t, then for a function $f(s(t))$

$$\frac{df}{dt} = \frac{df}{ds}\frac{ds}{dt} = \frac{df}{ds}v \tag{12}$$

is obtained using the chain rule where $v > 0$ is assumed. By using (12), the models can be transformed as desired.

The approach in this work is numerical and therefore the model equations should be made discrete. The quantization step in position is constant and equal to h. The control signals are considered piece-wise constant during a discretization step. Denote

$$\begin{aligned} x_k &= x(kh),\ u_k = u(kh) \\ \alpha_k &= \frac{1}{h}\int_{kh}^{(k+1)h} \alpha(s)ds \end{aligned} \tag{13}$$

where the road slope α_k is set to the mean value over the discretization step. The trapezoidal rule is used to make the truck model (8) discrete. If a gear shift occurs during a step, a second-order Runge-Kutta method is used for a time step equal to the delay τ_{shift} to modify the initial values and the step length. The system dynamics is finally

$$x_{k+1} = f(x_k, u_k, \alpha_k) \tag{14}$$

where $f(x_k, u_k, \alpha_k)$ is given by (8).

The discretized problem is incorporated into the algorithm and thus affects the algorithm complexity. The simplest Euler method do, however, not yield satisfactory results due to truncation errors, see (Hellström, 2007). For this reason second-order methods were chosen.

3 Look-ahead control

Look-ahead control is a predictive control scheme with additional knowledge about some of the future disturbances, here focusing on the road topography ahead of the vehicle. An optimization is thus performed with respect to a criterion that involves predicted future behavior of the system, and this is accomplished through DP (Bellman and Dreyfus, 1962).

The conditions change during a drive mission due to disturbances, e.g., delays due to traffic, or changed parameters such as the vehicle mass. The robustness is increased by feedback and the approach taken here is therefore to repeatedly calculate the fuel-optimal control on-line. The principle is shown in Figure 1. At point A, the optimal solution is sought for the problem that is defined over the look-ahead horizon. This horizon is obtained by truncating the entire drive mission horizon. Only the first optimal control is applied to the system and the procedure is repeated at point B.

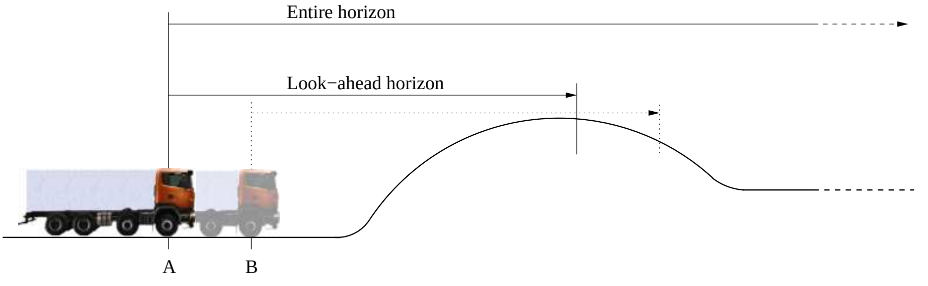

Figure 1: Illustration of the look-ahead control strategy.

This section will first deal with the identification of the control objectives. Based on these, a suitable criterion is devised and the tuning of its parameters is discussed. At the end, the DP algorithm is outlined.

3.1 Objective

The objectives are to minimize the energy and time required for a given drive mission. The vehicle velocity is desired to be kept inside an interval

$$v_{min} \leq \ v \ \leq v_{max} \tag{15}$$

where v denotes the vehicle velocity. These bounds are set with respect to the desired behavior of the controller. For example, the lower bound will be the lowest velocity the controller would deliberately actuate. The upper bound can be set by, e.g., safety reasons or legal considerations.

The brake system is assumed to be powerful enough to keep the upper bound in (15). On the other hand, the lower bound is not expected to be physically reachable on all road profiles. It is assumed though, that it is possible to keep a velocity, denoted by v_{lim}, which is greater than zero at all times. If Equation (15) were to be used, it would not be certain to find any feasible solution. Therefore the constraints on the vehicle speed v are expressed as follows:

$$0 < \min \{v_{min}, v_{lim}(s)\} \leq \ v \ \leq v_{max} \tag{16}$$

3.2 Criterion

The fundamental trade off when studying minimization of energy required for a drive mission is between the fuel use and the trip time. The fuel use on a trip from $s = s_0$ to $s = s_f$ is

$$M = \int_{s_0}^{s_f} \frac{1}{v}\dot{m}(x, u)ds \tag{17}$$

where $\frac{1}{v}\dot{m}(x,u)$ is the mass flow per unit length as function of the state x and control u. The trip time T is simply

$$T = \int_{s_0}^{s_f} \frac{dt}{ds} ds = \int_{s_0}^{s_f} \frac{ds}{v}. \tag{18}$$

To weigh fuel and time use, the cost function proposed is

$$I = M + \beta T \tag{19}$$

using (17) and (18) and where β is a scalar factor which can be tuned to receive the desired trade off.

Criterion (19) is then made suitable for discrete DP by dividing the look-ahead horizon into N steps of length h (m) and transforming the cost function. Denote

$$m_k = \int_{kh}^{(k+1)h} \dot{m}(x,u)ds, \quad t_k = \int_{kh}^{(k+1)h} \frac{ds}{v},$$
$$a_k = |v_k - v_{k+1}| \tag{20}$$

and the cost function can be expressed as

$$J = \sum_{k=0}^{N-1} \zeta_k(x_k, x_{k+1}, u_k, \alpha_k) \tag{21}$$

where

$$\zeta_k = [1, \beta, \gamma] \begin{bmatrix} m_k \\ t_k \\ a_k \end{bmatrix}$$
$$k = 0, 1, \ldots, N-1 \tag{22}$$

and β, γ are scalar penalty parameters for controlling the properties of solutions. The difference in the criterion between neighboring discretization points is typically very small. In order to receive a smoother control, the term a_k is added with a small value of γ.

3.3 Penalty parameters

The size of the factor γ is chosen for smoothing but still such that the term γa_k becomes considerable smaller than the others.

One way to determine the parameter β, i.e. the trade off between fuel and time, is to study a stationary solution to the criterion in Equation (19). Assume that a gear is engaged and there exists at least one control $\hat{u}$, for which (2) holds and that gives a stationary velocity $\hat{v}$, i.e. $dv/dt = 0$. From the equations (1), (8), and Table 1 it is concluded that $\hat{u}$ can be written as

$$\hat{u} = c_1\hat{v}^2 + c_2\hat{v} + f(\alpha) \tag{23}$$

where, for a given gear, c_1 and c_2 are constants and $f(\alpha)$ is a function corresponding to the rolling resistance and gravity, and thus being a function of the road slope α. With (11) and (12), the fuel flow is written as

$$\frac{1}{v}\dot{m}(x,u) = c_4 \hat{u}_f \tag{24}$$

where c_4 is the proportionality constant. The cost function (19) is thus

$$\hat{I}(\hat{v}) = \int_{s_0}^{s_f} \left(c_4 \left(c_1 \hat{v}^2 + c_2 \hat{v} + f(\alpha) \right) + \frac{\beta}{\hat{v}} \right) ds \tag{25}$$

where the integrand is constant with respect to s if constant slope is assumed. A stationary point to $\hat{I}$ is found by setting the derivative equal to zero,

$$\frac{d\hat{I}}{d\hat{v}} = \int_{s_0}^{s_f} \left(c_4 \left(2c_1 \hat{v} + c_2 \right) - \frac{\beta}{\hat{v}^2} \right) ds = 0. \tag{26}$$

Solving the equation for β gives

$$\beta = c_4 \hat{v}^2 \left(2c_1 \hat{v} + c_2 \right) \tag{27}$$

that can be interpreted as the value of β such that a stationary velocity $\hat{v}$ is the solution to (26). Note that the value of β neither depends on the vehicle mass m nor the slope α. The calculated β will thus give the solution $\hat{v}$ of the criterion for any fixed mass and slope as long as there exists a control $\hat{u}_f$ satisfying the bounds in (2).

3.4 Preprocessing

The ambition with the present work is a real-time algorithm and hence the complexity plays an important role. The subset of the state space over which the optimization is applied, the search space, is one determining factor for the complexity. If the search space is reduced without loosing any solutions, obvious gains are made. A preprocessing algorithm is therefore developed with this aim.

Since DP is used in an predictive control setting, the current velocity can be measured and used for limiting the set of possible initial states. In order to handle terminal effects, the final velocities are also constrained. By using the model and traversing the horizon forward and backward before the optimization is started, the search space is downsized, see (Hellström, 2007).

The preprocessing algorithm gives, for each stage, an interval of velocities which are to be considered. For every stage the interval $[v_{lo}, v_{up}]$ is discretized in constant steps of δ. This makes up a set V_k,

$$V_k = \{v_{lo}, v_{lo} + \delta, v_{lo} + 2\delta, \ldots, v_{up}\}. \tag{28}$$

3.5 DP algorithm

To summarize, the optimal control problem at hand is the minimization of the objective,

$$\min_{u,g} \sum_{k=0}^{N-1} \zeta_k(x_k, x_{k+1}, u_k, \alpha_k)$$

where ζ_k is given in (3.2). The system dynamics is given by

$$x_{k+1} = f(x_k, u_k, \alpha_k) \quad k = 0, 1, \ldots, N-1$$

according to (14). The constraints are

$$0 < \min\{v_{min}, v_{lim}(kh)\} \leq \ v_k \ \leq v_{max} \ \forall k$$

according to (16). Due to the predictive control setting, the initial state x_0 is given.

With a given velocity, only a subset of the gears in the gearbox is feasible. If the operating region of the engine is defined with bounds on the engine speed $[\omega_{e,min}, \omega_{e,max}]$, it is easy to select the set of feasible gears. Only gears with a ratio that gives an engine speed in the allowed range are then considered. In a state with velocity v, the set of usable gears G_v is thus defined as

$$G_v = \{g \mid \omega_{e,min} \leq \omega_e(v, g) \leq \omega_{e,max}\} \tag{29}$$

where $\omega_e(v, g)$ is the engine speed at vehicle velocity v and gear number g.

Braking is only considered in the algorithm if the upper velocity bound is encountered. Braking without recuperation is an inherent waste of energy and therefore braking will only occur when the velocity bounds would otherwise be violated. This reduces the complexity since the number of possible control actions lessens.

A state x is made up of velocity v and gear number g. The possible states in stage k are denoted with the set S_k and it is generated from the velocity range V_k given in (28) and the set of gears G_v given in (29). This yields

$$S_k = \{\{v, g\} \, | v \in V_k, g \in G_v\} \, . \tag{30}$$

At a stage k, feasible control actions $u_k^{i,j}$ that transform the system from a state $x^i \in S_k$ to another state $x^j \in S_{k+1}$ are sought. The control is found by an inverse simulation of the system equations, see e.g. (Fröberg and Nielsen, 2008). Here this means that for a given state transition, x_k to x_{k+1}, the control u_k is solved from (14). Interpolation is thereby avoided. If there are no fueling level u_f and gear u_g that transforms the system from state x^i to x^j at stage k, there are two possible resolutions. If there exist a feasible braking control u_b it is applied, but if there is no feasible braking control the cost is set to infinity.

The algorithm is outlined as follows:

1. Let $J_N(i) = 0$.

Table 2: Truck specifications.

Component	Type	Characteristics
Engine	DC9	Cylinders: 5
		Displacement: 9 dm^3
		Max.torque: 1,550 N m
		Max.power: 310 Hp
Gearbox	GRS890R	12 gears
Vehicle	-	Total weight: 39,410 kg

2. Let $k = N - 1$.
3. Let
$$J_k(x^i) = \min_{x^j \in S_{k+1}} \left\{ \zeta_k^{i,j} + J_{k+1}(x^j) \right\}, \; x^i \in S_k.$$
4. Repeat (3) for $k = N - 2, N - 3, \dots, 0$.
5. The optimal cost is J_0 and the sought control is the optimal control set from the initial state.

4 Trial run

The experiments are performed on the highway E4 between the cities of Södertälje and Norrköping in Sweden, see Figure 4. The truck used is a SCANIA tractor and trailer, see Figure 2. The specifications are given in Table 2.

Figure 2: The vehicle used in the experiments.

Following in this section, the experimental setup and road slope data are presented. The last part of the section will present some results from the trial runs that have been undertaken.

Table 3: User parameters

Parameter	Function	Value
h	Step length	50 m
N	Number of steps	30
$h \cdot N$	Horizon	1500 m
δ	Velocity discretization	0.2 km/h
v_{min}	Min. allowed vel.	79 km/h
v_{max}	Max. allowed vel.	89 km/h

4.1 Setup

The information flow in the experimental setup is shown schematically in Figure 3. Due to adjustments for the demonstrator vehicle, gear shifts were not

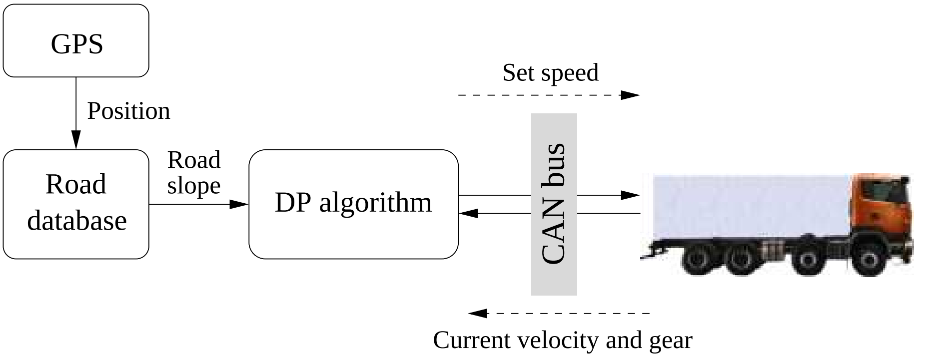

Figure 3: Information flow.

directly controlled by the algorithm. This is handled by including a prediction model of the gear control system and take it into account when calculating the running costs. In the optimization algorithm, a shift that is not predicted is assigned an infinite cost. As depicted in Figure 3 the algorithm controls the vehicle by adjusting the set speed to the conventional CC. The fueling level is therefore only controlled indirectly. The standard CC available from SCANIA is used, which is basically a PI-regulator. All communications are done over the CAN bus.

The algorithm parameters used are stated in Table 3 and the penalty factors are shown in Table 4. The factors are adjusted in order to receive a stationary solution in the middle of the desired velocity interval (15). All software run on a portable computer with an Intel Centrino Duo processor at 1.20 GHz and 1 Gb RAM. With the stated parameters, a solution on a road stretch of level road is calculated in tenths of a second on this computer.

The truck has a legislative speed limiter at 89 km/h. Propulsion above this limit is not possible. When the truck accelerates due to gravity above 89 km/h,

Table 4: Penalty factors

Factor	Penalizes	Value
	Fuel use	1.0
β	Time use	6.2
γ	Velocity changes	0.1

the brake control system is activated at a set maximum speed. In the trial run this limit is set to be 91 km/h.

Database

The slope in front of the vehicle for the length of the look-ahead horizon is needed to be known. It is expected that such data will be commercially available soon. But for now, the road slope along the trial route is estimated off line prior to any experiments. This is done by aid of a non-stationary forward-backward Kalman filter (Sahlholm et al., 2007). The estimated slope and calculated altitude are shown in Figure 4. The measurements were obtained at 20 Hz from a GPS unit. The filter inputs are vertical and horizontal velocity of the vehicle, altitude and the number of reachable GPS satellites.

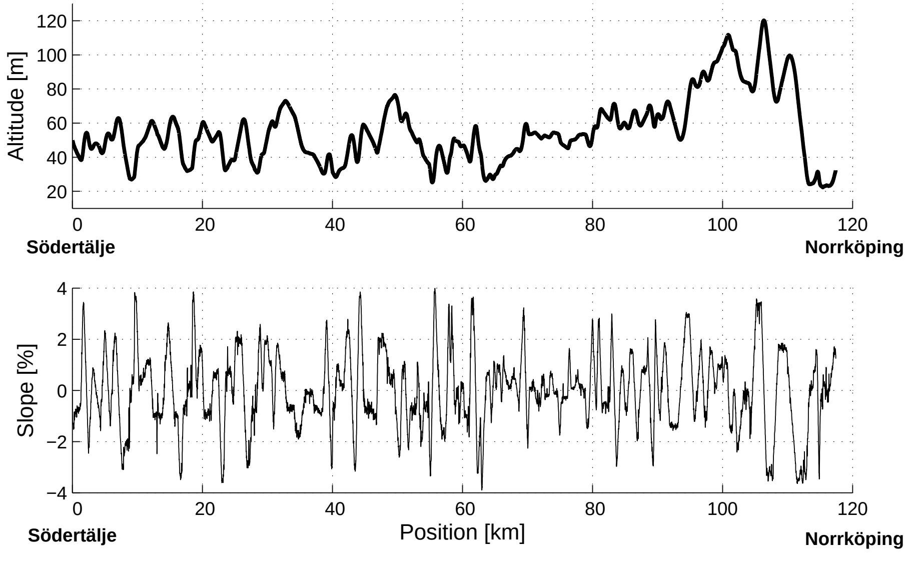

Figure 4: Estimated road topography.

4.2 Performance

In total, five comparative trial runs were made. All runs were done in light to moderate traffic, and each consisted of one drive with look-ahead control and one with standard cruise control. The algorithm parameters, see Table 3 and 4, were the same for all runs. The trip time thus became about the same for all drives with the look-ahead control. The set point for the CC was chosen in order to achieve a trip time close to the one obtained with look ahead.

4.3 Overall results

The relative changes in fuel consumption and trip time (Δfuel, Δtime) are shown in Figure 5 and Figure 6 for each direction on the trial road. A negative value means that the look-ahead controller (LC) has lowered the corresponding value. The set point for the CC increases along the horizontal axis. The left-most result is maybe the most convincing since it reduces both fuel use and trip time in both directions. The average results in both directions that are made

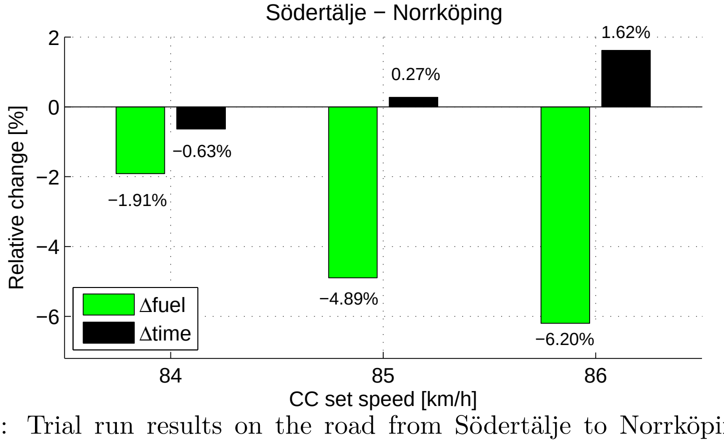

Figure 5: Trial run results on the road from Södertälje to Norrköping with varying cruise controller (CC) set speed.

with the same set speed are also calculated. For these mean values the fuel consumption was lowered with 3.53%, from 36.33 L/100km to 35.03 L/100km, with a negligible reduction of the trip time (0.03%) in comparison with the CC. Also interesting to note is that the mean number of gear shifts on this route decreases from 20 to 12 with the LC.

4.4 Control characteristics

With the intention to give a representative demonstration of more detailed controller characteristics, two road segments have been chosen. The first is a 2.5 km segment close to Södertälje and is named *the Järna segment*. The second

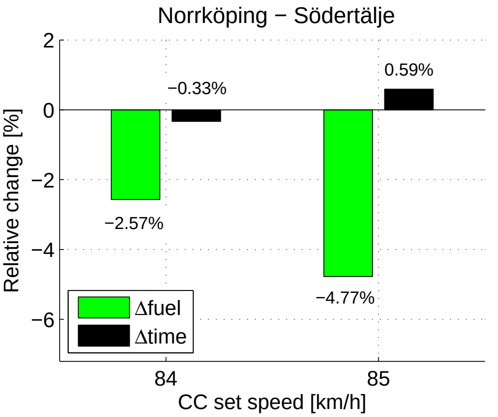

Figure 6: Trial run results on the road from Norrköping to Södertälje with varying cruise controller (CC) set speed.

one is a 3.5 km segment about halfway on the trial route and called *the Hållet segment*.

Each figure, see, e.g., Figure 7, is divided into four subfigures, all having the position as the horizontal axis. The road topography is shown at the top and the coordinates for the start and final position are also given on the horizontal axis. The next subfigure shows the velocity trajectories for the LC and the standard CC. The third part shows normalized fueling (acc) and retarder (brake) levels with thick and thin lines, respectively. At the bottom, both the engaged gear number and the fuel use are shown. Data related to the LC are displayed in solid lines and data associated to the CC are displayed with dashed lines in these figures. Above the figures, the time and fuel spent on the section are shown together with the relative change (Δfuel, Δtime) in these values between the two controllers. A negative value means that the value is lowered by the LC.

The Hållet segment

Figures 7 and 8 show the Hållet segment. In Figure 7, the LC accelerates at 500 m prior to the uphill that begins at 750 m. At the top of the hill at 1750 m, the LC slows down in contrast to the CC. The truck is thus let to accelerate by gravity alone. The CC will, however, use a non-zero fueling as long as the truck is going slower than the set point. The LC slow down reduces the need for braking later in the downslope and thereby the inherent waste of energy is lessened. From the fuel integral at the bottom, it is seen that the LC consumes more fuel the first 1.5 km owing to the acceleration, but then gains.

The trip in the other direction, see Figure 8, gives similar features. A gain of speed at 250 m and then a slow down at the top of the hill at 2250 m. In both directions, time as well as fuel are saved.

Note that the sections in Figure 7 and 8 are not exceptionally steep. The uphill and downhill slope is at most about 4% for short intervals. However, they become significant for the truck due to the large vehicle mass.

The Järna segment

In Figures 9 and 10 the Järna segment is shown. Figure 9 shows that the LC begins to gain speed at 200 m and thereby avoids the gear shift that the CC is forced to do around 1 km. At 1400 m, the LC slows down and lets the truck accelerate in the downslope.

In Figure 10, a drive in the other direction is shown. The LC accelerates at 500 m and starts to slow down at 1400 m. The slow down lessen the braking effort needed at about 2000 m.

5 Conclusions

The control algorithm was proven to perform well on board in a real environment. Using the standard cruise controller as an inner loop and feeding it with new set points is advantageous considering robustness against model errors and disturbances.

The gearbox consists of a set of discrete gears and there is no propulsion force during a gear shift. Taking these facts into account renders a challenging optimization problem. A discrete dynamic programming algorithm is devised where the search space is reduced by a preprocessing algorithm. The algorithm computes a solution in tenths of a second on a modern laptop computer and this allows evaluation in a real environment on board a truck.

The trial runs show that significant reductions of the fuel consumption can be achieved. A fuel consumption reduction of about 3.5% on the 120 km route without an increase in trip time was obtained. The mean number of gear shifts was reduced with 42% due to shifts avoided by gaining speed prior to uphills.

The look-ahead control mainly differs from conventional cruise control near significant downhills and uphills where the look-ahead control in general slows down or gains speed prior to the hill. Slowing down prior to downhills is intuitively saving fuel. There is, however, no challenge in saving fuel by traveling slower, so if the vehicle is let to slow down at some point, the lost time must thus be gained at another point. Accelerating prior to uphills is one way which, at least for shorter hills, gives a higher velocity throughout the hill and will reduce the need for lower gears. These strategies are intuitive but the crucial issue is the detailed shape of the solution and its actuation such that a positive end result is obtained, and this is shown to be handled well by the algorithm.

A final comment is that the controller behavior has been perceived as comfortable and natural by drivers and passengers that have participated in tests and demonstrations.

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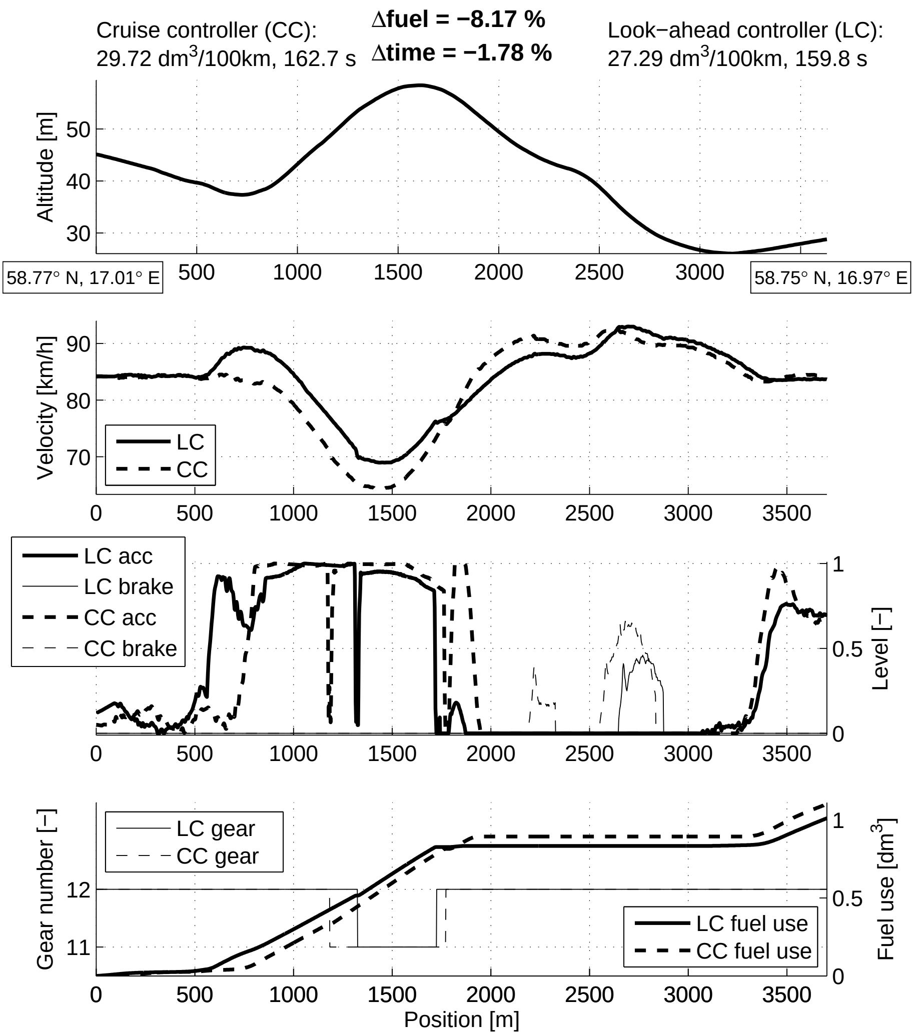

Figure 7: The Hållet segment. The LC accelerates at 500 m prior to the uphill and slows down at 1750 m when the top is reached.

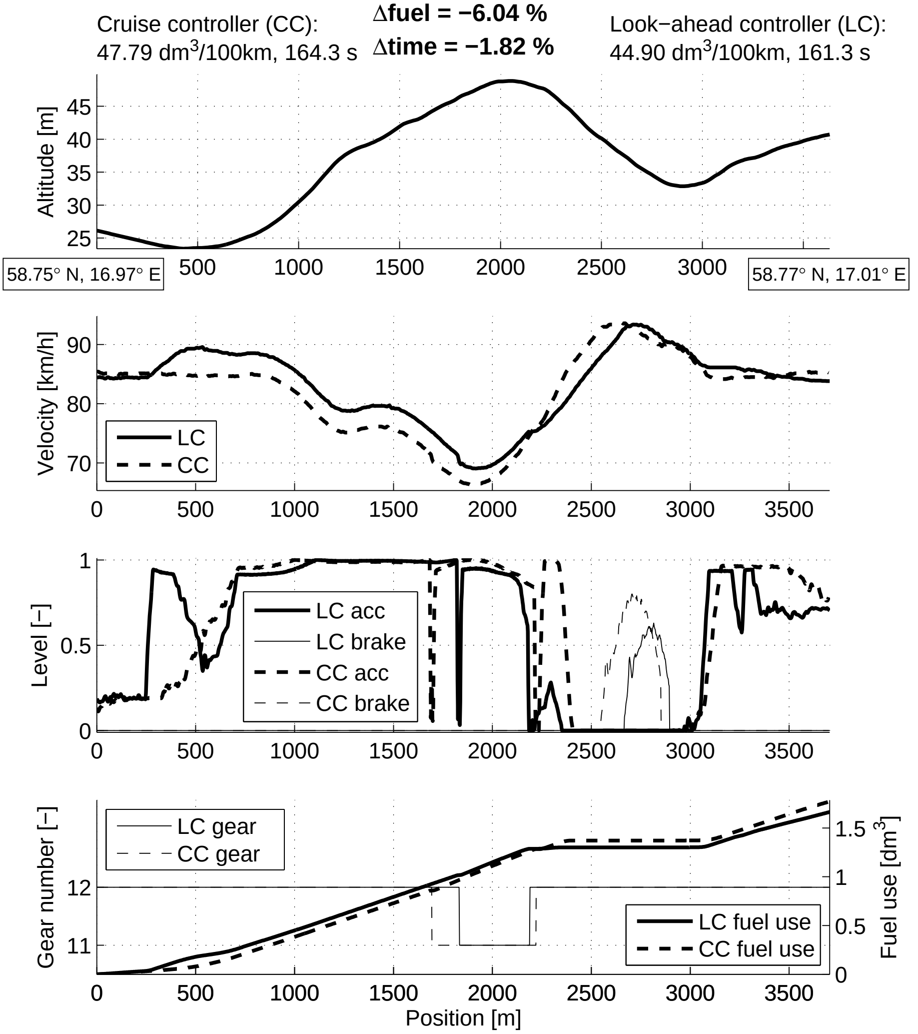

Figure 8: The Hållet segment. The LC gains speed at 250 m prior to the uphill and slows down at 2250 m prior to the downhill.

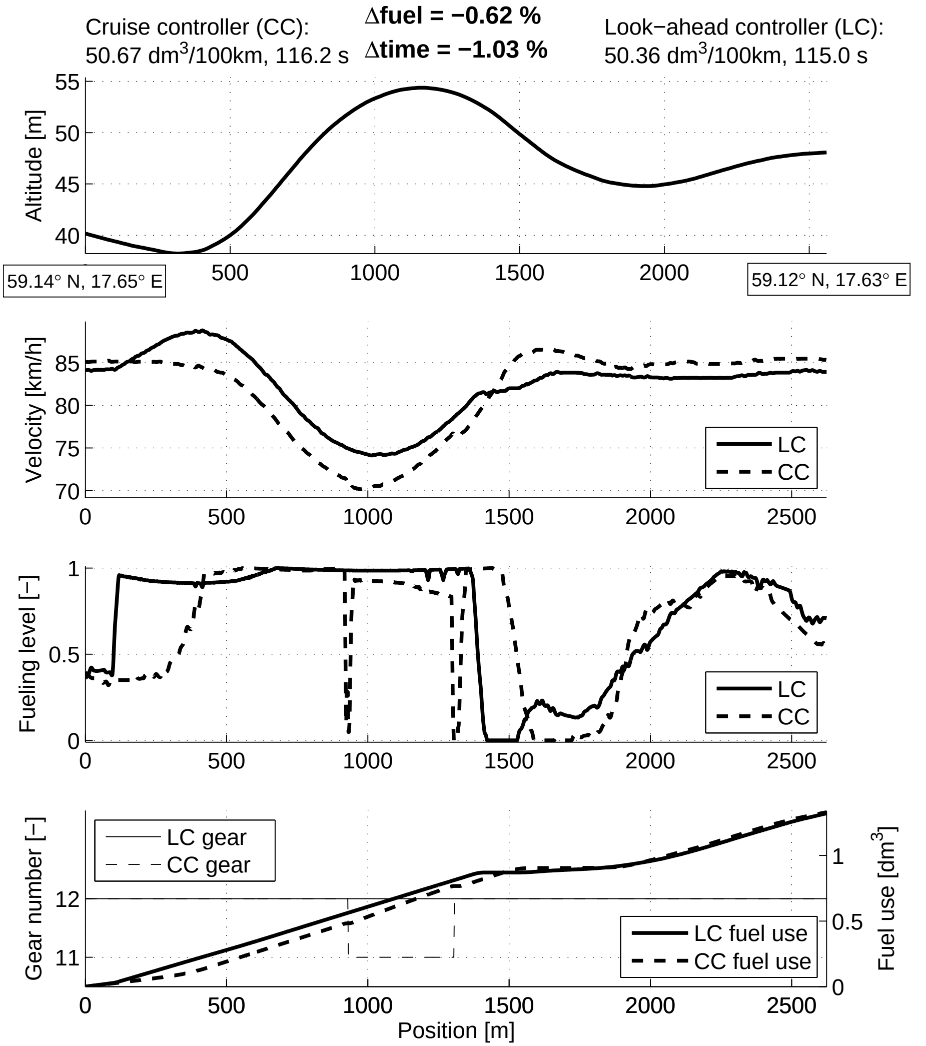

Figure 9: The Järna segment. The LC gains speed at 200 m prior to the uphill and avoids a gear shift. At 1400 m the LC slows down and the truck is let to accelerate in the downslope.

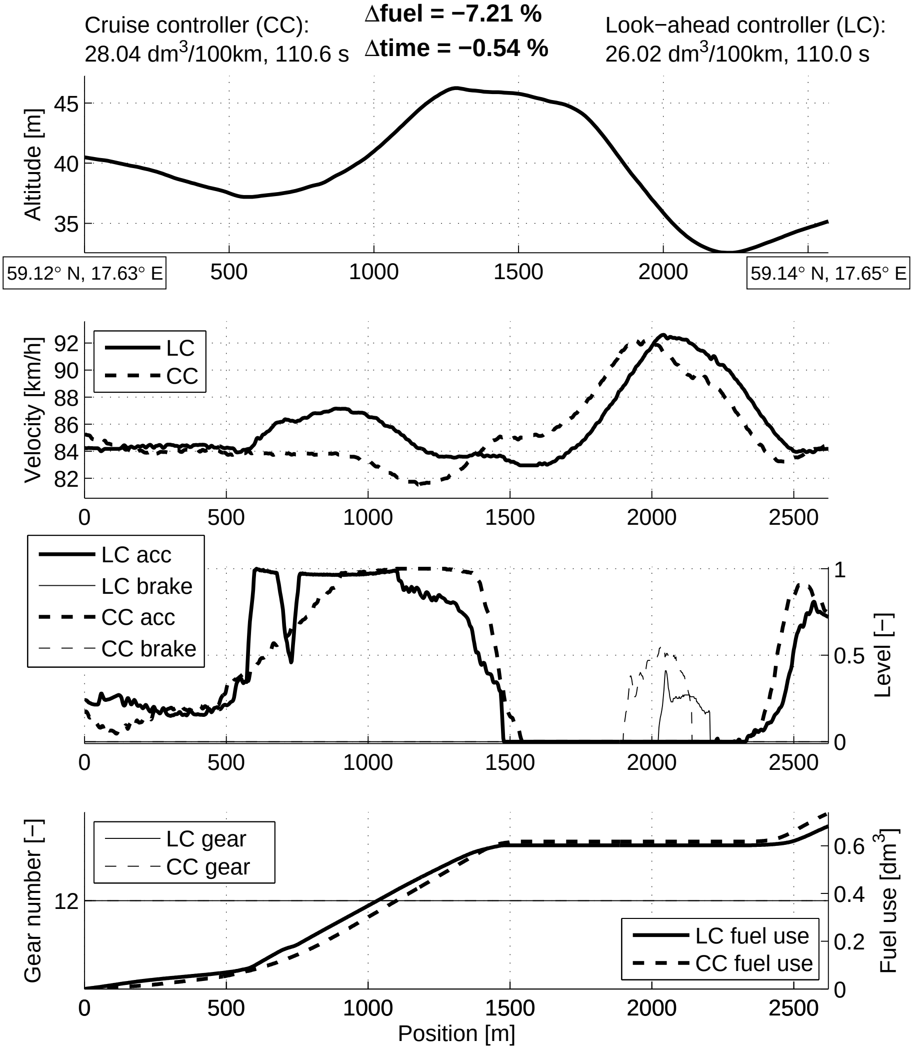

Figure 10: The Järna segment. The LC accelerates at 500 m and slows down at 1400 m thereby reducing the braking effort needed later on.

Paper 2

Look Ahead Control - Consequences of a Non-Linear Fuel Map on Truck Fuel Consumption[1]

2

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Abstract

Consequences of non-linearities in specific fuel consumption, sfc, of a heavy truck combustion engine are studied with focus on so small road gradients that constant speed is optimal if the engine torque has an affine relation to fueling. A quasi-static analysis gives valuable insights into the intrinsic properties of minimization of fuel consumption. Two objective functions are shown to give different optimal velocity trajectories on a constant road gradient, when the non-linearity in sfc is significant, a notation which is quantified. For a significant non-linearity, when a constraint is set to keep a final time, switching between two characteristic speeds is optimal. Alternatively, if consumed time, in addition to fuel consumption, is part of the objective function, then keeping to one constant speed is optimal also for significant non-linearities. However, the different optimal solutions still show similarities, since for a certain significant non-linearity a specific speed range determined by the characteristic velocities is shown to be unobtainable for both optimality criteria. Similar results are obtained for a full dynamic model including a realistic fuel map and other realistic constraints.

[1]This is an extended version of (Ivarsson et al., 2008), Optimal Speed on Small Gradients – Consequences of a Non-Linear Fuel Map, presented at the *IFAC World Congress 2008*

1 Introduction

Look ahead control deals with the problem of driving a vehicle in an optimal way that benefits from information of the road ahead. Since it is nowadays possible to know the topography ahead using GPS and a map data base, such fuel optimal controls are being deployed for on-board vehicle control. One early work in fuel optimal control is (Schwarzkopf and Leipnik, 1977), and others have followed, e.g. (Hooker et al., 1983) and (Terwen et al., 2004). It has been shown, by e.g. (Hellström et al., 2009; Hellström, 2007; Fröberg et al., 2006; Chang and Morlok, 2005) that the fuel optimal solution, given an affine relation between engine torque and fueling, is to keep a constant speed when the road gradient is sufficiently small, i.e. in gradients in which the vehicle is able to keep a constant speed (Fröberg et al., 2006). For a heavy truck, the hills are significant even in normal high-way topography, and it is impossible to keep constant speed. In such cases, it has been shown in real experiments, see (Hellström et al., 2009), that it is possible to significantly reduce fuel consumption by controlling the speed.

Given the success in fuel reduction for the above schemes, it is natural to ask if even further gains can be obtained when considering not only the approximation of an affine relation between torque and fueling, but the actual non-linearities of a realistic heavy truck fuel map that illustrates how engine efficiency varies for a range of engine speeds and loads. The engine efficiency is described by the sfc, the specific fuel consumption [g/kWh]. In a traditional combustion engine, without any electric control system, the sfc of the fuel map typically has a concave shape with one operating point giving the best engine efficiency, i.e. the lowest sfc. Modern engines may have characteristic peaks in sfc, as shown in Figure 1, because of control strategies that today are possible to implement as for example after treatment of exhaust gases, optimization of gas flows and engine cooling control that are now common for combustion engines of heavy trucks.

Intuitively the mapped data in Figure 1 gives the appearance that certain points are more beneficial than others. Qualitative and quantitative analysis of this situation is the topic of the present paper. Before going into the line of investigation, a motivating example is presented.

1.1 Motivating example

The tractor with trailer that will be studied in this paper, has a total mass of 40 000 kg, and it requires a torque of approximately 40% of the maximum torque on level road at the highest gear.

Consider for this truck a drive mission that with constant speed would give that the engine operates on an sfc-peak in the fuel map. It is then intuitive that it might be possible to save fuel by operating in one higher speed and one lower speed while still covering the distance in the same time as with constant speed. To show that this intuitive way of driving renders fuel savings with realistic vehicle parameters, examples have been constructed by dynamic programming, as described in (Hellström et al., 2006). One example in a small gradient where constant speed (v_0) would have been optimal, had the engine torque been affine, is shown in Figure 2. Instead non-constant speed is beneficial, to avoid the sfc-peak encountered on the operation line of stationary speeds in Figure 1.

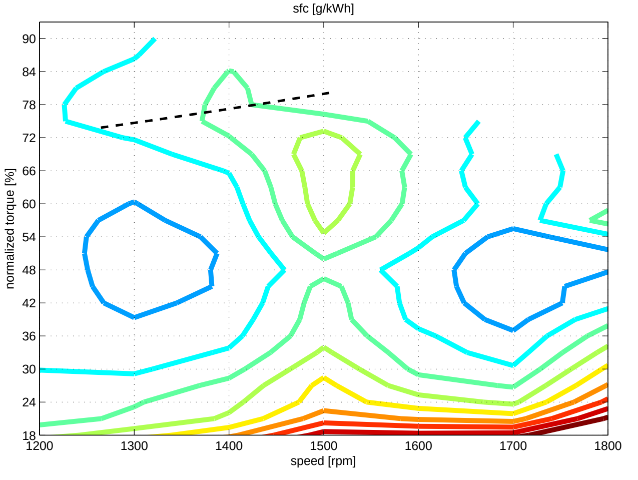

Figure 1: Fuel map showing sfc [g/kWh] of a combustion engine. The disadvantageous speed 1500 rpm requires a higher sfc than 1300 or 1700 rpm. The dashed line is the operation line of stationary speeds for a small uphill gradient.

The fuel consumption that is rendered by the varying velocity trajectory is compared to the fuel consumption of the regular cruise control in Figure 2. The reduction of fuel consumption is about 1% in this driving mission. The varying trajectory completes the road stretch in the same time as the regular cruise control does.

1.2 Line of investigation

The observation from the example is that the optimal strategy avoids a disadvantageous speed by operating above and under it, and thus a non-constant speed is sometimes optimal also for small gradients where constant speed would have been optimal if not considering the non-linear fuel map. In fact, it has been noted in the optimal control achieved in simulations according to (Hellström et al., 2006), using a non-linear fuel map, that a significant disadvantageous speed is never obtained, and that this is true regardless of how heavily total time of the driving mission is weighted into the objective function.

The example immediately poses a number of questions, like how to characterize and quantify the disadvantageous speed range around the disadvantageous

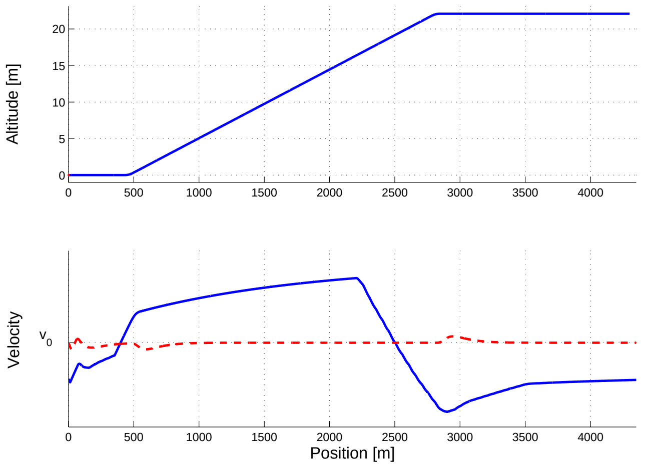

Figure 2: Upper plot: altitude profile. Lower plot: Solid line - optimal control, see (Hellström et al., 2006). Dashed line - cruise control with constant set speed, v_0.

speed. Further, to quantify the critical degree of sfc non-linearity that changes the control strategy from being constant speed. It turns out that a quasi-static analysis, where the fuel cost of accelerating and decelerating is neglected, gives a lot of insight and captures main characteristics. Therefore, after introducing the truck model and two natural criteria in Section 2, then Section 3 presents a quasi-static analysis that takes off from the intuition to switch between two velocities if a certain final time is requested and if this final time corresponds to a disadvantageous mean velocity. A simplified model of sfc is used, allowing analytical treatment. In Section 4 the dynamic truck model is returned to including transient costs, and the full fuel map is used in the optimization. The solutions are here obtained numerically. Interestingly they show a lot of the behavior obtained in the quasi-static analysis of Section 3. Implications and conclusions are drawn in Section 5 and Section 6.

2 Problem formulation

2.1 Model

The truck model used here is a basic longitudinal model. However, it captures the important characteristics to be able to predict fuel consumption correctly. The foundation for the modeling work is found in (Kiencke and Nielsen, 2005; Sandberg, 2001).

The model represents a stiff driveline with engine, transmission, final gear, wheels and chassis, based on Newton's second law of motion. The braked engine torque is denoted T_e, the vehicle speed, v, and the total driving resistance, F_{res}. The radius of the wheel is denoted r and the total transmission ratio, i, which is assumed to be constant, a relevant assumption for small gradients. The model is accordingly

$$m\dot{v} = \frac{i}{r}T_e - F_{res} \tag{1}$$

where F_{res} is a sum of rolling resistance (F_r), air drag (F_a) and gravitational force (F_g). These forces are modeled as

$$F_r(\alpha) = c_r m g \cos\alpha \tag{2}$$

$$F_g(\alpha) = m g \sin\alpha \tag{3}$$

$$F_a(N) = \frac{1}{2}c_w A_a \rho_a v(N)^2 \tag{4}$$

where c_r and c_w are constants, m is the total mass of the vehicle, A_a is the maximum vehicle cross section area, ρ_a is the air density, g is the gravitational acceleration and N is the engine speed [rpm].

The vehicle speed [m/s], v, is a direct function of N assuming no slip or elasticities

$$v = \frac{r2\pi}{i60}N \tag{5}$$

The consumed fuel mass per meter [mg/m], $\frac{1}{v}\dot{m}_f$, is a function of engine fueling [mg/stroke], δ;

$$\frac{1}{v}\dot{m}_f = \frac{\delta n_{cyl} i}{n_r r 2\pi} \tag{6}$$

where the number of cylinders is denoted n_{cyl} and the revolutions per stroke, n_r. The fueling is in the problem at hand chosen to be described as a function of specific fuel consumption, sfc [g/kWh], and T_e [Nm] where sfc is a function of T_e and N (see Figure 1) as

$$\text{sfc} = f_{\text{sfc}}(T_e, N) \tag{7}$$

Thus, engine fueling is according to

$$\delta = \frac{\text{sfc} T_e n_r 2\pi}{n_{cyl} 3600} \tag{8}$$

The reduction of fuel consumption on a specific driving mission is defined as

$$\Delta m_f = \frac{m_f - m_{f,v_0}}{m_{f,v_0}} \tag{9}$$

where m_{f,v_0} is fuel consumption when driving with constant speed (v_0) and m_f is the fuel consumption of an alternative speed profile, completing the traveled distance in the same time as the constant speed.led distance in the same time as the constant speed.

2.2 Optimality criteria

Two different optimality criteria are used in the attempt of finding the optimal speed profile that generates the lowest fuel consumption and still completes the road stretch in a certain final time, t_f. A time constrained cost function is the first that comes to mind as it gives a direct reflection of the objective. The time constrained objective function is previously used in e.g. (Fröberg et al., 2006). Next, an objective function that weights final time to consumed fuel is studied. The weighted obejctive function gives a simpler optimization and is previously used in e.g. (Hellström et al., 2006). The two optimality criteria are presented in the following subsections.

Time constrained optimality

The objective function of the time constrained optimality, I, is an integral of consumed fuel mass per meter over the traveled distance,led distance, s, i.e. the total consumed fuel mass. The objective function and its associated constraints are according to

$$I = \int_0^s \frac{1}{v}\dot{m}_f ds \tag{10}$$

$$t_f = \int_0^s \frac{1}{v} ds = \frac{s}{v_0} \tag{11}$$

$$s = \int_0^s ds \tag{12}$$

Weighted optimality

The weighted objective function, J, is according to

$$J = \int_0^s \left(\frac{1}{v}\dot{m}_f + \beta \frac{1}{v} \right) ds \tag{13}$$

where the integrals from (10) and (11) are combined. Instead of having the final time, t_f, as a constraint it is here weighted into the objective function with a weighting factor, β.

3 Quasi-static analysis

In the quasi-static analysis of this section it is assumed that the changes in speed are instantaneous, which implies that neither acceleration nor deceleration is included. This is a good approximation if the studied road stretch is long and the road gradient is constant. The driving scenario that will be used as an example throughout the section has a constant road gradient of approximately 1%, and the operation line in Figure 1 shows loads corresponding to stationary speeds of this driving scenario. The operation line traverses an sfc-peak for the speed range plotted.

A quasi-static analysis using a model of the fuel map gives valuable insights into the intrinsic properties of the problem. It turns out that the time constrained optimality gives rise to a solution where switching between two speeds is optimal instead of keeping constant speed, if the fuel map is significantly nonlinear (where the precise meaning will be given later). The weighted optimality leads to one optimal speed in stationary conditions, but also here two velocities play a crucial role in defining an unobtainable speed range.

In the stationary conditions that are considered in this section the acceleration, $\dot{v}$, in (1) is equal to zero. The equations (1) - (5) thereby result in torque as a quadratic polynomial of speed.

The sfc is here assumed to be a function of speed alone, i.e. $\text{sfc} = f_{\text{sfc}}(N)$. This is a valid assumption in a quasi-static analysis where loads and speeds of the driving scenario considered are constrained to the operation line shown in Figure 1. The function is assumed to be unimodal and symmetric around N_{sfc_0}, i.e. $\text{sfc}(N_{\text{sfc}_0}) = \text{sfc}_0$ where sfc_0 is the maximum sfc.

Fuel map model

The model used to exemplify the fuel map is

$$\text{sfc}(N) = \frac{C_d}{1 + C_i(N - N_{\text{sfc}_0})^2} + (\text{sfc}_0 - C_d) \tag{14}$$

where C_d, C_i and sfc_0 besides N_{sfc_0} are model parameters used to fit the model to the sfc of the fuel map along the operation line in Figure 1. Figure 5 shows the sfc model of this driving scenario (N_{sfc_0} is 1400 rpm). A characteristic parameter ratio of the sfc model is $\frac{C_d}{\text{sfc}_0}$. It interprets the height of the sfc elevation, and thus quantifies the degree of non-linearity, since

$$\frac{\text{sfc}_0 - \text{sfc}(N)}{\text{sfc}_0} \rightarrow \frac{C_d}{\text{sfc}_0} \quad \text{as } N \rightarrow \pm\infty \tag{15}$$

3.1 Time constrained optimality

In this subsection the optimality criterion in use is according to (10) - (12), where fuel consumption is minimized whilst final time is constrained. It is realized that minimizing fuel consumption is equal to minimizing $\int_0^s (T_e \text{sfc})\, ds$ as (6) - (8) describes the consumed fuel mass per meter, and $\frac{i}{r3600}$ is constant. Due to the quasi-static assumptions of this section further reformulations of the optimality criteria can be performed. This is the topic of the next subsection.

Reformulating the problem

In a quasi-static analysis the optimal control according to time constrained optimality, is to switch between no more than two speeds. The optimality criterion, (10) - (12) is thus rewritten into

$$\tilde{I} = \mathrm{sfc}(N_1)T_e(N_1)s_1 + \mathrm{sfc}(N_2)T_e(N_2)s_2 \tag{16}$$

$$t_f = \frac{s_1}{v_1(N_1)} + \frac{s_2}{v_2(N_2)} = \frac{s}{v_0(N_0)} \tag{17}$$

$$s = s_1 + s_2 \tag{18}$$

where s_1 and s_2 are non-negative.

The statement that an optimal control can be found by switching between no more than two speeds can be shown by studying an optimal solution where k different speeds, $N_1, \ldots, N_k$, are used to propel the truck over the distances $s_1, \ldots, s_k$, as in Figure 3. Such a solution can be used to construct another

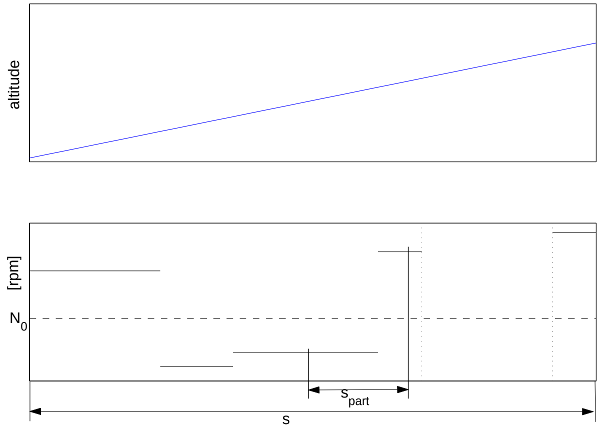

Figure 3: An optimal control which switches between many velocities is always replaceable to an optimal control that switches between no more than two speeds on a constant gradient.

driving strategy with only two speeds generating the same average speed, N_0, and the same fuel consumption. First, it is always possible to find a subinterval of the total distance s where only two speeds are used and where the average speed is the same as for the original optimal solution. This interval is denoted s_{part} in the figure. Now, consider a strategy with the same distribution of velocities on the entire traveling distance s, as in s_{part}. This is a strategy with only two speeds that fulfills the time constraint.

The second step is to show that it also has the same fuel consumption as the original optimal solution. This can be realized in the following way: First, assume that the average fuel consumption is smaller in the interval s_{part} than for the optimal solution on interval s. By using the same distribution of velocities on the entire interval s, a lower fuel consumption is obtained, which contradicts the fact that the original solution is optimal. In the same way, if the average fuel consumption is larger on the interval s_{part} than on the interval s, then we obtain a smaller fuel consumption in average on the interval $s \backslash s_{part}$ and another contradiction. Hence, the only possibility is that the distribution of velocities in interval s_{part} is optimal and it is sufficient to use this distribution with only two velocities in order to obtain an optimal solution.

Necessary conditions for stationary solution

To find the stationary solutions, the problem is formulated as an optimal control problem, see e.g. (Bryson and Ho, 1975). Hence, the objective function is augmented with the constraints using the Lagrange method resulting in

$$
\begin{aligned}
H = \mathrm{sfc}(N_1)T_e(N_1)s_1 + \mathrm{sfc}(N_2)T_e(N_2)s_2 + \\
+ \lambda_1 \left(\frac{s}{v_0(N_0)} - \frac{s_1}{v_1(N_1)} - \frac{s_2}{v_2(N_2)} \right) + \\
+ \lambda_2 (s - s_1 - s_2) - \mu_1 s_1 - \mu_2 s_2 \quad (19)
\end{aligned}
$$

where λ_1, λ_2, μ_1 and μ_2 are Lagrange multipliers and $\mu_i \geq 0$ if $s_i = 0$, $\mu_i = 0$ if $s_i > 0$ for $i = 1, 2$. The necessary conditions for a stationary value are

$$\frac{\partial H}{\partial N_i} = 0 \quad (20)$$

$$\frac{\partial H}{\partial s_i} = 0 \quad (21)$$

$$\frac{\partial H}{\partial \lambda_i} = 0 \quad (22)$$

$$\frac{\partial H}{\partial \mu_i} = 0 \quad (23)$$

where (23) is relevant only when the inequality constraints are active (i.e. $s_i = 0, \quad i = 1$ or $i = 2$). The equations (22) reflect the equality constraints. When the inequality constraints are inactive then (20) – (21) are calculated according to

$$\mathrm{sfc}'(N_i)T_e(N_i) + \mathrm{sfc}(N_i)T_e'(N_i) + \lambda_1 \frac{1}{v_i(N_i)^2} v_i'(N_i) = 0 \quad (24)$$

$$\mathrm{sfc}(N_i)T_e(N_i) - \frac{\lambda_1}{v_i(N_i)} - \lambda_2 = 0 \quad (25)$$

where $i = 1, 2$ and $v_i'(N_i)$ is constant, see (5).

Switch velocities and Switching zone

The optimal solution, N_1^* and N_2^*, as well as the Lagrange multipliers, λ_1 and λ_2, are obtained from the four equations (24) – (25). Figure 4 illustrates the optimal solution for varying N_0 (reflecting constraint on final time), based on

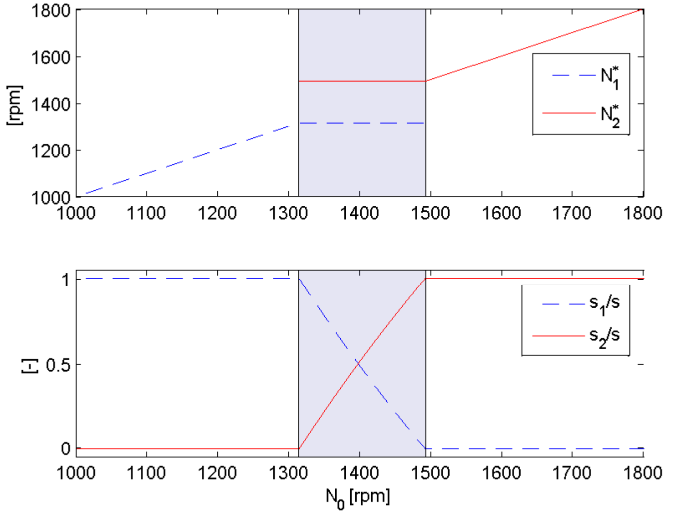

Figure 4: Optimal solution for varying N_0 (N_{sfc_0}=1400 rpm). The upper plot shows the optimal speeds, i.e. switch velocities in the switching zone (shaded area) and constant speed outside the switching zone. The lower plot shows the distances traveled with each speed.

the sfc model (14) shown in Figure 5 where N_{sfc_0} is 1400 rpm.

Switch velocities is used to denote N_1^* and N_2^* whenever $N_1^* \neq N_2^*$. The switch velocities are independent of the required final time for all final times which corresponds to an average speed, N_0, according to $N_1^* \leq N_0 \leq N_2^*$ (see upper plot of Figure 4 where the optimal solutions are constant with respect to N_0 in the shaded area).

Switching zone is used to denote the speed range that enforces two speeds, the switch velocities, as the optimal solution. In the upper plot of Figure 4 the switching zone is the shaded speed range. For a required final time corresponding to a mean speed, N_0, which is not in the switching zone, i.e. far from significant non-linearities, constant speed is optimal, $N_1^* = N_2^* = N_0$. The boundary values of the switching zone are thus the switch velocities, N_1^* and N_2^*.

When the optimal speeds have been found, the corresponding distances, s_1 and s_2, are determined from the constraints (22). To fulfill the constraints, the distances, s_1 and s_2, are adjusted (see lower plot of Figure 4). If the required final time is set so that $N_0 \to N_i^*$ then $s_i \to s$.

Figures of the studied driving scenario

If the required average speed is the disadvantageous speed, $N_0 = N_{\mathrm{sfc}_0} = 1400$ rpm, the optimal solutions for the studied driving scenario are the switch velocities, $N_1^* = 1314$ and $N_2^* = 1492$ rpm. The fuel consumption is reduced by -0.72%, when the distances travelled on each speed are set to fulfill the final time constraint. How the optimal speeds are distanced from the sfc-peak is shown in Figure 5.

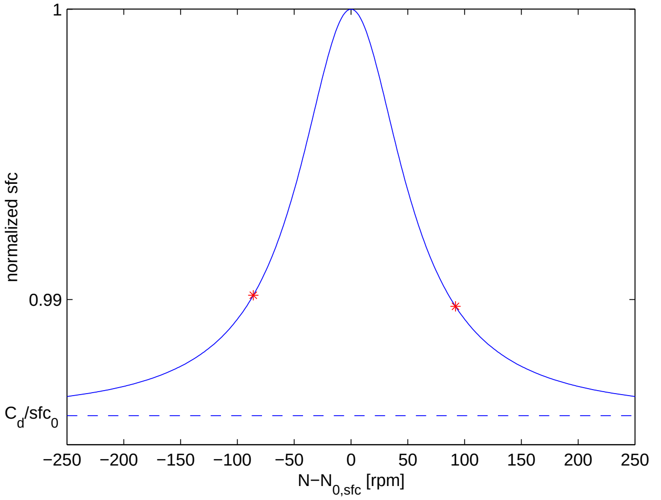

Figure 5: The $*$'s point out the switch velocities, N_1^* and N_2^* (N_{sfc_0}=1400 rpm).

Numerical optimization using the full fuel map, Figure 1, instead of the model (14) gives the switch velocities N_1^*=1292 and N_2^*=1500 rpm for the same driving mission. This is quite close to the result that was obtained with the sfc model. The fuel reduction of -0.76% is also similar to what was expected with the sfc model, confirming that the model captures the characteristics of the fuel map.

3.2 Weighted optimality

The assumption of a driving mission with constant gradient leads to one optimal speed for the weighted optimality. In order to find the optimum, the weighted objective function (13) is rewritten, by using (6) - (8) for describing fuel mass per meter, into

$$J = \int_0^s \left(C_0 T_e \mathrm{sfc} + \beta \frac{1}{v} \right) ds \tag{26}$$

where $C_0 = \frac{i}{r3600}$. The variables sfc, T_e and v are all functions of N. The stationary solutions of (26) are given by $\frac{dJ}{dN} = 0$. Due to the constant road

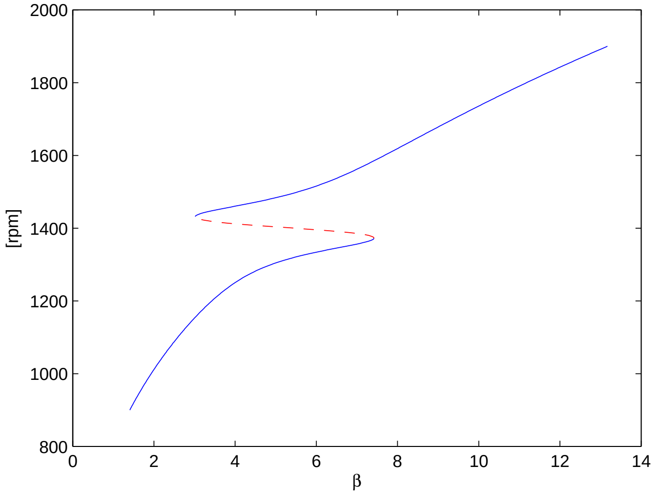

Figure 6: Stationary solutions, $\hat{N}$. Solid - local minima. Dashed - local maxima.

gradient, the stationary solutions to the objective function can also be found by studying the minimum of the integrand of the objective function (which will be denoted F, i.e. (26) is $J = \int_0^s F ds$). Thereby, the necessary condition is $\frac{dF}{dN} = 0$ which gives

$$C_0 f'(N) - \beta \frac{1}{v(N)^2} v'(N) = 0 \tag{27}$$

where $f(N) = \text{sfc}(N) T_e(N)$ and $v'(N)$ is constant, see (5). The stationary solution of (27), $\hat{N}$, a function of β if studied locally, is plotted in Figure 6.

Analysis of stationary solutions

For a certain weighting factor β there is, as stated previously, only one optimal solution. However, for some weighting factors there are more stationary solutions than one. To understand the characteristics of the stationary solutions the following calculations are performed.

The necessary condition of stationarity (27) is a function, denoted K, dependent on β and $\hat{N}$, i.e.

$$K(\hat{N}(\beta), \beta) = \frac{dF}{dN} = C_0 f'(\hat{N}(\beta)) - \beta \frac{1}{v(\hat{N}(\beta))^2} v' = 0 \tag{28}$$

where $v' = \frac{r 2\pi}{i 60}$ is a positive constant and achieved from (5). Also $\frac{dK}{d\beta} = 0$ has to be fulfilled since $K(\hat{N}(\beta), \beta)$ is constant zero. The derivative of K with respect

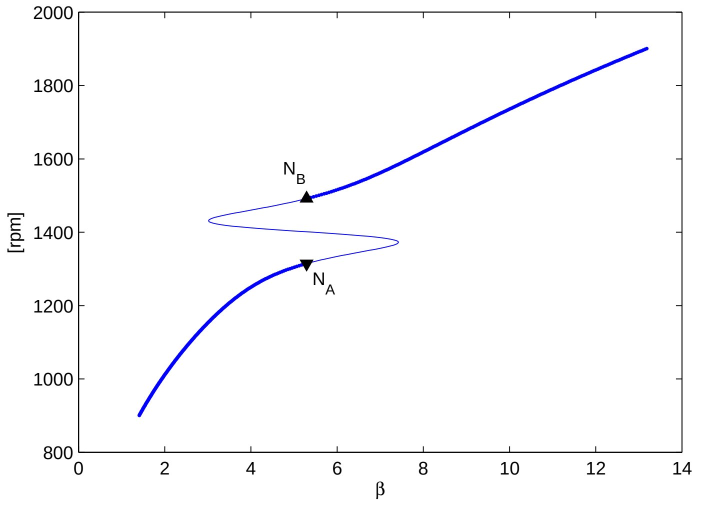

Figure 7: Stationary solutions, $\hat{N}$, where the global minimum, $\hat{N}^*$, is marked with fat lines, and the boundary velocities, N_A and N_B.

to β is calculated as

$$\frac{dK}{d\beta} = \frac{\partial K}{\partial \hat{N}} \frac{d\hat{N}}{d\beta} + \frac{\partial K}{\partial \beta} \tag{29}$$

and $\frac{dK}{d\beta} = 0$ gives the following

$$\frac{d^2F}{dN^2} \frac{d\hat{N}}{d\beta} = \frac{1}{v^2} \frac{r2\pi}{i60} \tag{30}$$

Since $\frac{1}{v^2}$ is always positive, $\frac{d^2F}{dN^2}$ and $\frac{d\hat{N}}{d\beta}$ must always have the same sign. This means that whenever the slope of $\hat{N}(\beta)$ in Figure 6 is positive, $\frac{d\hat{N}}{d\beta} > 0$, then $\frac{d^2F}{dN^2} > 0$ and there is a local minimum of the objective function.

Unobtainable speed range and boundary velocities

For some β there are two local minima. By comparing the objective function values of the local minima, the global minimum, $\hat{N}^*$, is found to be the speeds plotted with fat lines in Figure 7. The characteristic of the global minimum, is the jump at a certain β (at $\beta = 5.3$ in the studied driving scenario) from one speed, N_A, to another, N_B, which occurs for significant non-linearities.

Boundary velocities is used to denote N_A and N_B. The boundary velocities are called so since they are on the verge of the discontinuity, which will be studied later. Figure 8 shows the weighted objective function calculated for the weighting factor β that renders the same objective function value at the boundary velocities (i.e. $\beta = 5.3$). The same figure shows the objective function for two β that give a higher and a lower optimal speed respectively, than the boundary velocities.

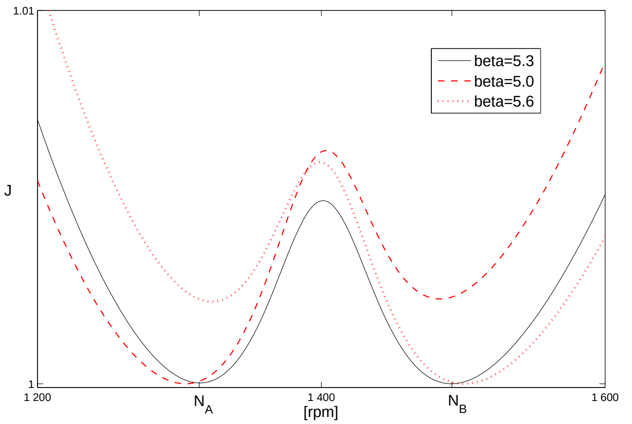

Figure 8: Objective function (26) for the studied constant road grade with different β.

The equations that give the boundary velocities can be summarized as

$$C_0 f'(N_A) - \beta \frac{1}{v(N_A)^2} v' = 0 \tag{31}$$

$$C_0 f'(N_B) - \beta \frac{1}{v(N_B)^2} v' = 0 \tag{32}$$

$$C_0 f(N_A) + \frac{\beta}{v(N_A)} = C_0 f(N_B) + \frac{\beta}{v(N_B)} \tag{33}$$

where (31) and (32) generate two stationary solutions (N_A and N_B) for the same β and the last equation says that these solutions shall give the same objective function value.

Unobtainable speed range under J is used to denote the speed range $(N_A \;\; N_B)$ of the discontinuity of the global minimum. There is no weighting factor β such that the trade-off between time and fuel consumption will result in a speed in this range.

3.3 Relation between the characteristic velocities

The optimal solution will differ depending on the applied optimality criteria. The optimal solution of the weighted objective function is always a constant speed that is never within the unobtainable speed range under J, $(N_A \;\; N_B)$. The time constrained optimality conditions leads to an optimal solution that switches between the switch velocities, N_1^* and N_2^*, if the required final time is compatible to a mean velocity, N_0, within the switch zone $(N_1^* \;\; N_2^*)$.

However, there are similarities between the two optimality criteria which become clear if (31) – (33) are rewritten with $f(N_i) = \text{sfc}(N_i)T_e(N_i)$ and λ_2 is eliminated from the four equations (24) – (25). The weighting factor is found to be $\beta = -\lambda_1 C_0$, and hence $N_1^* = N_A$ and $N_2^* = N_B$. This shows that the switch zone, $(N_1^* \;\; N_2^*)$, for the time constrained optimality and the unobtainable speed range under J, $(N_A \;\; N_B)$, for the weighted optimality are equal.

3.4 Significant non-linearity

The degree of non-linearity in sfc influences the optimal solution and it is therefore important to classify the non-linearities as significant or insignificant dependent on the optimal solution characteristics. A non-linearity in sfc is defined to be significant if the optimality criteria has multiple stationary solutions, as in Figure 6. The significance of a non-linearity is an effect of the sfc model parameter setting. An unspecified combination of model parameters that describes the degree of sfc non-linearity, denoted γ, is set up for the general calculations in the first subsection. These calculations result in the critical threshold, γ_c, i.e. the value for which the non-linearity turns into being significant. When $\gamma < \gamma_c$ the non-linearity is insignificant and the optimal solution is one constant speed equal to the required mean speed. In the second subsection, the model parameter setting of the studied sfc model, (14), is considered. The combination of model parameters that describes the non-linearity is $\gamma = \frac{C_d}{\text{sfc}_0}$, see (15), for which the critical threshold is determined. Subsequently, the reasons why a low engine efficiency, on the peak of an insignificant non-linearity, is beneficial compared to switching speeds is discussed.

Calculating the critical threshold

The critical threshold, γ_c, is calculated by studying the necessary conditions for stationary solutions of the weighted objective function, i.e. $\frac{dF}{dN} = 0$ shown in Figure 6. For the critical parameter setting, the maximum of the objective function (close to N_{sfc_0}) turns into a minimum and the three solutions in Figure 6 degenerate into a triple root. Continuity is thus achieved in $\hat{N}^*(\beta)$, plotted with fat lines in Figure 7, and there are no unobtainable speeds under J. If the equation $\frac{dF}{dN} = 0$ has a triple root it follows that also the second and the third derivative of the cost function are equal to zero. Summarized, the equations to be solved to find the critical threshold are according to

$$\frac{dF}{dN} = 0 \tag{34}$$

$$\frac{d^2F}{dN^2} = 0 \tag{35}$$

$$\frac{d^3F}{dN^3} = 0 \tag{36}$$

The equations (34) – (36) are functions of the unknown variables N, β and γ, for a given driving mission with a constant gradient. The speed N, close to N_{sfc_0}, and the corresponding weighting factor β, are solved for as well as the critical threshold, $\gamma = \gamma_c$.

Critical threshold for the studied parameter setting

For the studied sfc model (14) the degree of non-linearity is described by $\gamma = \frac{C_d}{\text{sfc}_0}$ since this parameter ratio interprets the height of the sfc-elevation, see (15), and thus describes the degree of non-linearity. The model parameter C_i is also of great interest, interpreting the inclination of the sfc-peak, and of course γ_c is dependent of C_i. The curve in Figure 9 illustrates the critical threshold, γ_c. If the model parameters are above the critical curve, the non-linearity is significant. The $*$ above the curve points out the model parameters, $\frac{C_d}{\text{sfc}_0}$ and C_i, of the studied sfc model, which is a significant non-linearity.

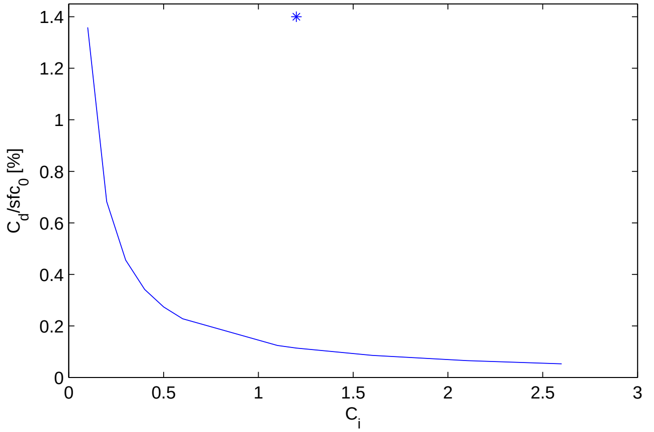

Figure 9: The curve illustrates γ_c, the critical threshold. The $*$ shows the parameters of the sfc model considered in the quasi-static analysis.

The dependency of $\frac{C_d}{\text{sfc}_0}$ to C_i along the curve in Figure 9 is expected, a low elevation requires a large inclination to make the non-linearity significant and vice versa. Moreover, the critical threshold, γ_c, is dependent on e.g. the load, due to the influence of engine torque in the cost function. The higher the load, e.g. the road gradient, the lower is the critical threshold. In Figure 9 the road gradient is constant, the same as in the driving scenario of the quasi-static analysis.

Discussion of insignificant non-linearities

If the model parameters C_d and/or C_i are set to zero then the engine torque is affine in fueling, see (14), and constant speed is of course optimal in this analysis, as it is also in a varying small gradient (Hellström et al., 2009; Hellström, 2007; Fröberg et al., 2006; Chang and Morlok, 2005). Moreover, it has been shown that also for some small, insignificant non-linearities, constant speed is still optimal and switching speeds is not beneficial (see Figure 9 where γ_c is nonzero).

The reason why a non-linearity in the fuel map, with a low engine efficiency, does not always generate an optimal solution that switches between a high and a low speed is that air drag, F_a, over the total distance increases by switching speeds compared to keeping a constant speed. The increased air drag is a consequence of air drag's quadratic dependency to speed, see (4). The increased air drag is an increased energy loss of the vehicle and for some small non-linearities the small gain that is obtained by switching speeds is not enough to level up the energy loss due to increased air drag.

4 Dynamic analysis

Now return to the full dynamic system (1) – (8). This means that accelerations and decelerations are fully included, and that the mapped sfc is used as a look-up map. In this section, time constrained optimality (10) – (12) is considered. Recall that, also for the full dynamic system, constant speed is optimal on small gradients if the engine torque is affine in fueling and engine speed, see e.g. (Fröberg et al., 2006). Moreover, in large gradients under the assumptions of affine engine torque, acceleration/deceleration in an optimal manner is performed by using maximum/minimum torque, according to e.g. (Fröberg et al., 2006).

The purpose now is to investigate the consequences of the non-linear fuel map in a full dynamic analysis and to make comparisons to the results and insights from the quasi-static analysis in the previous section. The fuel map studied (Figure 1) has significant non-linearities in a quasi-static perspective, such that γ, for some driving scenarios is larger than the critical threshold, γ_c, obtained by (34) - (36) in Section 3.4.

It will in this section be shown that the fuel map contains non-linearities that are significant also from a dynamic perspective, meaning that a non-constant speed profile still is optimal for some driving scenarios, even though transients are considered. Additionally, the characteristics of the transients are studied in terms of optimal torque/speed profile. After some introductory remarks on the constraints and on the computational scheme, there is a sequence of subsections treating in order a constant gradient, a simple road profile, and a real road profile.

Constraints

Maximum and minimum engine torque are engine specific functions. Realistic experimental values are used as plotted in Figure 10, and they are modeled as

$$T_{e,max}(N) = c_1 N^2 + c_2 N + c_3 \tag{37}$$
$$T_{e,min}(N) = c_4 N + c_5 \tag{38}$$

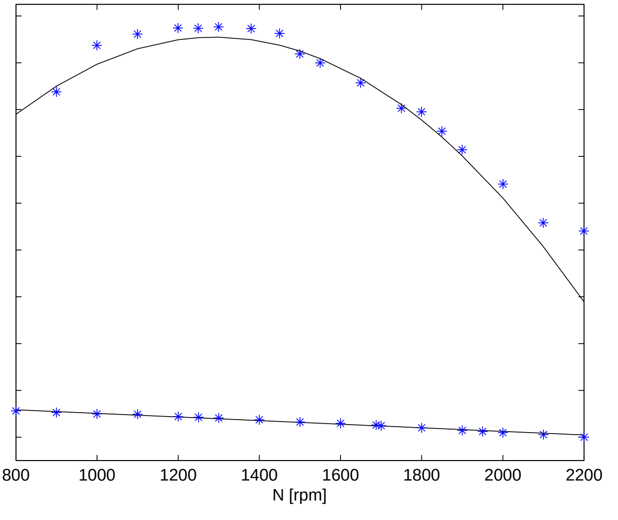

Figure 10: Maximum and minimum torque.

The constraints of the optimization are thus:

- the constraint of final time (11); i.e. to finish the driving mission in at most the time that corresponds to the constant average speed $v_0(N_0)$
- to control the torque within its limits; $T_{e,min} \leq T_e \leq T_{e,max}$, according to (37) and (38)
- not to decrease the kinetic energy over the driving mission, $v_{final} \geq v_{start}$

Computational scheme

To perform the numerical computations, the system dynamics (1)-(7) is written in the form

$$\dot{v} = f(v, \alpha, T_e) \tag{39}$$

and a transformation to distance dependence is done according to

$$\frac{dv}{ds} = \frac{dv}{dt}\frac{dt}{ds} = \frac{1}{v}\dot{v} \tag{40}$$

An Euler forward discretization approximates the dynamics of the system according to

$$v_{i+1} = v_i + \frac{\Delta s}{v_i} f(v_i, \alpha_i, T_{e,i}) \tag{41}$$

The optimization is implemented by the use of Matlab (MathWorks, 2007) using its non-linear optimizer *fmincon*.

4.1 Constant gradient

In order to study the behaviour, the investigation starts with the simplest case, namely constant small gradients either uphill or downhill. A small uphill inclination is shown in Figure 11 where the optimal control gives a fuel consumption reduction of -2.4%. The torque/speed trace of this drive cycle is shown in the fuel map in Figure 12.

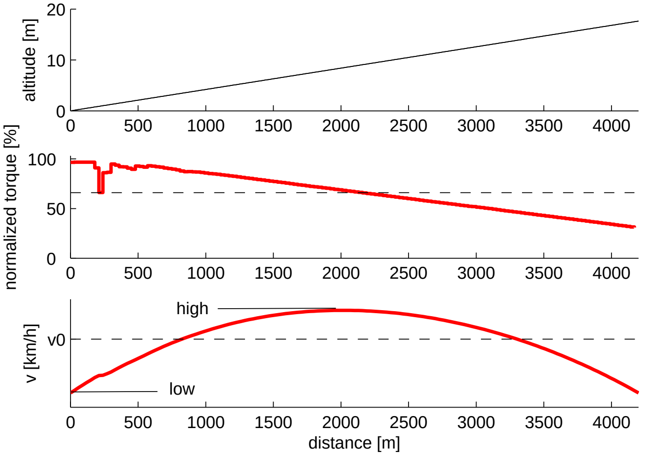

Figure 11: Solid - optimal control. Dashed - constant speed control. $\Delta m_f = -2.4\%$ on a 4200 m road stretch.

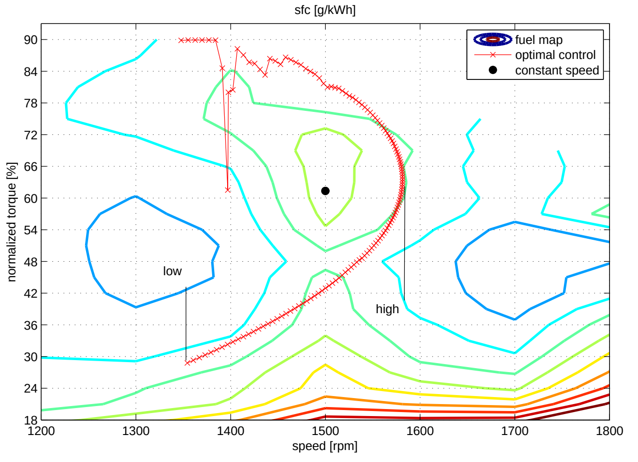

Figure 12: Altitude profile and optimal control according to Figure 11.

Constant small downhill gradients require a longer road stretch for a varying speed trajectory to be advantageous for the considered fuel map. Figure 13 shows the optimal speed trajectory for a slight downhill gradient on an 8000 m road stretch. The optimal control gives a fuel reduction of -1% on this road stretch. Figure 14 shows the torque/speed trace of this driving mission.

Observations

The first observation from the simulations shown in Figures 11 – 14 is that fuel is saved. For the fuel map of this paper, the largest savings are possible to make in an uphill gradient (partly because the fact that, for a higher load, less is required for a non-linearity to be significant, see Section 3.4). Further, the savings in Figures 11 – 14 are obtained by switching between two velocities (marked high and low in the figures). It can be seen that it is not optimal to accelerate with maximum torque and not to decelerate with minimum torque. The accelerations and decelerations are performed in different manners depending on the gradient. However, it is obvious that the optimal strategy is to avoid elevations in the fuel map which can be seen both in Figure 12 for the slight uphill and in Figure 14 for the slight downhill. It is quite natural that the motion around the disadvantageous speed in these figures depend on the fuel cost for acceleration and deceleration. It is also natural that the importance of this cost depends on road topography in combination with the characteristics of the fuel map. One way of how the transition cost affects the optimal solution is studied in the next subsection. Further insight is there gained in the phenomena of transients being less important on a longer road stretch compared to a shorter stretch with small gradients.

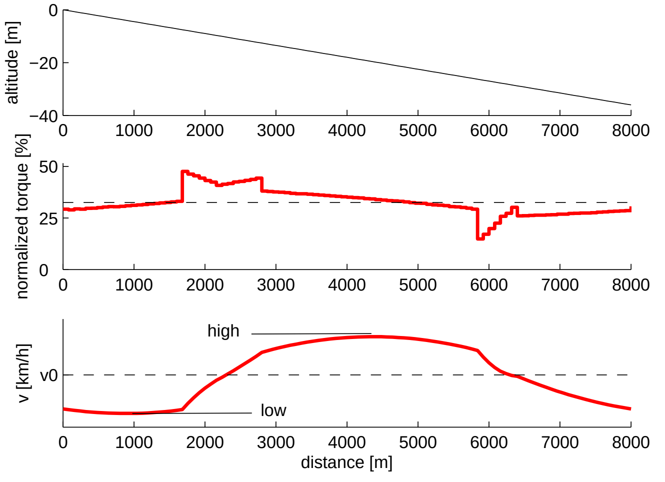

Figure 13: Solid - optimal control. Dashed - constant speed control. $\Delta m_f = -1\%$ on a 8000 m road stretch.

Transient costs in quasi-static analysis

In Figures 11 – 14 it is obvious that also in this section when acceleration and deceleration are regarded, two velocities (marked high and low in the figures) are characteristic for the optimal control, as it is in the quasi-static analysis in Section 3. Thus, in a qualitative manner the solutions have the similarity that they are of a switching type, but here the switching is not instant but instead obey the dynamics of the system. One may note that the high and low speed levels are not equal to the switch velocities, quantitatively, and this is a natural consequence of the cost that originates from changes in speed.

To see what the high and low speed levels are compared to the quasi-static switch velocities (N_1^* and N_2^*), the quasi-static analysis is extended with the cost of acceleration/deceleration, denoted $\Phi(N_h, N_l)$, where the high and low speeds are denoted N_h and N_l respectively. It is clear that Φ is larger the larger the distance between N_h and N_l is since it implies a larger transient. This cost, Φ, is added to the quasi-static time constrained objective function, $\tilde{I}$ in (16). The new objective function is divided by the distance of the complete road stretch, s. The resulting objective function is

$$\tilde{I}_\Phi = \mathrm{sfc}(N_h)T_e(N_h)\bar{s}_h + \mathrm{sfc}(N_l)T_e(N_l)\bar{s}_l + \frac{\Phi(N_h, N_l)}{s} \qquad (42)$$

where $\bar{s}_h = \frac{s_h}{s}$ and $\bar{s}_l = \frac{s_l}{s}$ are normalized distances. The associated constraints

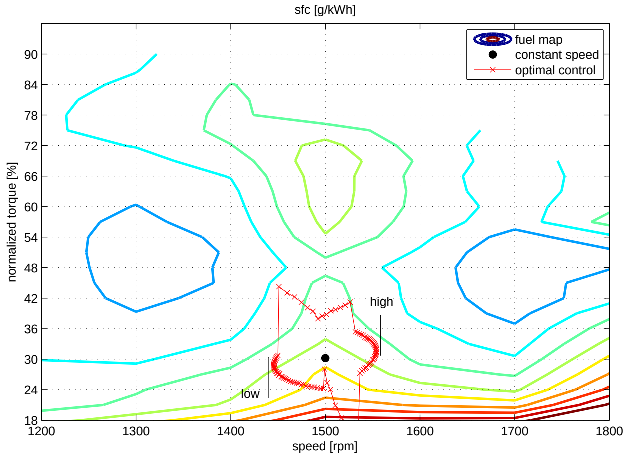

Figure 14: Altitude profile and optimal control according to Figure 13.

are

$$\frac{1}{v_0} = \frac{\bar{s}_h}{v_h} + \frac{\bar{s}_l}{v_l} \tag{43}$$

$$1 = \bar{s}_h + \bar{s}_l \tag{44}$$

Recall that if $\Phi = 0$ then (42) turns into (16), i.e. quasi-static time constrained optimality with the switch velocities N_1^* and N_2^* as solution independent of the length of the road stretch, s. If $\Phi \neq 0$ then the solution, the high and low speed levels, N_h and N_l, are dependent on the total distance, s. By alternating between a high and low speed not far from each other, the transition cost, Φ, is decreased and thus a lower value of the objective function is obtained. Thus, for short distances the optimal high and low velocities will be closer to the required mean speed, N_0, compared to the switch velocities. For longer road stretches, i.e. larger s, the magnitude of Φ/s in (42) influences the optimal solution less, and the optimal high and low velocities are more distanced from each other. Finally, if $s \to \infty$ it follows from (42) that the optimal high and low velocities equal the switch velocities.

4.2 Simple road profile

An arbitrary driving mission is not restricted to a constant gradient. Instead the full fuel map can be traversed, and as it most often has several disadvantageous areas then several sfc-peaks should be avoided. A first step towards understanding this situation is to study the simple case of two gradients. In this section, the two constant gradients used in Section 4.1 are adjoined using an altitude profile that follows the standards of the Swedish Road Administration for vertical curves, see (Swedish Road Administration, 2004), namely a second order curve with restricted curvature.

Figure 15 shows the result of the optimization, and the torque/speed trace of the same optimization is shown in Figure 16.

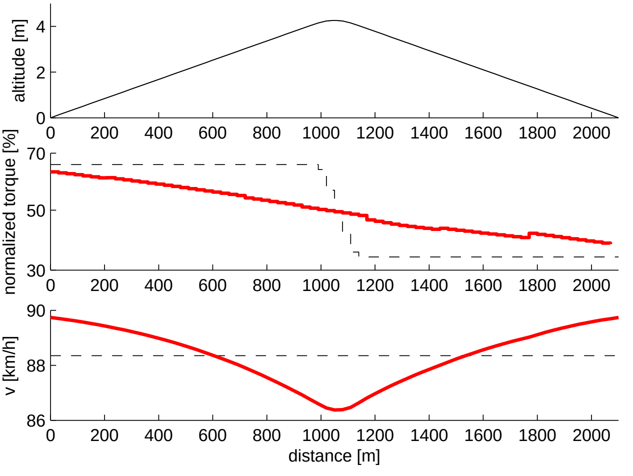

Figure 15: Solid - optimal control. Dashed - constant speed control. $\Delta m_f = -1.6\%$ on a 2100 m road stretch.

The result is that fuel is saved, but there are also some interesting observations to be made in Figure 16 about the behavior. The vertical line represents the optimal control for a fuel map with insignificant non-linearities, being constant speed for these small gradients. For this vertical line it can be seen that there are two disadvantageous areas, and that the optimal control is beneficial since it avoids these areas. One may note that this is done by acceleration downhill and retardation uphill.

4.3 Real road profile

A real road segment will now be used to illustrate the behavior in a driving mission traversing the fuel map with its sfc-peaks that are beneficial to avoid. Comparisons will be made with (Hellström et al., 2009) where look ahead control was implemented in a truck and real test runs were made on the highway between Södertälje and Norrköping, Sweden. This road stretch has significant hills and fuel consumption reduction was therefore achieved, compared to the conventional cruise control, by reducing the need for braking and gear shifting. One part of this road stretch (south of Nyköping) mainly has small gradients and is shown in Figure 17. A 3350 m part of the road segment in Figure 17 (from shortly after 4 km to about 8 km) has been simulated both with constant speed

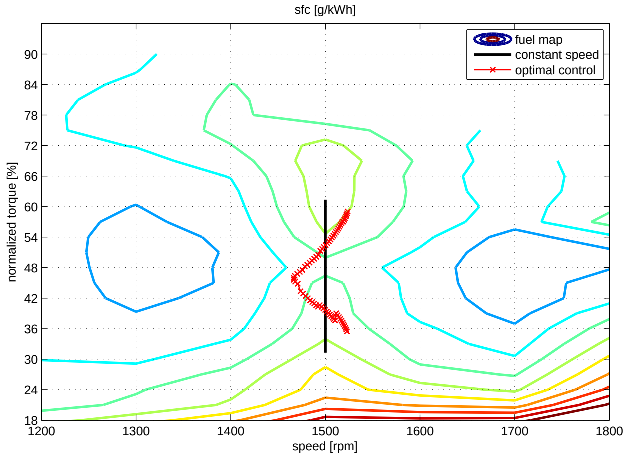

Figure 16: Altitude profile and optimal control according to Figure 13.

(that would be optimal given an affine relation between engine torque and fueling) and optimal control based on the fuel map considered in this paper. The results are shown in Figure 18.

The gradient on this road stretch varies (-1% $< \alpha <$ 1%) along the distance, which implies that a wide range of engine torque is required to keep constant speed, and a couple of sfc-peaks are encountered. The optimal control avoids these peaks, and the result is a significant fuel saving. In this example, and this is typical for the considered fuel map, the optimal control avoids the sfc-peaks with a behavior much like the one in Figure 16, namely acceleration in small downhill gradients and deceleration in small uphill gradients.

5 Implications

The previous section shows potential to reduce fuel consumption on real roads with small gradients even when transients are considered, if the non-linearity is significant in a dynamic perspective, as described in the opening of Section 4. Then as in Figures 11 and 13 it is beneficial to switch between a high velocity, N_h, and a low velocity, N_l. The optimal velocity trajectory is calculated from the time constrained optimality (10) - (12), but the consequences of transients and costs associated to that are found by studying the expanded quasi-static objective function $\tilde{I}_\Phi$ in (42). It can be concluded that to achieve significancy of non-linearities for a fully dynamic system, the reduction of sfc in a higher/lower speed needs to be as large as to overcome the transient cost as well as to counteract the increased air drag. The intuition to avoid a disadvantageous average speed, by operating above and under it, is still confirmed in several simulations,

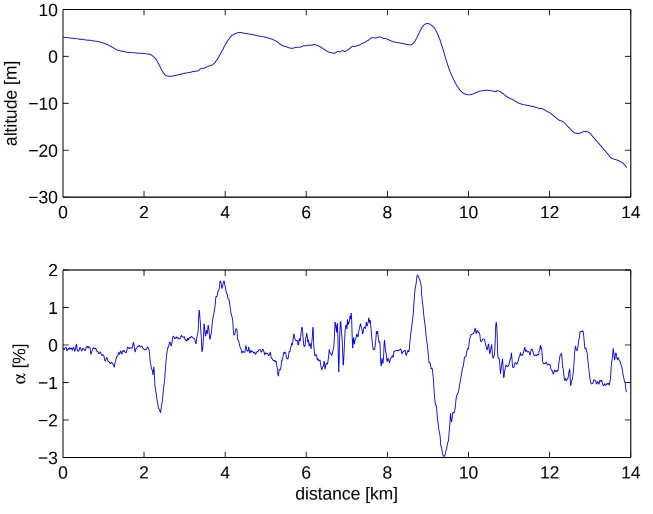

Figure 17: Road profile south of Nyköping.

e.g. in Figure 18, even as the full dynamic system is considered. Since the fuel savings are substantial for fuel maps with non-linearities of the same order as the one studied here, it is interesting to further analyze the implications for the proposed look ahead control on real roads.

5.1 Look ahead horizon

Still considering time constrained optimality, one may note that the transient costs, according to the above reasoning, has consequences for the choice of look ahead horizon. The transition cost, Φ, together with the total distance of a small gradient, affect the optimal control, where it is clear that, the shorter the horizon, the bigger the impact of acceleration and deceleration. This is realized by studying the objective function $\tilde{I}_\Phi$ in (42), which gives a narrower speed range between the high and low speed if Φ/s is large, i.e. if s, the look ahead horizon, is small. If s is large, the gap between N_h and N_l becomes larger. To obtain the full potential fuel saving on a constant gradient, a sufficiently long look ahead horizon should be used in order to enable the high and low velocities to come close to the switch velocities which is beneficial besides the possibility of switching speeds as seldom as possible. It should also be kept in mind that the longitudinal dynamic of a heavy truck is quite slow and that optimal accelerations can last for more than a kilometer (see e.g. Figure 13).

A varying road grade gives a larger possibility of choosing the most beneficial time to accelerate/decelerate, as shown in Figure 15 and Figure 18. Thus, a beneficial look ahead horizon should be as long as to include variations in the

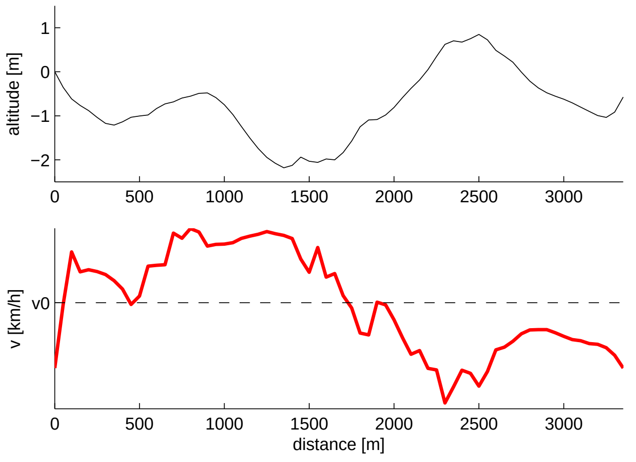

Figure 18: Solid - optimal control. Dashed - constant speed control ($\Delta m_f = -1.1\%$).

topography, to enable economical acceleration and deceleration.

5.2 Weighted optimality

The advantage of the time constrained optimality is that the required final time is always guaranteed, but a disadvantage compared to weighted optimality is that it requires more computational effort. If the final time is important but not as critical as the cost of fuel consumption, then weighted optimality (13) may be preferable due to its simpler form.

Recall from Section 3.2 that weighted optimality with a fixed parameter β gives a solution so that the speed trajectory can be seen as a function of β. Recall also that in the quasi-stationary analysis this gives raise to unobtainable speeds as depicted in Figure 7.

The first observation, that speed is a function of β, leads to the natural idea to use some technique to tune the parameter β to obtain a desired average velocity and thus a desired trip time. In the tuning process it is necessary to take into account information such as the desired trip time, road profile and total mass of the vehicle. Similar tuning problems have been studied in (Sciarretta and Guzzella, 2007), where the objective is to minimize fuel consumption in a hybrid vehicle, and thus the final state of charge must be weighted into the objective function.

In the case with significant non-linearities in the fuel map the second observation is relevant. In the stationary case there are discontinuities in the optimal velocity as a function of β (as shown in Figure 7). Thus, there might be discontinuities in the trip time as a function of β for a given drive mission with a known road profile. Hence, it is not always possible to find a β such that the desired trip time is obtained. If the trip time is essential in such a case it is necessary to use time constrained optimality with its switching behavior. Nevertheless, when considering a longer look ahead horizon and a varying road profile an interesting behavior can be observed when using weighted optimality. The required final time, which correlates to a disadvantageous mean speed, is in this case achieved due to the varying road slope, despite the use of a constant β. In fact, such an example is shown in Figure 2, and interestingly enough a switching like behaviour can be observed due to the varying road gradient. Thus, the problem with non-feasible trip times for weighted optimality becomes less significant with a longer look ahead horizon and a more varying road profile due to this type of switching behavior.

6 Conclusions

Look ahead control has been considered in the case where the fuel map expressed in sfc does not have the conventional concave shape, but has significant non-linearities. The interesting case that has been in focus is roads with small gradients in which the vehicle is able to keep a constant speed, as defined in (Fröberg et al., 2006). When engine torque is affine in fueling, constant speed is fuel optimal on such road segments. For the case of a significantly non-linear fuel map it has been shown that fuel savings in the order of percents may be otained on the same road segments by varying the velocity.

The investigation was performed for two cases, namely time constrained optimality and weighted optimality. They both consider fuel optimality while respecting trip time, but the first does so strictly and the second indirectly by the cost assigned to a time loss. It turns out that they both avoid disadvantegous speeds, but in different ways. Nevertheless, even though the behavior are different the two solutions have common characteristics. The quasi-static analysis in Section 3 gave a good understanding for how time constrained optimality leads to switching between two characteristic velocities, whereas in the weighted optimality the same characteristic velocities instead determine an unobtainable speed range. It was also clarified how the existence of characteristic velocities is related to the degree of non-linearity, in terms of threshold of significant non-linearities quantified in (34) – (36), to change the control strategy from being constant speed. Additionally, for a significant non-linearity the multiple stationary solutions were sorted as local minima or maxima in Section 3.2 by studying (30).

The results were verified in Section 4 for the full dynamic system including a realistic fuel map and other realistic constraints, like maximum and minimum torque. If strict time enforcement is not needed, then the choice of optimality criterion becomes a choice between managing β-tuning in weighted optimality and having more computations in time constrained optimality. When considering longer distances with varying road gradient the difference between the two optimal solutions tends to lessen.

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Paper 3

Impacts of AMT Gear-Shifting on Fuel Optimal Look Ahead Control[1]

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3

Abstract

A fuel optimal gear shift control has been studied, when look ahead information is available, and the impact of the automated manual transmission (AMT) gear-shifting process is analayzed. For a standard discrete heavy truck transmission, answers are found on when to shift gears, prior to or when in an uphill slope. The gear-shifting process of a standard AMT is modeled, not considering the comfort details, in order to capture the fuel and time aspects of the gear shift. A numerical optimization is performed by dynamic programming, minimizing fuel consumption and time by controlling fuel injection and gear. Since a standard AMT does not have look ahead information, it sometimes gears down unnecessarily and thus gives a significantly higher fuel consumption compared to the optimal control. However, if gearing down is inevitable, the AMT gear-shifting strategy, based on engine thresholds, is a well-functioning gear control so that the optimal control only gives marginal additional savings. To attain the possible fuel reductions it is shown that the reduced propulsion of an AMT gear-shifting process, and the resulting vehicle retardation, must be considered. The point of shifting gears must be chosen to ensure an adequate engine speed in order to get a sufficient engine power after the gear shift, even as the truck is decelerated during gear shift.

[1]This is a technical report at the Departement of Electrical Engineering, Linköpings Universitet, LiTH-ISY-R-2883

1 Introduction

It has been shown that an optimal velocity profile for a heavy truck not only ensures a high utilization of the energy in a topography with significant hills, but also that gear-shifting is less likely to occur, see (Hellström et al., 2006, 2009). However, for long uphill slopes with large road gradients, gearing down becomes inevitable since the maximum engine torque on the direct gear is not high enough to manage propeling the vehicle within an acceptable speed range.

Heavy trucks with an automatically controlled manual gear box, i.e. an automated manual transmission (AMT), are common in long haulage and the impact of its gear-shifting process on optimal gear control will be studied. In Figure 1 a typical AMT gear-shifting process is illustrated, in terms of engine torque and speed. An AMT system does not contain a clutch or torque con-

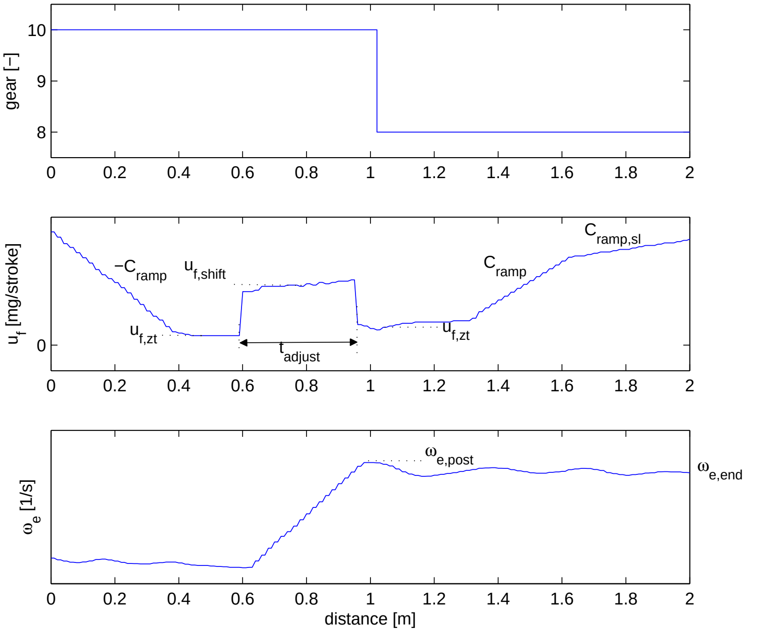

Figure 1: A gear shift from 10th to 8th gear of a 37 tonnes truck in a 6.5% uphill gradient.

verter. Instead engaging and disengaging of gears are enabled by engine torque control (as shown in Figure 1), which is performed differently depending on driving scenario. This leads to a lower propulsive force for a couple of seconds and, accordingly, the traction of the vehicle is affected by the AMT gear-shifting process. The low propulsive work during a gear shift leads to that vehicle speed is, for a short period of time, only determined by the driving resistance. The driving resistance varies with the road gradient and truck speed, and consequently the truck is affected differently depending on when the gear shift is executed.

Due to the characteristics of the AMT gear-shifting process, and the fact that the truck is retarded differently during gear shifts, the occasion of shifting gears does affect comfort, performance and fuel consumption. A fuel optimal gear-shifting strategy that respects final time is many times requested, however it is not intuitive to find. The objective is thus to find the fuel optimal gear-shifting strategy. Firstly, the gear-shifting process of a standard AMT is modeled, not considering the comfort details, in order to capture the fuel and time aspects of the gear shift. Thereafter, a numerical optimization is performed by dynamic programming, minimizing fuel consumption and time by controlling fuel injection and gear.

1.1 Outline

The numerical optimization is performed by dynamic programming, since a gear-shifting process that varies with the road profile and the status of fueling, speed and current gear is easily implemented with this method. To enable the optimization, models are set up to capture the characteristics of fuel consumption in highway driving and accordingly in Chapter 2 and 3 the road, the truck and specifically the gear shift model is presented. In Chapter 4 the model is adapted to the numerical optimization and a verification of the gear shift model is given. Chapter 5 presents a discussion regarding energy and fuel consumption when climbing a hill and how this relates to trip time. In Chapter 6 and 7 results are shown and a discussion of the AMT gear-shifting process is presented. In Chapter 8 conclusions are drawn.

2 Problem formulation

2.1 Road model

Uphill slopes with significant road gradients are studied in order to find roads that enforce gear shifts. Swedish highways are well documented by the Swedish Road Administration (2004), and are thereby chosen as the road model of this paper. Highways are if possible constructed to have a road gradient, α, less than 6%, which classifies the road to a high standard. Road gradients up to 8% are allowed as well but the road is then classified as low standard. However, for the purpose of studying heavy truck gear-shifting, 6% is an interesting road gradient. In the studied road profile, shown in Figure 2, the 6% road gradient is kept constant for a road segment long enough to impose gear-shifting (approximately between 1000 and 2000 m).

Highways consist of road segments of constant gradients adjoined by vertical

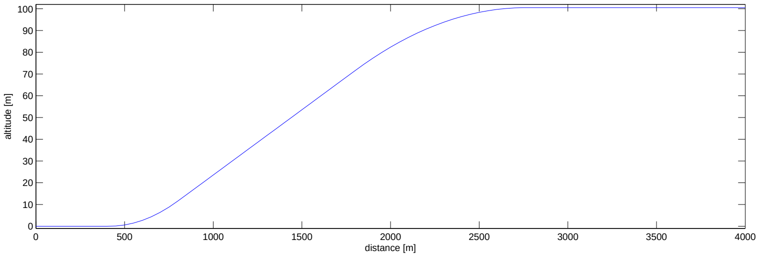

Figure 2: Road stretch for which gear-shifting strategies are studied.

curves. The relative altitude Δz in a vertical curve is a parabolic function according to

$$\Delta z = \frac{s^2}{2R} \tag{1}$$

where s is the horizontal distance and the curve radius, R, is set so as to assure line of sight as well as to enhance the aesthetics of the road in the surrounding terrains. The road that is analyzed, shown in Figure 2, is classified as a high standard highway also in the aspect of radii of vertical curves, with $R_{concave} = 6500$ m and $R_{convex} = 16000$ m. In Swedish highways the maximum vehicle mass of a heavy truck, m, is 60 tonnes. Optimizations will be performed for different vehicles, 17-60 tonnes, on the same uphill slope.

2.2 Truck model

It has been shown in (Hellström et al., 2008) that the optimization of a truck model using distance as the independent variable, is performing well if kinetic energy, $\frac{m_{tot}}{2}v^2$, is considered as a state rather than speed, v. The kinetic energy changes with the total force of the system, F_{tot}, i.e. the difference between propelling force and driving resistance, and infinitesimally this change is according to

$$\frac{m_{tot}}{2} dv^2 = F_{tot} ds \tag{2}$$

Since the mass, m_{tot}, is constant, v^2 instead of the kinetic energy is defined as a state of the system, giving the system dynamics according to

$$\frac{dv^2}{ds} = \frac{2}{m_{tot}} F_{tot} = 2 \frac{r_w}{J_l + m r_w^2 + \eta(g) i(g)^2 J_e} \Big(i(g)\eta(g) T_e(\omega_e, u_f) - \\ - r_w \left(F_a(v^2) + F_r(m, \alpha) + F_N(m, \alpha) \right) \Big) \tag{3}$$

following the modeling described in (Kiencke and Nielsen, 2005; Sandberg, 2001). The notation of the driving resistance, $F_a(v^2) + F_r(m, \alpha) + F_N(m, \alpha) = \frac{1}{2} c_w A_a \rho_a v^2 + m g c_r \cos\alpha + m g \sin\alpha$, and the driveline inertia, J_l and J_e in (3) is according to (Hellström et al., 2009). Engine torque, T_e, is assumed to be affine in fueling, u_f, and engine speed, ω_e,

$$T_e(\omega_e, u_f) = T_{e,ind}(u_f) - T_{e,loss}(\omega_e) = c_f u_f - (c_\omega \omega_e + c_c) \tag{4}$$

The state vector, x, consists of the squared velocity, v^2, and the currently engaged gear, g,

$$x = \left[v^2, g\right]^T \tag{5}$$

The gear is chosen from a discrete set, $g \in [1 \quad 2 \quad \dots \quad 12]$, according to a standard transmission of a heavy truck. The gear ratio corresponding to each gear, $i(g)$, includes the constant gear ratio of the final drive. The highest gear, $g = 12$, implies the lowest gear ratio, $i(12)$ i.e. the gear ratio of the final drive alone.

The velocity is constrained to $v \leq 89$ km/h as this is the maximum speed of a heavy truck. Since only uphill gradients are considered, this constraint will always be possible to comply with, without braking. Braking is a waste of energy, and accordingly braking will not be considered as a control signal. The control signals are thus engine fueling, u_f [mg/stroke], and gear selector, u_g,

$$u = [u_f, u_g]^T \tag{6}$$

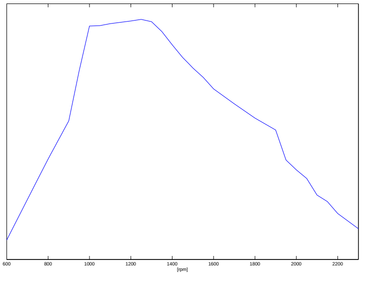

Figure 3: Realistic maximum fueling, $u_{f,max}$.

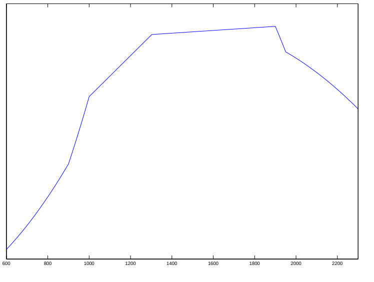

Figure 4: Realistic maximum engine power, $P_{e,max}$.

The constraint on maximum fueling, $u_{f,max}(\omega_e)$, shown in Figure 3, is based on realistic mapped data instead of a commonly used polynomial estimation, since it has been shown that the characteristics of $u_{f,max}$ is important for optimal gear-shifting control, (Fröberg and Nielsen, 2007; Fröberg, 2008). The maximum engine power, $P_{e,max} = T_e(\omega_e, u_{f,max})\omega_e$, is plotted in Figure 4. The important characteristics of these curves is that the highest fueling level, and consequently highest engine torque, is obtained between approximately 1000 and 1300 rpm and the highest engine power is obtained between approximately 1300 and 1900 rpm.

As fuel consumption is to be minimized, the mass fuel flow, $\dot{m}_f$ [mg/s], must be calculated. The mass fuel flow is determined, as in (Hellström et al., 2009), by multiplying fueling, u_f, and the number of cylinders, n_{cyl}, with engine speed, $\omega_e = \frac{i(g)}{r_w} v$ and by adjusting for number of revolutions per stroke n_r according to

$$\dot{m}_f(x,u) = \frac{n_{cyl}}{2\pi n_r} \frac{i(g)}{r_w} v u_f \tag{7}$$

2.3 Objective function

To minimize fuel consumption while trying to keep to highway speeds, the following cost function is used

$$I = \int_0^{s_f} \left(\frac{1}{v} \dot{m}_f + \beta_c \frac{1}{v} \right) ds \tag{8}$$

where $\frac{1}{v}\dot{m}_f$ [mg/m] is weighted with $\frac{1}{v}$ [s/m] reflecting consumed fuel and time when integrated over the total distance s_f. The weighting factor, β_c, is set so as to fulfill a request on final time. Minimization of (8) leads to optimal fueling and gear-shifting and consequently an optimal vehicle and engine speed profile.

3 Gear-shifting

In a standard automated manual transmission (AMT), a gear shift is performed differently depending on driving scenario. This is due to that the requirements of performance are differently balanced with fuel consumption and comfort depending on driving scenario. However, for an engine as the one modeled (4), fuel optimal uphill driving (which also respects trip time) is restricted to no more than two different driving scenarios, see (Fröberg et al., 2006), according to

$$u_f = \begin{cases} u_f(v_{const}) & \text{low load} \quad - \quad \text{constant speed in small road gradients} \\ u_{f,max} & \text{max load} \quad - \quad \text{prior to or in a large road gradient} \end{cases}$$

The two driving scenarios differ a lot and consequently, the gear-shifting process in a standard AMT differ between the two driving scenarios. Anyhow, by verifying the model in different road gradients it will be shown (Section 4.2) that the proposed gear-shifting model holds for both driving scenarios in order to minimize fuel consumption.

3.1 Realistic gear shift model

The gear-shifting model to be presented is as close as possible to how a standard AMT gear shift (Figure 1) is performed. The ramps and times of this model, describe the gear shift with a sufficent accuracy for the objective of minimizing fuel and time. For comfort reasons the gear shift is sometimes longer lasting. However, a gear shift is always possible to be performed as quickly as modeled and since only fuel consumption and time are part of the objective function, the comfort aspect is not taken into account. The gear-shifting process is completed in about two seconds. The different parts of the gear-shifting process will be modeled in the following subsections with time as independent variable.

Ramping engine fueling

The slopes of the fuel ramps may be adjusted due to comfort but since this aspect is not taken into account, the slopes are set to be the steepest choice of a standard AMT. A quick ramping is beneficial in order not to have long interruption of engine torque that would give a lower mean speed over the traveled road stretch. The fueling is ramped according to

$$u_f = C_{ramp}(t - t_0) + u_{f,0} \tag{9}$$

where C_{ramp} is negative when ramping down and positive when ramping up and $u_{f,0}$ is the start fueling level, i.e. in a large uphill road gradient $u_{f,0} = u_{f,max}$ when ramping down and $u_{f,0} = u_{f,zt}$ when ramping back up (see Figure 1).

The turbine of the turbocharger looses speed and thereby the charging pressure is decreased during a gear shift. The low charging pressure persists for some time even when the new gear is engaged, i.e. a turbo lag, and thereby results in a disability of acheiving maximum torque immediately post a gear shift. A high fueling with a low charging pressure generates black smoke and to avoid this, a smoke limiter mode constrains fueling while the turbocharger regains effect. The influence of turbo lag is shown in Figure 1, where $C_{ramp,sl} < C_{ramp}$.

To capture the characteristics of the turbo lag, the fueling ramp is restricted to the lower slope $C_{ramp,sl}$ if u_f comes close to $u_{f,max}$ when ramping up after a gear shift. Depending on the loss of air mass flow in different driving situations, the smoke limiter mode is activated differently. However, a reasonable simplification is to assume that this behaviour is independent of driving scenario and the smoke limiter mode is thus assumed to be active when $u_f > pu_{f,max}$ (p is a constant $0 < p < 1$).

Zero torque fueling level

The zero torque fueling level, $u_{f,zt}$, is set so that the gear can be easily engaged or disengaged, as there is no torque in the gear box. This is performed by matching the engine acceleration to the propeller shaft acceleration. The fueling level, when this is achieved, is a low fueling level, generating a low indicated engine torque, $T_{e,ind}$, that except for matching the acceleration of the propeller shaft also evens up the engine losses, $T_{e,loss}$. This is described by the dynamics of the disengaged engine, according to

$$J_e\dot{\omega}_e = T_e(\omega_e, u_{f,zt}) = T_{e,ind}(u_{f,zt}) - T_{e,loss}(\omega_e) \tag{10}$$

for a certain engine acceleration, $\dot{\omega}_e$, given by (11).

$$\dot{\omega}_e = \frac{\dot{v}}{r_w} i(g) \tag{11}$$

The truck acceleration, $\dot{v}$, is in the gear-shifting process a retardation caused by driving resistance in the driving mission considered. Recall from (4) that indicated torque, $T_{e,ind}$, is asumed to be a linear function of fueling, u_f, and engine losses, $T_{e,loss}$, are assumed to be affine in engine speed, i.e. $T_{e,loss}(\omega_e) = c_\omega \omega_e + c_c$.

This gives that Equations (10)-(11) describe the zero torque fueling level, $u_{f,zt}$, as a function dependent on engine losses and vehicle retardation, according to

$$\begin{aligned} u_{f,zt} &\propto J_e\dot{\omega}_e + T_{e,loss}(\omega_e) \Rightarrow && (12)\\ u_{f,zt} &\propto c_1 i(g)\dot{v}(\alpha, v, m, J_e, J_l, i(g)) + c_\omega \frac{vi(g)}{r_w} + c_c && (13) \end{aligned}$$

and the zero torque fueling level is thus a function of road gradient, speed, gear ratio, vehicle mass and drive line inertia. In order to find a simpler model of the zero torque fueling level it is assumed that $\dot{v} \propto c_a\alpha + c_b v^2$, which reflects the most important characteristics of the vehicle retardation during a gear shift, giving an increased retardation in large road gradients and for high air drag. This leads to the final model for the zero torque fueling level, as

$$u_{f,zt} = c_\alpha i(g)\alpha + c_{v2} i(g) v^2 + c_\omega \frac{vi(g)}{r_w} + c_c \tag{14}$$

In the zero torque fueling model (14), c_α and c_{v2} are negative, and c_ω and c_c are positive. There is of course a natural constraint as $u_f > 0$. The zero torque fueling level is assumed to be kept for a constant time prior/post engagement/disengagement.

Adjusting engine speed

To switch from one gear ratio to another, the engine must be accelerated when gearing down or decelerated when gearing up to reach the new engine speed. The time it takes to reach the new engine speed is denoted t_{adjust}. When gearing down the acceleration is performed by injecting fuel. When final time is critical the fueling level is set as high as possible, but since the charging pressure is low while shifting gears, this fueling level can not be near maximum fueling. The control signal of a standard AMT is often engine torque, instead of engine fueling, and to accelerate the engine, a constant torque is requested, $T_{e,shift}$. This gives an engine acceleration according to (10) implicating an increase in engine speed which is affine in time, since a constant $T_{e,shift}$ gives a constant acceleration, $\dot{\omega}_e$. In this model, having fueling as the control signal, the fueling level, $u_{f,shift}$, is approximated to be constant and is always set as high as possible since time is to be minimized.

When gearing up, the engine is decelerated with no injected fuel according to

$$J_e \dot{\omega}_e = -T_{e,loss}(\omega_e) - T_{e,gas}(u_{VGT/EB}) \tag{15}$$

where $u_{VGT/EB}$ is a braking control signal generated by closing the exhaust brake and/or VGT (variable geometry turbine) and consequently restricting the exhaust gas flow. The shift time is longer when gearing up compared to gearing down, since $u_{VGT/EB}$ is not as powerful in decelerating as diesel injection is to accelerate the engine. The engine friction $T_{e,loss}(\omega_e)$ is approximated by an affine function in speed, see (4).

The time constant, $\frac{c_\omega}{J_e}$, for the studied engine is approximately equal to t_{adjust} in the experimental test data when gearing up one step. When gearing up two or three steps, the exponential effects could influence the deceleration. However, in reality the restriction of exhaust gas flow is such that engine speed is close to an affine function of time. Thus, the model of engine deceleration is as shown in Figure 5.

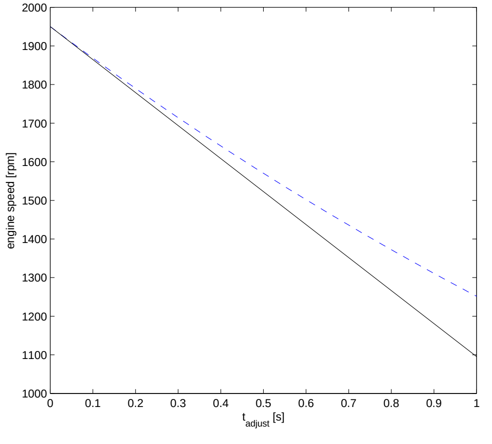

Figure 5: The model of a realistic engine deceleration as an affine function (solid) and engine speed as an exponential function solving (15) for $T_{e,gas} = 0$.

The difference in engine speed prior and post gear shift is proportional to the difference in gear ratios, $\Delta w_e \propto \Delta i(g) = i(g_{post}) - i(g_{pre})$, if the vehicle speed is constant, since $w = \frac{vi}{r}$. The difference in gear ratios, $\Delta i(g)$, is approximated to have a linear dependency to the number of steps between the gears. Figure 6 shows this dependency for the 6 highest gears in the studied gear box. The time for adjusting engine speed, t_{adjust}, is consequently approximated to be proportional to the difference in gear, $g \in [1 \quad 2 \quad \ldots \quad 12]$, both when gearing

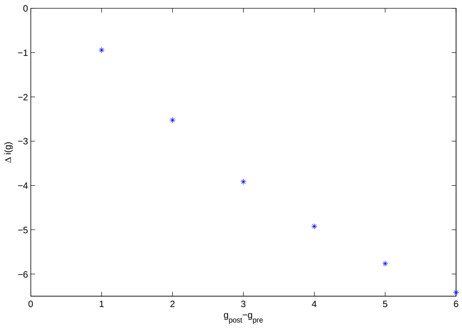

Figure 6: Dependence of difference in gear ratios, $i(g_{post}) - i(g_{pre})$, to number of steps between gears, $g_{post} - g_{pre}$, (where $g_{pre} = 6$).

up and down according to

$$t_{adjust} = \begin{cases} C_{down}(g_{post} - g_{pre}) & \text{when gearing down} \\ C_{up}(g_{pre} - g_{post}) & \text{when gearing up} \end{cases} \tag{16}$$

where C_{up} is larger compared to C_{down}.

Skipping some gear steps and shifting double or triple gear steps at once is common in heavy truck driving. The summed up t_{adjust} for all gear shifts over the total distance is, however, independent of whether the gear shifts have been performed as single or multiple steps. Anyhow, the summed up time for ramping over the total distance decreases by shifting multiple gear steps coincidentally. The shift time over the total distance is, consequently, dependent on what driving scenario the gear shifts are performed in and if multiple gear steps are shifted coincidentally, since this affects the total time for ramping.

4 Model adapted for optimization

In order to use the model in the numerical optimization, some adaptions of the gear shift model must be made. As stated in Section 3, to set up the fueling profile of a gear shift, the current engine and vehicle speed must be known. In Section 4.1 it will be shown that in the dynamic programming the speed profile during gear shifts is simulated with the fueling profile of that gear shift as an input. The consequence is that the fueling profile will be based on speeds from past time steps. In Section 4.2 the fueling profile of the gear shift model, with these adaptions, is verified by comparison to experimental test data.

4.1 Numerical optimization

To implement dynamic programming, the objective function (8) is discretized with the step length, Δs. The consumed fuel over a distance, $\frac{1}{v}\dot{m}_f$, is proportional to $u_f i(g)$, see (7), and accordingly the objective function becomes

$$J = \sum_{j=0}^{n-1} \left(u_{f,j+1} i(g_{j+1}) + \beta \frac{1}{v_{j+1}} \right) + \zeta_n \tag{17}$$

where $n = \frac{s_f}{\Delta s}$ and ζ_n is the cost of the last step set as

$$\zeta_n = \begin{cases} \infty & \text{if} \quad v_n = v_0 \\ 0 & \text{if} \quad v_n \neq v_0 \end{cases}$$

Calculating cost when no gear shift is performed

The slow dynamics of a heavy truck allows a step length, Δs, as long as 50 meter. Fueling and road gradient are assumed constant over the step length. To calculate the cost to go from step j to $j+1$, fueling must be computed. When no gear shift is carried out, the dynamics of the system is approximated with the trapezoidal method according to

$$\frac{v_{j+1}^2 - v_j^2}{\Delta s} = \frac{1}{2} \left(f_j + f_{j+1} \right) \tag{18}$$

where $f = \frac{2}{m_{tot}} F_{tot}$ is according to (3) from which fueling, $u_{f,j+1}$, easily can be extracted, being an argument to both f_j and f_{j+1}.

Calculating cost when gear shift is performed

To evaluate the cost for a step that includes a gear shift, the specific fueling profile of that gear shift is set up according to the model, (9)-(16). As the gear-shifting model is described in time rather than distance, the following dynamics is used to describe the system during the gear shift

$$m_{tot} \frac{dv}{dt} = F_{tot} \tag{19}$$

With the engine fueling control of the gear-shifting process as an input, the gear-shifting velocity profile is simulated in the time domain by the use of the Runge Kutta method of the second order, as it is better suited compared to (18) when the proceeding state is not known. This method is according to

$$k_1 = \Delta t f(x_k, v_k) \tag{20}$$
$$k_2 = \Delta t f(x_k + \Delta t, v_k + k_1) \tag{21}$$
$$v_{k+1} = v_k + \frac{1}{2}(k_1 + k_2) \tag{22}$$

where $f = \frac{1}{m_{tot}} F_{tot}$. The time step, Δt, is not constant but is set to reflect the different phases of the gear-shifting process, e.g. ramping, zero torque, speed adjusting.

Some fueling levels (e.g. $u_{f,zt}$, $u_{f,max}$) in the gear shift fueling profile are dependent on speed. Since the speed profile is not known prior to the set up of fueling profile, speed is approximated to a speed previously evalutated, i.e. $u_{f,zt}(v) = u_{f,zt}(v_k)$, $u_{f,max}(\omega_{e,end}) = u_{f,max}(\omega_{e,post})$. Maximum fueling, $u_{f,max}$, is linearly interpolated from mapped data (see Figure 3) for a certain speed. The initial and final fueling levels depend on driving scenario. Whether maximum fueling is used or not is estimated prior to the dynamic programming, and determined iteratively.

After simulating the gear shift, the fueling of the remaining distance of the step length is calculated in the same way as for a step without a gear shift (18), using the resulting speed post the gear shift as initial velocity ($v_j = v_{k+1}$). During, and for a time after, a gear shift a standard engine is controlled differently, resulting in a little higher fueling to acheive a given engine torque. This is reflected by using another set of coefficients of (4) to describe engine effieciency.

4.2 Verification of gear shift model

Verification of the gear shift model is performed for gear shifts in different road gradients involving different gear ratios by comparing the gear shift model (9)-(16) to experimental test data of a standard AMT in a 37 tonnes truck. In Figure 7 a two step gear shift (from 10th to 8th gear) in a steep, 7%, uphill road gradient is shown. Since this is an example of gearing down, extra fueling

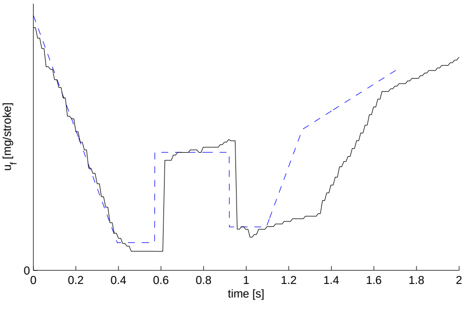

Figure 7: A gear shift from 10th to 8th gear in a 7% uphill road gradient, model (dashed) and experimental test data (solid).

is required during t_{adjust} to accelerate the engine to its new higher speed. The slopes and times of the fueling profile coincide for the model and the experimental data with only some exceptions, and these are due to comfort features in the AMT test data. For example, to avoid oscillations in the powertrain, the ramping up from the zero torque fueling level is not as immediate as the model predicts. This leads to another mismatch, a low predicted fueling at the end of the gear-shifting process, i.e. $u_{f,max}(\omega_{e,end})$. The maximum fueling level is underestimated since the modeled engine speed is overestimated. The higher engine speed, compared to the test data, is both due to the shorter total shift time of the model, but also due to numerical issues since $\omega_{e,end}$ is approximated to $\omega_{e,post}$ for the model.

Figure 8 displays a one step gear shift (from 10th to 11th gear) in a 2% road gradient. Both Figure 8 and Figure 7 illustrate a gear shift in an uphill road

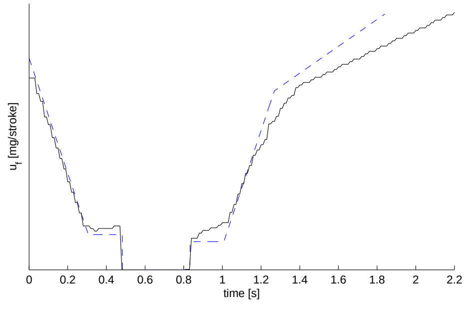

Figure 8: A gear shift from 10th to 11th gear in a 2% uphill road gradient, model (dashed) and experimental test data (solid).

gradient, and thereby the fueling ramps are similar. However, in Figure 8 the road gradient is on the verge of being small enough for the vehicle to keep the set speed, and thus maximum torque is not demanded neither before nor after the gear shift. Due to the AMT assumption of low load, the smoke limiter ramp has a lower slope rate in the experimental test data compared to the model. This is due to a choice of the AMT giving an alternative gear-shifting process where a quick ramping is not endorsed in low load. As the modeled gear-shifting process always gives priority to quick ramping, the mismatch between test data and the model is expected.

Shifting while keeping constant speed is not a common scenario in a standard AMT. However, Figure 9 shows a gear shift in a downhill road gradient where no fueling is needed to keep constant speed. The gear shift shown is a consequence of the driver's demand of accelerating at this point. In Figure 9, the increased fueling after the gear shift does not generate a fuel optimal velocity profile, and is not modeled. The ramp prior to the gear shift is similarly not considered. However, it can be seen that the modeled zero torque fueling levels are well fitted to the test data. It can be concluded that the model is well fitted regardless if the switching of gears is performed in low load (as in Figure 9), max load (as in Figure 7) or on the verge between low load and max load (as in Figure 8). The model (9)-(16) is accordingly well suited for use in the numerical optimization. As described in Section 4.1, a predetermined speed is used when setting the fueling profile. This is found to be an acceptable simplification since the model still captures the characteristics of the gear shift. It can also be stated that the gear-shifting process is marginally dependent to road gradient, only through the

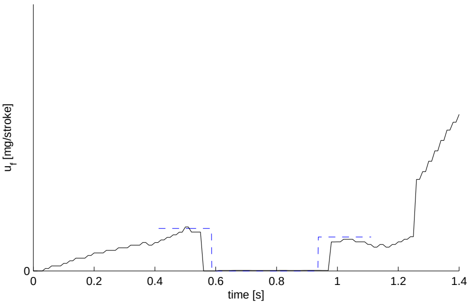

Figure 9: A gear shift from 10th to 11th gear in a −2% downhill road gradient, model (dashed) and experimental test data (solid).

zero torque levels (14). Greater impact is made by shifting gears in high or low load, and if the gears are shifted by single steps or multiple steps coincidentally.

5 Climbing a hill

In the subsections that follow it is discussed in what way engine and vehicle speed influence fuel consumption and final time when climbing a hill. The reasoning will give a foundation for interpreting the results of Section 6.

Fuel consumption

To find the lowest fuel consumption, engine efficency must be considered. A low gear ratio, and thus a low engine speed, is always beneficial in small road gradients as it leads to a low engine loss, $T_{e,loss} = c_\omega \omega_e + c_c$. Low engine losses give a higher output engine torque, T_e, for a certain fueling level, u_f, and accordingly a high engine efficiency, η_e, see (23).

$$\eta_e = \frac{T_e}{T_{e,ind}} = \frac{T_{e,ind} - T_{e,loss}}{T_{e,ind}} = \frac{c_f u_f - (c_\omega \omega_e + c_c)}{c_f u_f} \tag{23}$$

However, in large road gradients a high wheel torque is required, to keep the truck velocity within an acceptable speed range, which leads to a need of shifting to a higher gear ratio. Consequently, a higher engine speed is inevitable even though it results in a worse engine efficiency. The gear that gives a sufficient wheel torque, is denoted lowest necessary gear and for a given uphill, in terms of road gradient and length of slope, the lowest necessary gear for a certain truck can always be calculated given look ahead information. Thereby, a high engine efficiency is ensured by not gearing down unnecessarily.

Energy consumption

Not only engine efficiency determines the total fuel consumption, since also the total energy required to complete the road stretch can be decreased by optimal control. The required energy needed to complete the driving mission varies only with air drag, $F_a(v^2)$, over the distance travelled. The other energy consumers, i.e. gravitational force, rolling resistance and energy loss due to shifting gears are constant and independent of engine fueling or gear-shifting strategy (i.e. when the gear shift is performed or if the gear-shifting is performed by single or multiple steps). Gear shifts are often, in simpler models, associated with energy losses. However, for an AMT the gear shift only implies an energy loss when gearing down, as in Figure 1, where extra fueling is required during t_{adjust} when the driveline is disengaged. This energy does not go to propeling the vehicle but to accelerating the engine, and is consequently considered a loss. If the lowest necessary gear is predicted correctly, then the speed difference for which the engine must be accelerated is determined, and thus the energy loss due to shifting gears is constant over the driving mission.

In level road, constant speed is optimal as it minimizes air drag and hence driving resistance. In a driving mission with significant uphills, it is not possible to keep constant speed, but in order to minimize $\int_{s_0}^{s_f} F_a ds$, the truck speed over the total driving mission should vary as little as possible. However, there are restrictions in how much the velocity profile can be altered, and still finish with the requested final time. In large road gradients it is necessary to gear down to get the potential of adjusting to a higher truck speed, since this gives a higher engine power (see Figure 4). The point of gearing down in the slope affects the vehicle speed profile, and consequently the energy consumption.

Final time

A high engine power and a higher mean speed, acheived by a gear-shifting strategy that gears down early, of course also affects the final time. Another impact on final time is how total shift time depends on at what point the gear shift is performed. By shifting gears in low load a shorter total shift time is obtained and the propulsion is accordingly constrained during a shorter time.

The optimal results to be presented will show both vehicle velocity profiles that generate low air drag, and also engine speed profiles that vary over the road stretch, in order to be fuel efficient at some points and still to achieve a high power when so is needed.

6 Results

The numerical optimization uses the complete road stretch of the studied driving scenario (see Figure 2) as look ahead horizon and results in an optimal engine fueling and gear-shifting control. As described in Section 4.1, the final velocity, v_n, is forced to be equal to the initial velocity, v_0. The optimal vehicle and engine speed profiles as well as the resulting fuel consumption and final time are compared to a standard AMT. The standard AMT switches gears dependent on engine speed thresholds, and the thresholds are set differently depending on current driving scenario. In this analysis the truck with a standard AMT will be controled by an optimal engine fueling given the predetermined gear-shifting strategy, in order to distinguish the potential fuel consumption reductions due to gear-shifting strategies only. To find an optimal control that generates a final time which is comparable to the one obtained by the AMT-control, the weighting factor β of the objective function (17) is adjusted.

6.1 Not to gear down unnecessarily

In the 6% uphill considered, most heavy trucks must shift gears in order to climb the hill. However, in Figure 10, a driving scenario with a 17 tonnes truck is shown where the optimal driving is to stay with the highest gear whilst a standard AMT shifts gears. The optimal way of driving is set with the advantage

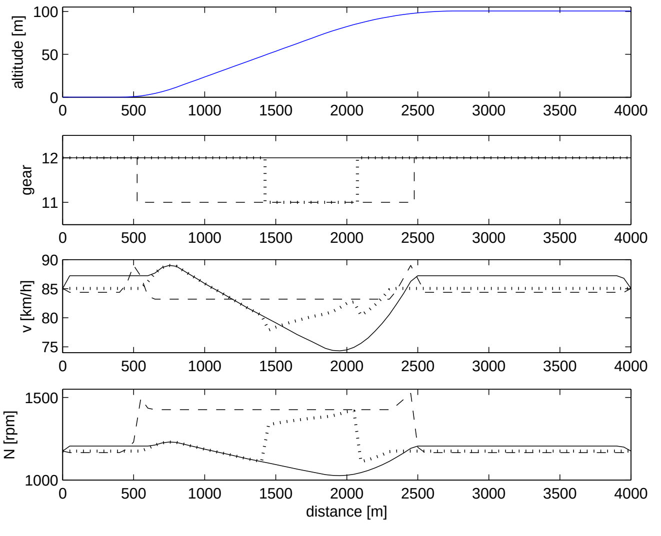

Figure 10: Truck of 17 tonnes: Optimal gear-shifting strategy (solid) reduces fuel consumption by 0.7% compared to a standard AMT (dotted) and by 2.2% compared to a manual gear-shifting strategy (dashed) while keeping the same mean speed.

of knowing the topography ahead and it can accordingly be predicted that gearing down is not necessary. The optimal control of Figure 10, i.e. not to gear down, results in a reduction of fuel consumption of 0.7% compared to a standard AMT, still finishing with the same final time. As gear-shifting is performed at specified engine speed thresholds in an AMT without look ahead information, and since these thresholds are set with a safety margin, the mistake of gearing down right before the top of the hill is not an uncommon scenario. The lowest engine speed for the optimal solution is about 1000 rpm, which is the lowest speed to secure a high fueling level, see Figure 3, and thereby a high engine torque. When the optimal solution reaches 1000 rpm, the road slope is leveled off and the truck starts accelerating, without having to shift gears.

Additionally in Figure 10, a usual way of shifting gears manually is shown, by shifting gears prior to the uphill. The fuel consumption of the optimal control is 2.2% less than what is acheived with this manual gear-shifting strategy, still finishing with the same final time. By gearing down, the AMT and the manual control reaches engine speeds that generate higher engine power, see Figure 4. This gives that the vehicle is, for these control strategies, able to keep a higher mean speed compared to the optimal control in the uphill slope. However, the optimal control has a higher speed on level road prior to and post the uphill, which gives that the control strategies acheive the same final time. To conclude, gearing down is always unbeneficial if a higher engine power is not needed in order to complete the road stretch in time. This is due to that the engine losses are low for lower engine speeds and consequently, the engine efficiency, η_e, is high, see (23). The transmission is modeled with realistic efficiencies, $\eta(g)$ (see Section 2.2). The fact that lower gears implicate lower transmission efficiency does not change optimal gear control, it only enforces the results in terms of higher fuel consumption reductions when keeping to higher gears. The same reasoning, not to gear down unnecessarily, is valid also for a heavier truck. However, in that case there is a need of gearing down a certain number of steps. It is thus beneficial to have the knowledge of the lowest necessary gear needed to climb a hill.

6.2 When to shift gears if necessary

For a truck of 30 tonnes, the lowest necessary gear is the 10th gear in the considered uphill, which is correctly chosen by a standard AMT, see Figure 11. In a driving scenario as this, where the AMT system chooses lowest necessary gear correctly, then the open question is when to shift to what gear. The optimal answer is based on the fact that the road gradient and the state, when the gear shift is performed, affect the gear-shifting process and the retardation of the truck during a gear shift. If several gear steps are passed over, i.e multiple gear steps are shifted at once, the total distance travelled while shifting gears is lower compared to the alternative of shifting gears by several single steps. Travelling less percentage of the complete road stretch with a constrained propulsion is beneficial, however this must be balanced with using an unbeneficial gear ratio for a longer time. In this section three different gear-shifting strategies, and their respective optimal velocity profiles, will be analyzed. The gear controls are different in terms of when gear shifts are performed and if multiple gear steps are shifted coincidentally or not.

Selecting cases

Two of the studied gear-shifting strategies are optimal since they minimize J (17). The first optimal gear-shifting strategy is denoted the β_{time}-case, as this gear-shifting strategy is the result when the weighting factor of (17), $\beta = \beta_{time}$, is such that final time is weigthed more heavily than fuel consumption. The second one is denoted the β_{fuel}-case, as this gear-shifting strategy is optimal when the weighting factor, $\beta = \beta_{fuel}$, is set to reflect that fuel consumption is more important than final time. Generally, changing β does not necessarily affect the gear-shifting strategy, but in this case the two gear-shifting strategies are quite diverse even though β_{fuel} is only thousands of a percent less than β_{time}. The β_{time}-case gears down by single steps and the first gear shift is performed very early, whereas the β_{fuel}-case gears down as late as when in the largest road gradient and then by two steps at the same time. When β is decreased, from β_{time} to β_{fuel}, a certain β is found for which there is a jump in final time, meaning that some final times are unobtainable when driving in an optimal manner according to (17), and one example is the final time acheived by the standard AMT. The jump in final time is in a way similar to the jump described in (Ivarsson et al., 2008) where non-linearities in fuel maps were analyzed. However this is not the reason to the jump observed here, since engine torque is affine in fueling. The jump in final time will instead be shown to be due to how vehicle retardation during gear shifts varies with the point of shifting gears. Both the β_{time}-case and the β_{fuel}-case of course generate low, and in fact almost identical, total costs, J. The difference is that the first-named acheives a low final time and the other a low fuel consumption.

A third gear-shifting strategy is presented that generates a high total cost, even though gears are shifted at points in between the optimal gear-shifting strategies. This gear-shifting strategy is the one that would have been optimal if gear-shifting had been instantaneous, i.e. if the truck would not have been decelerated while shifting gears. The gear-shifting strategy is denoted the *igs*-case, which is short for *instant gear shift.* The gear-shifting strategy of the *igs*-case is evaluated with the realistic gear shift model and for this model the *igs*-case is not optimal. It will be shown that, in order to find the optimal control for a real truck, it is important to use a gear shift model that includes time used to shift, to predict the vehicle retardation during gear shift. The following subsections will describe why the three different control strategies are beneficial (regarding the optimal controls of the β_{time}-case and the β_{fuel}-case) or not (regarding the *igs*-case) when using the realistic gear shift model.

Truck retardation while shifting gears

In Figure 11 the β_{time}-case is compared to a standard AMT. The β_{time}-case is 0.5% faster, and the fuel consumption is 0.1% less showing that it is possible both to reduce fuel and final time, even though the profit is small. It can be seen that the β_{time}-case gears down by one step just before the truck reaches

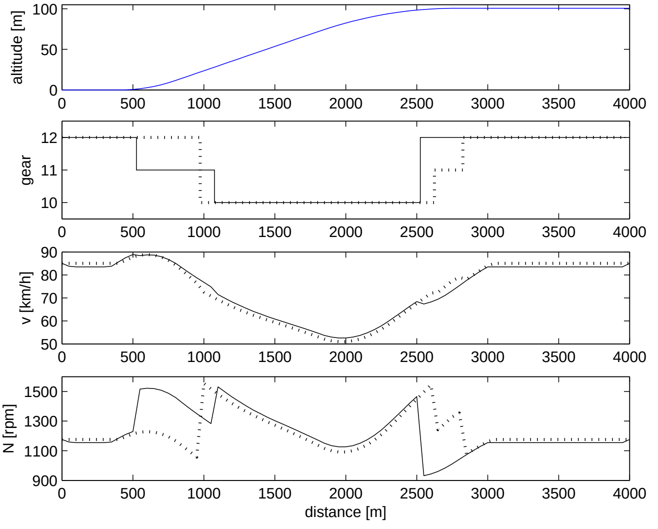

Figure 11: Truck of 30 tonnes: The β_{time}-case (solid) reduces fuel consumption by 0.1% compared to a standard AMT (dotted) while going 0.5% faster.

the steep uphill slope. This renders a slightly shorter total shift time compared to the AMT, due to the influence of road gradient on the gear-shifting process, see (14). The drop in vehicle velocity during shifting is also less by shifting gears in a smaller road gradient, both due to the slightly shorter total shift time but mostly due to a lower driving resistance. The drop in speed can be seen in Figure 11, both when gearing down just before reaching the large road gradients (at about 500 m), but the truck retardation is even more important in the 6% road gradient (at about 1000 m). By gearing down as early as is done by the β_{time}-case, i.e. before the road gradient becomes large, the truck can regain speed after the gear shift in order to start climbing the large road gradient with maximum speed.

Beneficial vehicle and engine speed profile

By the reasoning above it is understood that the gear-shifting process does affect vehicle speed directly due to a low propulsion during gear shift. But more importantly, by gearing down early a high engine speed and consequently a high engine power are obtained throughout the slope, and the β_{time}-case can accordingly keep a higher vehicle speed throughout the uphill compared to the AMT. The final time of the β_{time}-case is still quite close to the final time of the AMT and this is due to the lower speed on level road. The resulting vehicle speed profile is beneficial since the speed variation is less compared to the AMT.

This leads to a 0.4% reduction of total air drag, $\int_{s_0}^{s_f} F_a(v^2)ds$, compared to the AMT control. In parts of the driving mission with small gradients the optimal engine speed is also lower compared to the AMT, both because the β_{time}-case gears up early but also as a direct consequence of the low vehicle velocity. This leads to that the mean engine speed of the β_{time}-case over the total distance is about equal to the mean engine speed of the AMT and thus the total engine losses over the distance are about equal as well.

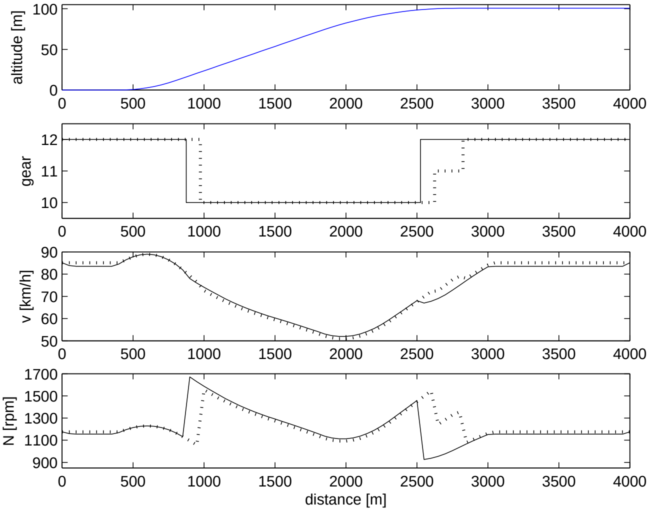

Figure 12: Truck of 30 tonnes: The β_{fuel}-case (solid) reduces fuel consumption by 0.4% compared to a standard AMT (dotted) while going 0.2% slower.

The final time of the β_{time}-case is a little lower compared to the AMT. To find a final time closer to that of the AMT, the weighting factor, β, is decreased from β_{time} to β_{fuel}. Even though β is decreased only by thousands of a percent, the resulting β_{fuel}-case (see Figure 12) seems far from the β_{time}-case. The β_{fuel}-case gears down by two steps as late as about 1000 m, when the road gradient is at its maximum, 6%. A standard AMT, without look ahead information, is more restrictive in shifting gears, and it gears down even later. The β_{fuel}-case also gears up earlier than a standard AMT and this results in a 0.2% higher final time compared to the AMT, with a fuel consumption reduction of 0.4%. A higher final time, and accordingly a lower mean speed, naturally gives a lower fuel consumption due to a lower air drag, $F_a(v^2)$, which means that it is difficult to distinguish how much of the fuel reduction that is an effect of optimal gear-shifting. The engine speed of the β_{fuel}-case is of course also lower than the engine speed of the AMT in general, leading to a good engine efficency. But, in the slope the engine speed is higher, giving a higher engine power when the road gradient is large. However, the higher power in the slope is not enough to get a final time equal to the one of the AMT. To reach the final time of the AMT, it would be intuitive that the optimal gear control would be to gear down somewhere in between what is done by the gear-shifting strategies of the β_{time}-case and the β_{fuel}-case, at least if there were no decelerations while shifting gears. Therefore, in the following subsection, for comparison the optimal gear-shifting strategy is found when the gear shifts are assumed to be instantaneous.

Instantaneous gear shifts

As stated earlier, for a real truck there is a deceleration during gear shifts which is different depending on at what point the gear shift is performed. To find what effect this has on optimal gear-shifting strategy, an optimization is performed for a gear shift model where the total shift time is reduced to $\frac{1}{20}$ of the realistic gear shift model, implicating very small divergence from maximum fueling when shifting gears. Since final time in that case only is dependent on available engine power in the slope, besides the speed on level road, the final time is easily adjusted by setting β. The resulting gear-shifting strategy, i.e. the *igs*-case which is shown in Figure 13, gears down by single steps in the steepest road gradient and gears up early, since this would give a high engine power in the uphill slope and a high engine efficiency on level road, if the gear shift would have been instantaneous.

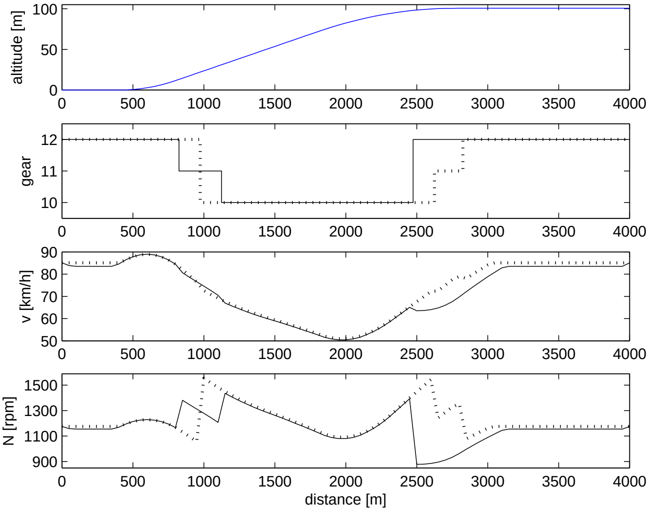

Figure 13: Truck of 30 tonnes: the *igs*-case (solid) reduces fuel consumption by 1.2% compared to a standard AMT (dotted) while going 2.2% slower.

However, the speed profiles shown in Figure 13 are the result of an optimization of engine fueling, by using the instantaneous gear-shifting strategy, but now returning to the realistic gear shift model, and by using $\beta = \beta_{fuel}$. This results in a final time that is 2.2% higher than a standard AMT. The high final time gives a high total cost since the fuel consumption is only 1.2% lower than the standard AMT. Thus, the gear-shifting strategy is not beneficial when a realistic gear shift model is assumed, and it can be concluded that in order to find the optimal gear-shifting strategy of a real truck, the vehicle retardation during gear shifts must be considered. Retardations during gear shifts and the resulting engine speed profiles will be studied for the different gear-shifting strategies in the following subsection.

Comparison of gear-shifting strategies

From just looking at the three gear-shifting strategies, it is difficult to tell which one is better than the other. But yet, it has been shown that they generate quite diverse results. The reason to the resulting fuel consumptions and final times is found by studying the figures in Table 1 and by comparing the engine speeds in Figure 14. In Table 1 it can be seen that the total cost, J, of the β_{fuel}-case

Table 1: Sensitivity analysis (in relation to the β_{time}-case).

Variable	β_{fuel}-case	*igs*-case
β	-0.00181%	-0.00181%
J	-0.000537%	0.129%
t_f	0.774%	2.72%
m_f	-0.293%	-1.01%
$\sum \Delta v$	-2.01%	58.6%

is close to the total cost of the β_{time}-case, even though the fuel consumption, m_f, and final time, t_f, differ to some extent. Figure 14 shows that the engine speed profiles of the β_{time}-case and the β_{fuel}-case are similar, only differing significantly while gearing down, i.e. between the distance of 500 and 1100 m. The small differences in engine speed in the rest of the road stretch are due to the difference in how much the truck speed drops during gear shifts, $\sum \Delta v = -2.01\%$, and the fact that the β_{time}-case regains some of the lost speed before starting to climb the hill. The similar engine speed profiles give that the engine power (affecting final time) and the engine losses (affecting fuel consumption) are similar for these gear-shifting strategies. They both have a high engine speed in the uphill slope and a low engine speed on level road, which was already found to be beneficial in comparison to the standard AMT. The optimality of high engine speeds, giving a high engine power, in long uphill slopes with large road gradients is also shown in earlier works, (Fröberg and Nielsen, 2007). The differences in final time and fuel consumption between the two gear-shifting strategies are mainly due to the different engine speed profiles between 500 and 1100 m.

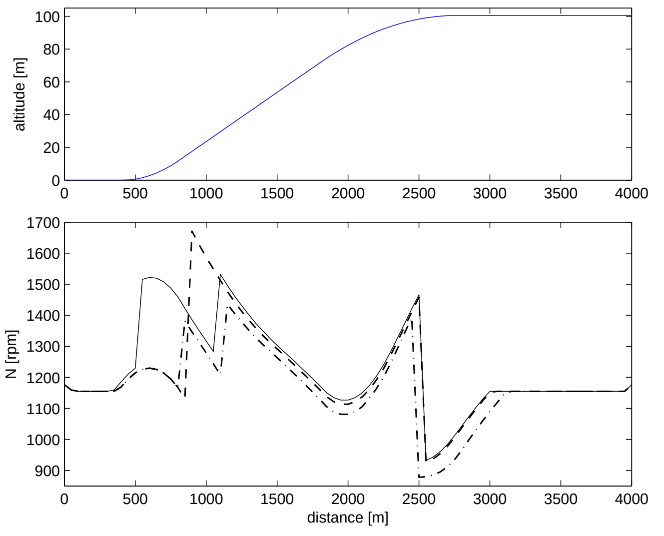

Figure 14: Engine speeds for β_{time}-case (solid), β_{fuel}-case (dashed) and *igs*-case (dotted-dashed).

The reasons to the high cost of the *igs*-case is understood by studying Figure 14 as well. For a long part of the travelled distance, the engine speed of the *igs*-case is lower compared to the other gear-shifting strategies. This gives a low engine power and accordingly a high final time. The fuel consumption is not low enough to even up the high final time. The reason to the low engine speed, and consequently the low engine power, is the high drop in truck speed during gear shifts, $\sum \Delta v = 58.6\%$ for the *igs*-case in relation to the β_{time}-case, see Table 1. The large truck retardation during gear shifts in the *igs*-case is due to that the gear-shifting points were chosen without considering the time used to shift, but when applied on the realistic gear shift model, the points of shifting gears turn out unfavorable and the truck looses speed. For the β_{time}-case and the β_{fuel}-case large retardations are avoided as gear shifts are performed in either small road gradients or by gearing down two steps at the same time.

There is another major reason to the high final time of the *igs*-case, as it gears up prior to the other gear-shifting strategies. But this is different since gearing up early gives a corresponding reduction of fuel consumption. Gearing up to the highest gear early leads to a reduced potential of acceleration, since the engine speed gets as low as 878 rpm, admitting only a low engine fueling, see Figure 3. If gearing up would have been performed at the same point as the other gear-shifting strategies, fuel consumption and time would differ less compared to the β_{time}-case. However the total cost would still be more than 0.1% higher compared to the β_{time}-case. Hence, gearing up early is not a reason to why the *igs*-case gear-shifting strategy is not optimal. The reason is instead the high vehicle retardation during gear shifts and the consequence of a low engine power in the large road gradient.

Characteristics of optimal gear control

The reasoning of Section 5 can be recognized in the optimal solutions:

- To gear down early to acheive a high engine power in large road gradients
- To gear up early to acheive a high engine efficiency in small road gradients
- To follow a velocity profile that generates a low driving resistance over the considered road stretch

After studying the characteristics of the optimal results, it can be added that it is important to choose the point of gear-shifting carefully, not to have large velocity drops during gear shifts but to ensure a high engine power after a gear shift in a large road gradient.

Further studies have shown that, even if β is increased by a factor 10, the earliest point of gearing down is just before the road gradient becomes large, in order to start climbing the hill with maximum speed and a lower gear. For any other cause, it is not optimal to gear down in small gradients, not even to enhance accelerations. Accordingly, the short shift time in small gradients and especially in low load does not even up the worse efficiency of keeping to a low gear a longer time.

6.3 Implications of a very heavy truck

For a truck of 60 tonnes, the lowest necessary gear is the 6th gear in the considered uphill, see Figure 15. The optimal control is in a way similar to the optimal controls of Section 6.2, as it gears down and back up earlier than the standard AMT. Similarities are also found in having a lower speed on level road and keeping a higher speed throughout the uphill. However, to reach the final velocity, maximum fueling must be kept from the distance of 100 m until the end of the driving mission. Accordingly, there are few openings in generating alternative vehicle speed profiles. A very heavy truck, which has slow longitudinal dynamics, would benefit from having a longer look ahead horizon, compared to a less heavy vehicle, to get the potential of varying the velocity in an optimal manner. The AMT and the optimal gear-shifting strategy shown in Figure 15, look very much alike if studied briefly, and the optimal reduction of fuel consumption is as small as 0.1% when the same final time is acheived. This means that, in the driving mission considered, the AMT is not far from the optimal control. The actual differences will be discussed in the following subsection.

The standard AMT in relation to optimal control

When studying the details, it can be seen that by gearing down early, in the 6% road gradient, the optimal engine speed is higher compared to the AMT

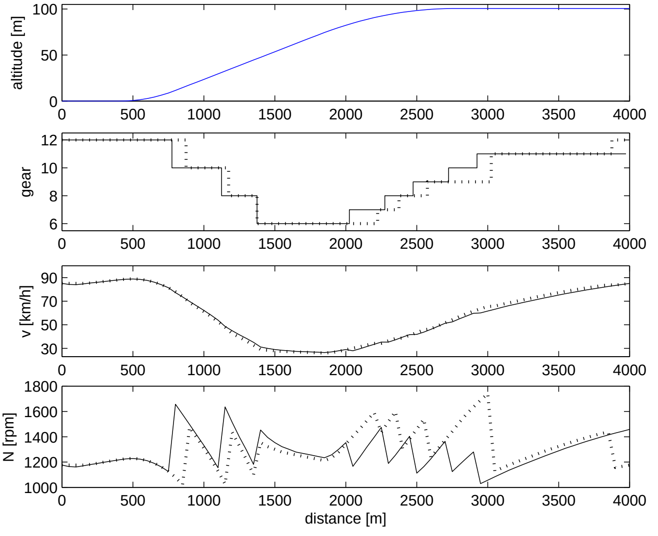

Figure 15: Truck of 60 tonnes: Optimal gear-shifting strategy (solid) reduces fuel consumption by 0.1% compared to a standard AMT (dotted) while keeping the same mean speed.

engine speed, giving a high engine power in large road gradients. The AMT gear-shifting is performed later as a precautionary measure as it does not know whether the slope will go on or level off. If the slope would level off the gear-shifting had, possibly, been performed unnecessarily. Moreover, when gearing up, at the end of the uphill slope, the scenario is just the opposite, the optimal engine speeds are lower compared to the engine speeds of the AMT. Low engine speeds are fuel efficient, and since the lowest engine speed is just above 1000 rpm a high torque is still ensured, see Figure 3. However, when a heavy vehicle climbs a hill with large road gradients for a long distance, there are very few possible gear-shifting strategies, i.e. there are few engine speed profiles that give the requested engine power to complete the road stretch within the requested final time. If a shorter trip time, compared to the AMT, is requested, then it is optimal to gear down just before the large road gradient in order to start climbing the hill with maximum speed and a lower gear, a gear-shifting strategy similar to the β_{time}-case of Section 6.2.

7 Discussion

7.1 Gear shift model

The gear shift model (9)-(16) is designed to describe the fueling profile of the AMT gear-shifting process when comfort is not considered. A conclusion from the verification of the model is that the gear-shifting process is not strongly dependent road gradient, but it varies to a larger extent with the initial and final fueling levels, aswell as with the need of acceleration or deceleration of the engine. This means that a simpler gear shift model could be sufficient to find optimal gear-shifting, as long as it reflects the mentioned characteristics such that the retardation of the truck while shifting gears can be calculated accurately, which has been shown to be essential in order to find the optimal gear control. Consequently, an instantaneous gear shift model is too simple and the time when engine speed is adjusted, t_{adjust}, must be set to reflect differences between gearing up and down and the number of steps that are shifted.

7.2 Gear shift comfort

The optimal control presented is optimal in the sense of minimizing fuel consumption and final time and the aspect of gear shift comfort is not considered. In a typical gear control system on the market today, ensuring a comfortable driving experience is also an important factor to satisfy the customer. According to e.g. (Huang and Wang, 2004) a discomfortable gear shift control can be quantified in terms of jerk, i.e. the derivative of acceleration, and a large jerk accompanied with a large acceleration is even more discomfortable. Gear shifting with an AMT can imply discontinuities in jerk, since the engine torque and thereby the vehicle acceleration is abruptly changed during the gear-shifting process. If gear-shifting would be performed in a more comfortable manner, by letting the gear shift take a little longer time compared to what is assumed in the gear-shifting model (9)-(16), then a longer distance travelled with a low propulsion implies that it is even more important to choose the right occasion to shift gears, to ensure a high engine power in the large road gradient. On the other hand, comfort can be kept even with short shift times if active damping is applied (Pettersson and Nielsen, 2000)

8 Conclusions

A fuel optimal gear shift control has been studied, when look ahead information is available, and the impact of the AMT gear-shifting process is analayzed. For a standard discrete heavy truck transmission, answers have been found to when to shift gears in an optimal manner, prior to or when in an uphill slope with large road gradients. Firstly, the gear-shifting process of a standard AMT is modeled, not considering the comfort details, in order to capture the fuel and time aspects of the gear shift. Thereafter, numerical optimization is performed by dynamic programming, minimizing fuel consumption and time by controlling fuel injection and gear. The results show, confirming earlier results with simpler gear shifting models, that it is optimal to keep maximum fueling prior to and throughout the slope, in order to have a high speed when in the slope. More importantly, the optimal solutions also give insight in optimal gear ratios and, if necessary, when to shift gears for a heavy truck with a standard AMT transmission.

The baseline of fuel optimal gear control is not to gear down, if the driving mission can be completed with an acceptable mean speed, without doing so. If a low final time is requested, then it is more beneficial to keep a higher speed on level road rather than gearing down to acheive a higher engine power. Since a standard AMT does not have look ahead information, it sometimes gears down unnecessarily and thus gives a significantly higher fuel consumption compared to the optimal control. If the uphill slope is such that gearing down is ineviteable, then there are three important factors that influence optimal gear-shifting strategy. Firstly, by gearing down early a high engine speed and hence a high engine power is ensured in large road gradients. However, the earliest point of gearing down is just before the uphill slope starts, in order to start climbing the hill with a lower gear and maximum speed. Even if time is weighted heavily, there is no other cause that justifies to gear down in small gradients, not even to enhance accelerations. Secondly, by gearing up early a low engine speed is obtained in small road gradients which is beneficial since the engine efficiency is higher for lower engine speeds. Thirdly, if vehicle speed varies, as it does in a significant uphill, then by decreasing the vehicle speed variation, the total air drag over the road stretch is reduced. The three factors mentioned, to gear down and back up early and to decrease vehicle speed variations, are all part of the optimal solutions.

If lowest necessary gear is chosen correctly by a standard AMT, only small reductions of fuel consumption are possible to obtain in comparison to the AMT. This means that a standard AMT is unlikely to shift gears in an optimal manner but on the other hand, if the lowest necessary gear is chosen correctly, the AMT system does not shift gears in a really bad manner either. To attain the possible fuel reductions it has been shown that the reduced propulsion of an AMT gear-shifting process, and the resulting vehicle retardation, must be considered. The point of shifting gears must be chosen to ensure an adequate engine speed in order to get a sufficient engine power after the gear shift, even as the truck is decelerated during gear shift. If, for comfort reasons, the gear shifts are longer lasting, then it is even more important to choose the optimal occasion to shift gears. A future work is to optimize the gear-shifting process, in terms of fuel consumption, performance and comfort.

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