Institutionen för systemteknik
Department of Electrical Engineering

Examensarbete

Computing The Ideal Racing Line Using Optimal Control

Examensarbete utfört i Fordonssystem
vid Tekniska högskolan i Linköping
av

Thomas Gustafsson

LITH-ISY-EX--08/4074--SE

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Title Computing The Ideal Racing Line Using Optimal Control

Författare Thomas Gustafsson
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Sammanfattning
Abstract

In racing, it is useful to analyze vehicle performance and driving strategies to achieve the best result possible in competitions. This is often done by simulations and test driving.

In this thesis optimal control is used to examine how a racing car should be driven to minimize the lap time. This is achieved by calculating the optimal racing line at various tracks. The tracks can have arbitrary layout and consist of corners with non-constant radius. The road can have variable width. A four wheel vehicle model with lateral and longitudinal weight transfer is used.

To increase the performance of the optimization algorithm, a set of additional techniques are used. The most important one is to divide tracks into smaller overlapping segments and find the optimal line for each segment independently. This turned out to be useful when the track is long.

The optimal racing line is found for various tracks and cars. The solutions have several similarities to real driving techniques. The result is presented as driving instructions in Racer, a car simulator.

Nyckelord
Keywords Optimal control, Racing, Car tuning

Abstract

In racing, it is useful to analyze vehicle performance and driving strategies to achieve the best result possible in competitions. This is often done by simulations and test driving.

In this thesis optimal control is used to examine how a racing car should be driven to minimize the lap time. This is achieved by calculating the optimal racing line at various tracks. The tracks can have arbitrary layout and consist of corners with non-constant radius. The road can have variable width. A four wheel vehicle model with lateral and longitudinal weight transfer is used.

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Sammanfattning

Vid utövning av bilsport är det användbart att kunna analysera bilens prestanda och olika körstilar. Detta för att prestera så bra som möjligt vid tävlingar. Vanliga metoder är simuleringar och testkörningar.

I denna uppsats används optimal styrning för att undersöka hur en bil ska köras för att uppnå kortast möjliga varvtid. Godtycklig banlayout kan användas. En kurva behöver inte ha konstant radie och banan kan ha variabel bredd. Fordonsmodellen har bland annat fyra hjul och tar hänsyn till longitudinell och lateral tyngdförflyttning.

Ett antal tekniker användes för att lättare nå ett resultat. Den viktigaste tekniken består av att dela in banan i kortare delar. Optimeringsproblemet kan sedan lösas separat för varje del. Detta visade sig vara användbart då långa banor användes.

Det optimala spåret hittades för olika banor och bilar, där flera likheter med riktiga körstilar hittades. Resultatet presenteras som körinstruktioner i en bilsimulator.

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First I would like to thank my supervisor Johan Sjöberg at the Department of Automatic Control, who suggested the topic of my thesis. Thanks for all the help and interesting discussions during my work. I want to thank my examiner Jan Åslund at the Department of Vehicular Systems. I want to thank Johan Åkesson at the Department of Automatic Control at Lund University for providing me with the Optimica software. Finally, I want to thank Markus Persson, my opponent.

Contents

Chapter 1

Introduction

1.1 Background

The ultimate goal of all racing teams is of course to win races. To win a race you need to reduce the lap times as much as possible. To achieve short lap times both a skilled driver and a good racing team providing the driver with a good car is required. The driver must drive in an optimal way to maximize the car performance at the current track. A lot of time and money are spent to analyze the car performance and driving styles. This is often done by simulations and test driving, whose purpose is both to find the car limits and to find out how to reach those limits. When a driver manages to achieve the shortest lap time possible, we will say that he has followed "the optimal line". We want to find this racing line theoretically. A method well suited for this problem is optimal control. This method has earlier been used in studies of single maneuvers in for example, Velenis et al. (2007a,b); Velenis and Tsiotras (2005). Entire tracks has also been studied in Casanova (2000). An alternative method using genetic algorithms is used in Mühlmeier and Müller (2003).

Optimal control is also used in other areas, such as, robotics, space flight and aviation.

1.2 Objective

The main goal is to obtain the racing line that minimizes the total lap time, with respect to a vehicle model and a track description. The track outline is based on a real racing circuit and the vehicle model has characteristics of a racing car. The result is to be presented as driving instructions in a car simulator. We must be able to use different cars and tracks. Combined with the optimal racing line we also want to find the optimal car setup.

1.3 Method

The problem to find the optimal racing line is solved using optimal control. This method is well suited for this kind of problem. The method is characterized by that a cost function is to be minimized with respect to a dynamic system and a set of constraints. In this application the cost function to minimize is time, the dynamic system is the vehicle model and the constraints are the track boundaries.

A set of additional techniques are used to assist in solving the optimal control problem. The most important technique consists of dividing the track into segments and evaluate them individually.

1.4 Thesis outline

Chapter 2 - Optimal Control: The necessary theory of optimal control is presented and the application of optimal control to compute the optimal racing line is discussed.
Chapter 3 - The Track: The track parameterization is described and methods to obtain the track layout is discussed.
Chapter 4 - The Vehicle: The vehicle model is presented along with the two different cars used in this thesis.
Chapter 5 - Additional Techniques: Here additional techniques are presented that are used in conjunction with optimal control.
Chapter 6 - Results: The optimal racing line for each track are presented together with an evaluation of the different techniques used together with optimal control.
Chapter 7 - Conclusions: This chapter contains a general discussion about the results.
Chapter 8 - Further Work: Here are improvements and possible further works suggested.

1.5 Notations

a_i	Normalized slip angle
A_b	Body front area
A_{wf}, A_{wr}	Front and rear wing area
B_{max}	Total brake torque available
B_f, B_r	Front and rear brake distribution
C_{fx}, C_{rx}	Front and rear wing drag coefficient
C_x	Body drag coefficient
C_{fz}, C_{rz}	Front and rear wing downforce coefficient
d	Distance from the vehicle center of gravity to the road center line
F_{wxi}	Longitudinal tire force in wheel axis system
F_{wyi}	Lateral tire force in wheel axis system
F_{xi}	Longitudinal tire force in vehicle axis system
F_{yi}	Lateral tire force in vehicle axis system
F_{zi}	Tire normal load
F_{ax}	Total aerodynamic drag
F_{azf}, F_{azr}	Front and rear wing aerodynamic downforce
G_r	Final drive gear ratio
J_z	Vehicle moment of inertia around yaw axis
J_{wf}, J_{wr}	Moment of inertia of front and rear wheel
k_i	Slip ratio
k_t	Track curvature
K_{diff}	Viscous differential constant
l_f	Distance between center of gravity and front axle
l_r	Distance between center of gravity and rear axle
M	Vehicle total mass
r	Track corner radius
R_f, R_r	Front and rear wheel radius
R_{sf}	Roll stiffness distribution
s	Driven distance measured along the road center line
s_i	Normalized slip ratio
S_{CF}	Time to distance scaling factor
t_f	Distance between front wheels
t_r	Distance between rear wheels
T_B	Brake torque
T_E	Engine torque
$T_{e_{max}}$	Maximum engine torque available
T_i	Wheel torque
u_{st}	Steering angle
u_{tb}	Throttle and brake control
V_x	Vehicle longitudinal velocity
V_y	Vehicle lateral velocity
W_b	Vehicle wheel base distance
w_{car}	Car width
w_t	Road width

x_1	Vehicle yaw angle
x_2	Angular velocity
x_3	Longitudinal velocity in vehicle axis system
x_4	Lateral velocity in vehicle axis system
x_5	Vehicle position x
x_6	Vehicle position y
x_7	Angular velocity of front left wheel
x_8	Angular velocity of front right wheel
x_9	Angular velocity of rear left wheel
x_{10}	Angular velocity of rear right wheel
x_t,y_t	Coordinate of the road center line
x_v,y_v	Coordinate of the vehicle center of gravity
α_i	Slip angle
ΔT_{diff}	Differential torque transfer
$\Delta_{F_{z_{long}}}$	Longitudinal weight transfer
$\Delta_{F_{zf_{lat}}}$, $\Delta_{F_{zr_{lat}}}$	Front and rear lateral weight transfer
φ_f, φ_r	Front and rear wing angle
ψ_t	Angle of the road center line tangent
ψ_v	Vehicle yaw angle
ρ	Air density
σ_i	Combined slip

Chapter 2

Optimal Control

In this chapter we will discuss the basic theory behind optimal control and how this is applicable in the case of calculating the optimal racing line.

2.1 Theory

The main goal of optimal control is to find a control history u and a trajectory x that minimize a criteria with respect to a dynamic model and a set of constraints. Formally, this can be written as:

$$\min J = \min_{u(t)} \int_{t_0}^{t_f} L\big(x(t), u(t)\big)dt, \quad t_0 \leq t \leq t_f \tag{2.1}$$

subject to:

$$\dot{x}(t) = f\big(x(t), u(t)\big), \quad x(t_0) = x_0 \tag{2.2}$$

$$u_{min} \leq u(t) \leq u_{max} \tag{2.3}$$

$$c\big(x(t)\big) \leq 0 \tag{2.4}$$

where (2.1) is the cost function to minimize, (2.2) is a dynamic system, (2.4) is the system constraints and (2.3) is the upper and lower bounds for the system control inputs.

2.2 Application

In this thesis, the optimal control problem has the following interpretation: the cost function is the lap time, the control history is the steering angle and applied throttle/brake, the dynamic system is a vehicle model and the constraints are the road boundaries. The goal is to minimize the lap time. This is not possible with the formulation in Section 2.1 since the lap time, or in other words, the final time

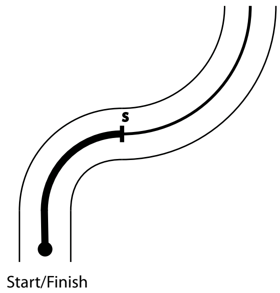


Figure 2.1. The track distance s.

t_f, is unknown. The problem is therefore modified by employing a technique used in Casanova (2000). We will use a distance s as independent variable instead of the time t. The distance s is the traveled distance from the track start as if you were measuring along the center of the road. This is illustrated in Figure 2.1. This distance is a natural choice because it makes it easier to parameterize the track and the final distance s_f is then a known constant, simply the track length. The new problem is written as:

$$\min_{u(s)} t_f, \quad s_0 \leq s \leq s_f \tag{2.5}$$

subject to:

$$\dot{x}(s) = S_{CF} f\big(x(s), u(s)\big), \quad x(s_0) = x_0 \tag{2.6}$$

$$u_{min} \leq u(s) \leq u_{max} \tag{2.7}$$

$$c(x(s)) \leq 0 \tag{2.8}$$

S_{CF} is transforming the vehicle model to a system where s is the new independent variable. This transformation is derived in (Casanova, 2000) and is written as:

$$S_{CF} = \frac{1-d/r}{V_x \cdot \cos(\psi_v - \psi_t) - V_y \cdot \sin(\psi_v - \psi_t)} \tag{2.9}$$

where d is the car distance from the road center line:

$$d = (y_v - y_t) \cdot \cos(\psi_t) - (x_v - x_t) \cdot \sin(\psi_t) \tag{2.10}$$

The lap time is then given by $t_f = \int_{s_0}^{s_f} S_{CF} ds$. Since it is desired that the entire car stays on road, the constraints $d(s)^2 \leq (\frac{w_t(s) - w_{\text{car}}}{2})^2$ are introduced. One could think about having slack variables so that the car is allowed to momentarily pass the road boundaries, but this is not investigated.

2.3 Grid

In this thesis, a so called direct method is used. This requires the problem to be discrete so that it can be solved using nonlinear programming. Since both the track and our vehicle model are specified as functions of the variable s, the problem is discretizised by dividing the track in a number of points, as illustrated in Figure 2.2. The actual discretization is done by a software called Optimica, see Section 5.4.2. The number of points is chosen by the user and they are evenly spaced throughout the track. Initially, 300 points per kilometer is used, this choice is discussed in Section 6.1.4. The discretization is described in more detail in Åkesson (2007).

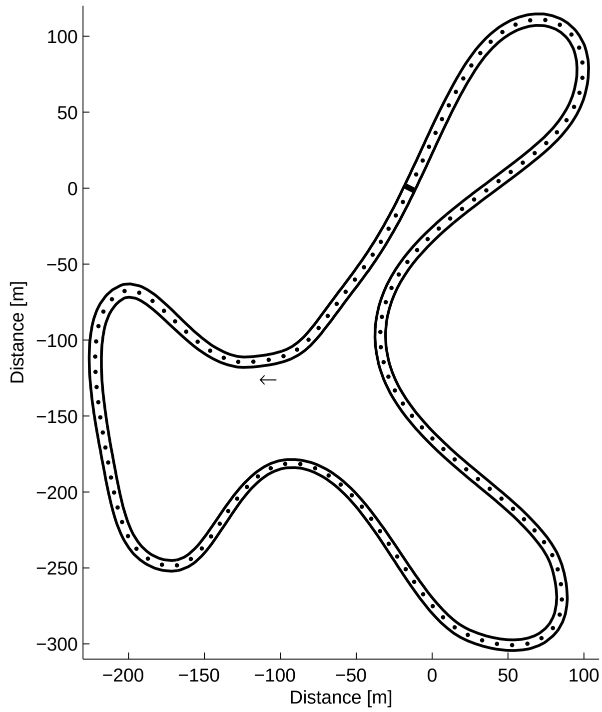


Figure 2.2. The grid

Chapter 3

The Track

The goal of the track model is to be able to describe any type of track layout with variable road width and corners of non-constant radius. The model is restricted to only describe a completely flat track, which means that no slopes or banked corners are present.

3.1 Parameterization

The track is described by the parameters k_t, ψ_t, (x_t, y_t) and w_t. The coordinate system is shown in Figure 3.1. The track curvature k_t, is calculated by $\frac{1}{r}$, where r is the radius of the curve. The angle of the center line tangent is described by ψ_t and w_t is the road width. (x_t, y_t) are the coordinates of the center line. All these parameters are functions of s, as described in Section 2.2. This is useful since it makes it possible to write the entire track as a function independent of time. The track data is obtained either from the simulator Racer, real photographs or simply from hand made drawings. The raw data are the coordinates of the left and right boundaries of the road. These are then used to calculate the track parameters. It is required by the solver that the functions are differentiable, which is achieved by using splines to represent the track parameters.

3.2 Tracks

Four tracks with different characteristics are used. Brands Hatch and Sviestad are real tracks while Fernstone and the Hairpin turn are only theoretical.

3.2.1 Fernstone

The layout of Fernstone is shown in Figure 3.2(a). The track was chosen mainly because of its smooth shapes and flat topology. The distance between the highest and lowest point is only three meters. This is useful since the track altitude is ignored in our models. The track length is 1468 meters and the width varies

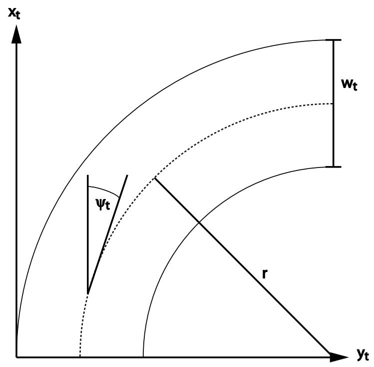


Figure 3.1. The track coordinate system.

between 7 and 11 meter. This track is obtained from Racer and the road layout can therefore be extracted directly.

3.2.2 Brands Hatch

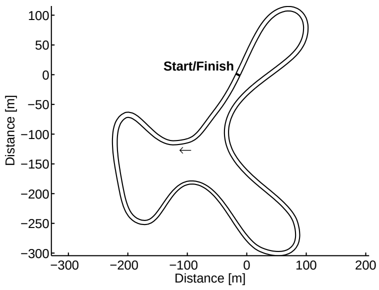
Brands Hatch is a more realistic track typical for racing. This track has long straights and both high and low speed corners. The track width varies between 9 and 21 meters. The layout is shown in Figure 3.2(b). The track is the longest considered with a length of 4015 meters. This should result in a larger problem to solve if we are to keep the same grid density. The maximum altitude difference is 30 meters but since it is distributed across the entire track it will hopefully have minimal effect. This track can also be extracted from Racer.

3.2.3 Sviestad

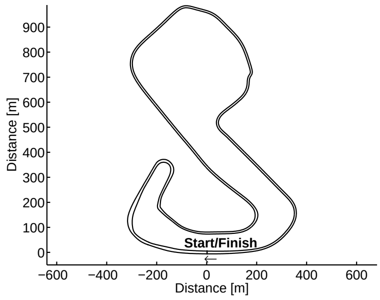
The Sviestad track was partly chosen to test the possibility to obtain track information without Racer. Only the first kilometer is used since it has an interesting set of corners. The track description is obtained from a photo taken from the air and its layout can be seen in Figure 3.2(c).

3.2.4 The Hairpin turn

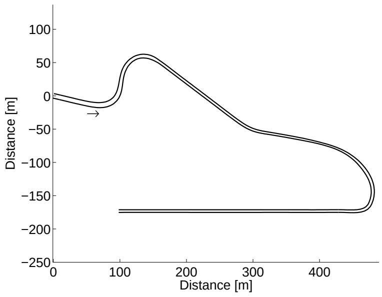
The Hairpin turn was created to examine the principles of driving through a 180 degree corner followed by a long straight. The track is shown in Figure 3.2(d). The main purpose was to find out when to use a so called late apex, described in Section 5.6.1. The track is completely artificial to allow us to change corner radius, road width and straight length.



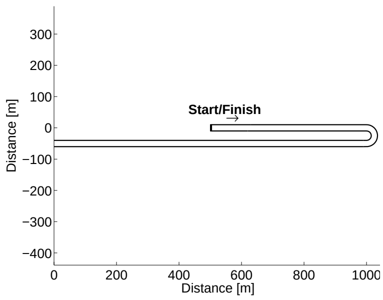
(a) Fernstone



(b) Brands Hatch



(c) Sviestad



(d) The Hairpin turn

Figure 3.2. The tracks.

Chapter 4

The Vehicle

In this thesis a vehicle model with four wheels is used to include lateral and longitudinal weight transfer. Aerodynamics of a front and rear wing is included in the model. The engine is modeled using a torque map extracted from Racer.

4.1 Model

The vehicle center of gravity is located at (x_v, y_v) in global coordinates. The vehicle orientation is specified by its yaw angle ψ_v, see Figure 4.1. The vehicle has its own coordinate system where the x-axis point forward and the y-axis to the driver's right hand side. The origin is located at the center of gravity. With the vehicle as the frame of reference, the longitudinal and lateral velocities are V_x and V_y, respectively.

The vehicle is controlled by u_{st} and u_{tb}, where u_{st} is the angle of the front wheels and u_{tb} is the applied throttle/brake. The controls have to belong to the intervals $-40 \leq u_{st} \leq 40$ and $-1 \leq u_{tb} \leq 1$. The limitations of u_{st} is introduced to prevent the solver from using angles that are physically impossible. Maximum brake is achieved at $u_{tb} = -1$ and maximum throttle at $u_{tb} = 1$. Notice that it is not possible to apply the brake and throttle at the same time.

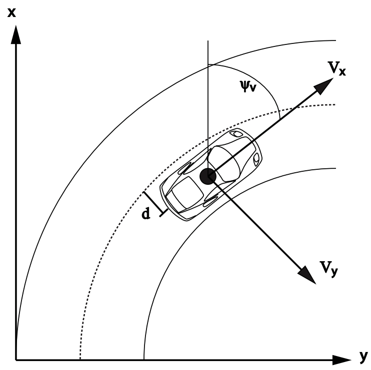


Figure 4.1. The vehicle coordinate system.

4.1.1 Equations

The vehicle dynamics are based up on a model presented in (Casanova, 2000) and are written as a state space model in (4.1). The parameters and variables are defined in Section 1.5.

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \left((F_{x1} - F_{x2})\tfrac{t_f}{2} + (F_{x3} - F_{x4})\tfrac{t_r}{2} + (F_{y1} + F_{y2})l_f - (F_{y3} + F_{y4})l_r\right)\tfrac{1}{J_z} \\
\dot{x}_3 &= (F_{x1} + F_{x2} + F_{x3} + F_{x4} - F_{ax})\tfrac{1}{M} + x_2 x_4 \\
\dot{x}_4 &= (F_{y1} + F_{y2} + F_{y3} + F_{y4})\tfrac{1}{M} - x_2 x_3 \\
\dot{x}_5 &= x_3 \cos(x_1) - x_4 \sin(x_1) \\
\dot{x}_6 &= x_3 \sin(x_1) + x_4 \cos(x_1) \\
\dot{x}_7 &= (T_1 - F_{wx1} R_f)\tfrac{1}{J_{wf}} \\
\dot{x}_8 &= (T_2 - F_{wx2} R_f)\tfrac{1}{J_{wf}} \\
\dot{x}_9 &= (T_3 - F_{wx3} R_r)\tfrac{1}{J_{wr}} \\
\dot{x}_{10} &= (T_4 - F_{wx4} R_r)\tfrac{1}{J_{wr}}
\end{aligned}
\tag{4.1}
$$

The forces on the front wheels are projected into the vehicle coordinate system:

$$
\begin{aligned}
F_{xi} &= F_{wxi}\cos(u_{st}) - F_{wyi}\sin(u_{st}), \quad i = 1,2 \\
F_{yi} &= F_{wxi}\sin(u_{st}) + F_{wyi}\cos(u_{st}), \quad i = 1,2
\end{aligned}
\tag{4.2}
$$

The forces on the rear wheels already have the correct direction:

$$
\begin{aligned}
F_{xi} &= F_{wxi}, \quad i = 3,4 \\
F_{yi} &= F_{wyi}, \quad i = 3,4
\end{aligned}
\tag{4.3}
$$

Since the car has rear wheel drive, the only torque applied on the front wheels are caused by braking:

$$
\begin{aligned}
T_1 &= \tfrac{T_B B_f}{2} \\
T_2 &= \tfrac{T_B B_f}{2}
\end{aligned}
\tag{4.4}
$$

The rear wheels are affected by the brakes and the engine, there is also a torque transfer between left and right side due to the differential:

$$\begin{aligned} T_3 &= \tfrac{T_E G_R}{2} + \tfrac{T_B B_r}{2} + \Delta T_{diff} \\ T_4 &= \tfrac{T_E G_R}{2} + \tfrac{T_B B_r}{2} - \Delta T_{diff} \end{aligned} \quad (4.5)$$

The total braking torque is distributed between the front and rear wheels and is given by:

$$T_B = B_{max} u_{tb} \qquad u_{tb} < 0 \quad (4.6)$$

The differential is a so called viscous differential where torque transfer is proportional to the speed difference between the left and right wheel:

$$\Delta T_{diff} = (x_{10} - x_9) K_{diff} \quad (4.7)$$

The engine torque only depends on throttle input and rear wheel RPM:

$$T_E = T_{e_{max}} (\tfrac{x_{10} - x_9}{2}) u_{tb} \qquad u_{tb} \geq 0 \quad (4.8)$$

The car has no actual gears but instead the gear ratios have been implemented into the engine torque map $T_{e_{max}}$, by calculating the resulting torque output as a function of the car longitudinal velocity. Different velocities give different torques according to the gears typically used at these speeds. An example is shown in Figure 4.2.

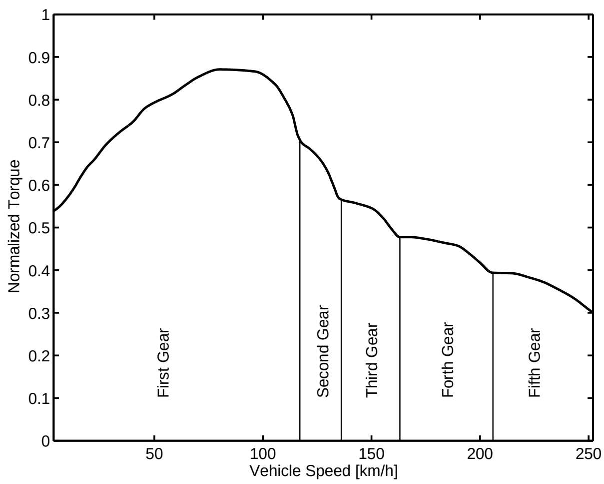


Figure 4.2. The torque output as a function the car speed.

The aerodynamic drag from the car body and wings is assumed to act upon the vehicle center of gravity:

$$F_{ax} = \tfrac{1}{2}\rho A_b C_x x_3^2 + \tfrac{1}{2}\rho A_{wf} \varphi_f C_{fx} x_3^2 + \tfrac{1}{2}\rho A_{wr} \varphi_r C_{rx} x_3^2 \quad (4.9)$$

The normal load on the tires is calculated as:

$$\begin{aligned}
F_{z1} &= \tfrac{F_{zf_{stat}}}{2} + \tfrac{F_{azf}}{2} - \tfrac{\Delta F_{z_{long}}}{2} + \tfrac{\Delta F_{zf_{lat}}}{2} \\
F_{z2} &= \tfrac{F_{zf_{stat}}}{2} + \tfrac{F_{azf}}{2} - \tfrac{\Delta F_{z_{long}}}{2} - \tfrac{\Delta F_{zf_{lat}}}{2} \\
F_{z3} &= \tfrac{F_{zr_{stat}}}{2} + \tfrac{F_{azr}}{2} + \tfrac{\Delta F_{z_{long}}}{2} + \tfrac{\Delta F_{zr_{lat}}}{2} \\
F_{z4} &= \tfrac{F_{zr_{stat}}}{2} + \tfrac{F_{azr}}{2} + \tfrac{\Delta F_{z_{long}}}{2} - \tfrac{\Delta F_{zr_{lat}}}{2}
\end{aligned} \tag{4.10}$$

The normal load resulting from aerodynamic downforce is calculated as if the wings where positioned directly above the front and rear wheel axle, respectively. This gives the expressions:

$$\begin{aligned}
F_{azf} &= \tfrac{1}{2}\rho A_{wf} C_{fz} \varphi_f x_3^2 \\
F_{azr} &= \tfrac{1}{2}\rho A_{wr} C_{rz} \varphi_r x_3^2
\end{aligned} \tag{4.11}$$

The static normal load is the force applied when the vehicle is standing still on a flat surface:

$$\begin{aligned}
F_{zf_{stat}} &= \tfrac{Mgl_r}{W_b} \\
F_{zr_{stat}} &= \tfrac{Mgl_f}{W_b}
\end{aligned} \tag{4.12}$$

The longitudinal and lateral weight transfer, (4.13) and (4.14), are approximated by the steady state behavior. These equations are derived in Casanova (2000):

$$\Delta F_{z_{long}} = \left(\tfrac{T_1+T_2}{R_f} + \tfrac{T_3+T_4}{R_r}\right)\tfrac{h_g}{W_b} \tag{4.13}$$

and

$$\begin{aligned}
\Delta F_{zf_{lat}} &= \tfrac{x_2 x_3 M}{t_f}\left(\tfrac{l_r h_{rf}}{W_b} + R_{sf}(h_g - h_{rc})\right) \\
\Delta F_{zr_{lat}} &= \tfrac{x_2 x_3 M}{t_r}\left(\tfrac{l_f h_{rr}}{W_b} + R_{sr}(h_g - h_{rc})\right)
\end{aligned} \tag{4.14}$$

The slip angles are written as:

$$\begin{aligned}
\alpha_1 &= -u_{st} + \tfrac{x_4 + l_f x_2}{x_3 + x_2 \frac{t_f}{2}} \tfrac{180}{\pi} \\
\alpha_2 &= -u_{st} + \tfrac{x_4 + l_f x_2}{x_3 - x_2 \frac{t_f}{2}} \tfrac{180}{\pi} \\
\alpha_3 &= \tfrac{x_4 - l_r x_2}{x_3 + x_2 \frac{t_r}{2}} \tfrac{180}{\pi} \\
\alpha_4 &= \tfrac{x_4 - l_r x_2}{x_3 - x_2 \frac{t_r}{2}} \tfrac{180}{\pi}
\end{aligned} \tag{4.15}$$

The slip ratios are written as:

$$\begin{aligned}
k_1 &= -(1 - \tfrac{x_7}{x_3 + x_2 \frac{t_f}{2}} R_f) \times 100 \\
k_2 &= -(1 - \tfrac{x_8}{x_3 - x_2 \frac{t_f}{2}} R_f) \times 100 \\
k_3 &= -(1 - \tfrac{x_9}{x_3 + x_2 \frac{t_r}{2}} R_r) \times 100 \\
k_4 &= -(1 - \tfrac{x_{10}}{x_3 - x_2 \frac{t_r}{2}} R_r) \times 100
\end{aligned} \tag{4.16}$$

The lateral and longitudinal forces acting on the tires are evaluated using the well-known Pacejka Magic Formula, see Pacejka (2006). These forces are only correct when acting alone. When both forces are present they will affect each other since the total grip generated by a tire is limited. This relationship can be modeled using the Pacejka Magic Formula but a simpler method is used instead. The lateral and longitudinal forces are combined using a method proposed by Beckman (2001a), which is similar to the model defined by Pacejka (2006). The objective is to achieve a model keeping the total tire force inside a traction ellipse. The lateral and longitudinal force should not be affected when acting alone. The slip ratio and slip angle are first normalized so that maximum grip is generated at a value of one which gives:

$$s_i = \frac{k_i}{k_{i_{max}}} \tag{4.17}$$

$$a_i = \frac{\alpha_i}{\alpha_{i_{max}}} \tag{4.18}$$

The variable σ_i measures the amount of total tire grip currently used. Maximum grip is achieved at a value of one.

$$\sigma_i = \sqrt{s_i^2 + a_i^2} \tag{4.19}$$

The combined slip, σ_i, is then used to calculate both lateral and longitudinal forces using Pacejka as if they were acting separately. By using s_i and a_i, the available grip can be distributed between the directions, according to:

$$\begin{aligned} F_{wxi} &= \frac{s_i}{\sigma_i} F_{wxi_{pure}}(\sigma_i k_{i_{max}}) \\ F_{wyi} &= \frac{a_i}{\sigma_i} F_{wyi_{pure}}(\sigma_i \alpha_{i_{max}}) \end{aligned} \tag{4.20}$$

4.2 Cars

Two completely different cars are used to find out if the optimal racing line is very vehicle dependent. The purpose is not to identify properties responsible of changing the line, but to investigate if the difference is noticeable.

4.2.1 Ferrari 333 SP

Ferrari 333 SP is a car specifically build for racing, with high aerodynamic downforce, low weight and a high performance engine. A sketch of the car is shown in Figure 4.3.

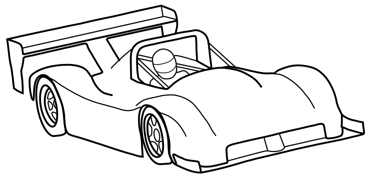


Figure 4.3. Ferrari 333 SP

4.2.2 Ferrari 360 Modena

This car is a standard sports car very different from the one above. It has almost twice the weight and yaw inertia. Furthermore the car has a substantially lower aerodynamic downforce due to the lack of wings, a higher center of gravity and about 20% less engine torque. A sketch of the car is shown in Figure 4.4.

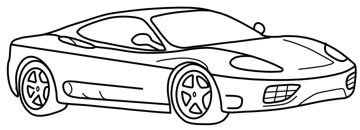


Figure 4.4. Ferrari 360 Modena

Chapter 5

Additional Techniques

We want to find the optimal line for the entire track directly without doing any changes to the problem specified in Section 2.2. This is however not always possible due to two problems. The first problem arises when a track is long. In these cases it is sometimes difficult to find a solution for the entire track at once. The Ipopt software either fails to converge to a solution at all or far too many iterations are required to find a solution within reasonable time. The second problem is to achieve a continuous lap where the beginning of a lap depends on the end of the previous lap. This is not done automatically by our originally defined problem where the initial and final states are either fixed or free but not necessarily equal. To reach our goals these problems will be solved by using a set of additional techniques.

5.1 Control input penalty

When longer road segments are to be optimized it is sometimes difficult to find a solution. This is characterized by a high number of required iterations or that no solution is found at all. Stability is increased by penalizing the control input derivatives. This is done by rewriting the cost function as:

$$J = t_f + \int_{s_0}^{s_f} w_1(\frac{d}{ds}u_{st})^2 + w_2(\frac{d}{ds}u_{tb})^2 ds \tag{5.1}$$

The parameters w_1 and w_2 are used to choose the degree of penalty to be applied. The amount of penalty is chosen so that convergence is achieved but as low as possible to minimize the changes to the final solution. The effects of introducing a penalty are discussed in Section 6.1.3. The reason for penalizing the controls can be seen in Figure 5.1. When studying the vehicle control inputs we can see that spikes or noise is present. Similar behavior is observed in Casanova (2000), where increased noise is accompanied by an increased number of required iterations. We can therefore suspect that difficulties to converge and control inputs with noise are somehow related.

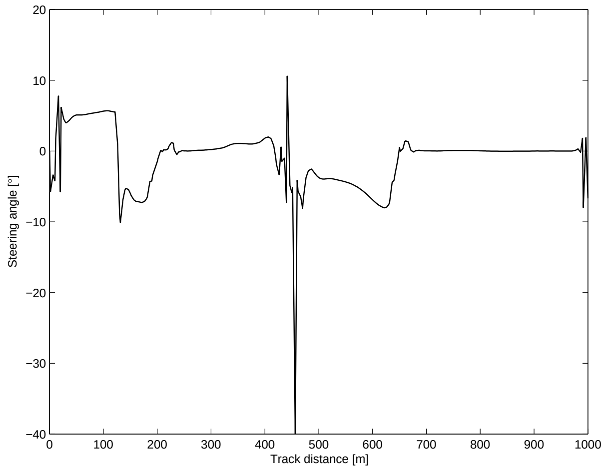


Figure 5.1. Steering input without penalty.

5.2 Decoupled road segments

This technique was developed to solve the problems that occur when the track is long and to achieve a continuous lap. The ideal racing line can easily be computed for shorter tracks. It could therefore be useful to divide the track into subsections and solve the problem for each of these segments independently, one at a time, as illustrated in Figure 5.2. When doing this we have to make sure that the same result is obtained as if the entire lap was solved at the same time. The main problem with this method is that the start and end points will not be part of the optimal lap. They are only based on the initial guess and do not take in to account the track outside the segment.

Imagine driving the car and somewhere on the track, you turn over to the side of the road and then you continue driving. Then, after a few corners you will reach the desired speed and the driven line is no longer affected by the stop you made earlier. It is therefore natural to make the assumption that we always can find a point further down the track that are independent of our current position and speed. This is a useful assumption that will allow us to overcome the starting point problem by simply start a couple of curves before the desired segment starts. This idea is tested in Figure 5.3, where the car position is forced to leave the optimal line. This is what would happen when starting in a joint between two sections. The two lines are however converging after a couple of corners and the car then follows the optimal line. By dividing the track in overlapping segments where the overlap is sufficiently long it should be possible to join the segments together seamlessly, see Figure 5.4 for an example.

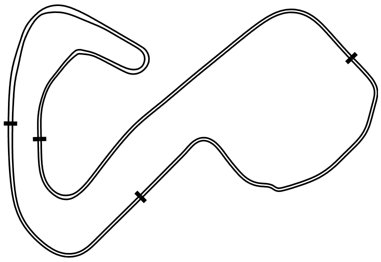


Figure 5.2. The track is divided into shorter segments.

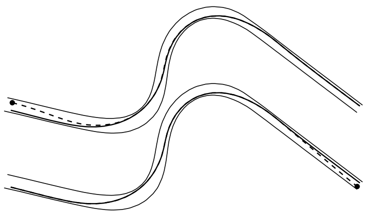


Figure 5.3. The figure shows the driven line (dashed), when the start or end point not is a part of the optimal line (solid).

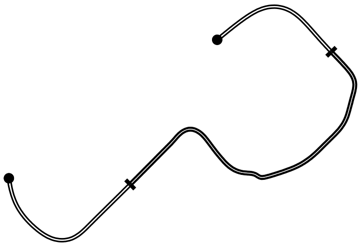


Figure 5.4. To obtain the ideal racing line for a segment, the optimization problem needs to be solved for a larger overlapping intervall.

5.3 Optimal Car Tuning

In racing, a lot of work is performed to make sure the car is properly tuned to give the fastest lap possible. This is done mainly by test driving and by simulations, which require a lot of time and money. In this section, a different approach is tested. The idea is not only to find the optimal racing line, but also to find the optimal car setup. It cannot entirely replace test drives due to model errors but it could probably decrease the number test drives required. The computed optimal setup will at least give a good hint on what setup to start with. However, it is also important to take into consideration that the car must be well-behaved and easy to drive since at the end it is supposed to be driven by a human. This requirement is not always satisfied for the optimal setup.

5.3.1 Method

To find the optimal setup, the model parameter to be tuned will be treated as constant control inputs. For some model parameters constraints are needed to ensure that no physically impossible values are attained. The solution to the optimal control problem will then be both the optimal racing line and the optimal car setup for the current track.

5.3.2 Optimized parameters

The parameters to be optimized are longitudinal weight distribution, brake bias, roll stiffness distribution, front and rear wing angles. All these parameters affect the car oversteer/understeer(OU)-properties. The optimally tuned car should then have the optimal balance between oversteer and understeer. In racing, it is common to have a slightly oversteered car. The purpose is to counteract the understeer often induced when braking.

Longitudinal weight distribution

In racing, the car is often designed to weigh less than the required weight limit. In this way, extra weight can be added to the car at a desired position in order to change the weight distribution. Moving weight forward will change the car towards understeer, according to Russ (2007); Wan (2000).

Brake Bias

The brake bias affects how the braking torque is distributed between front and rear wheels. The brake distribution is normally biased toward the front wheels. A typical brake bias is 60/40 which means that 60% of the brake torque is applied at the front wheels. Moving brake torque forward will change the car towards understeer.

Roll stiffness distribution

The roll stiffness distribution measures how roll stiffness is distributed between the front and rear wheels. This is determined by the stiffness of springs and anti-roll bars. Moving roll stiffness forward will change the car towards understeer. Note that the total roll stiffness of both the front and rear suspension is not considered in the model, only the ratio.

Wing angles

The front and rear wing angles affect aerodynamic downforce applied at the front and rear wheels. For higher angles, the downforce is increased, but also the drag. Therefore, the choice of angles is a compromise between a high top speed and a large grip. The ratio between front and rear downforce will affect the OU-properties of the car. Increasing the front wing angle or decreasing the rear wing angle change the car towards oversteer.

5.4 Software

There are a number of softwares used in this thesis. The basic workflow is illustrated in Figure 5.5.

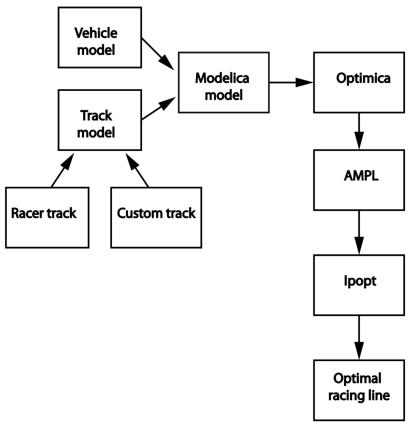


Figure 5.5. An overview of the software usage.

5.4.1 Racer

Racer is a free car simulator using a relatively advanced vehicle physics model, Gaal (2008). The user can create own cars and tracks which means that the vehicle model parameters and track properties are known to the user. In this work, Racer is partly replacing the need for real tracks and cars. There is also the ability to extract vehicle telemetry from Racer while driving the car. The most important data available are vehicle position, velocity, orientation, tire forces and slip. The goal is to use tracks and cars from Racer and also to use the simulator as an interface for presenting the result. This will be done as driving instructions visible to the driver.

5.4.2 Optimica

Optimica is a software that allows the user to formulate and solve optimal control problems. The system is modeled in the Modelica language while the cost function and constraints are formulated in the Optimica language, see Åkesson (2007). The problem is then discretizised and translated into AMPL code.

5.4.3 AMPL and Ipopt

AMPL is a high level programming language making it possible to describe optimization problems. AMPL uses an external solver supplied by the user. In this thesis, the Ipopt solver is used. Ipopt is designed to solve large nonlinear optimization problems and is particularly good at sparse problems, which is a feature of the problem obtained in this thesis.

5.5 Initial Guess

When solving our optimization problem an initial guess is needed as a starting point. The initial guess consists of both a control history and a trajectory. To avoid ending up in a local minimum far away from the global optimum, earlier studies have suggested that we need an initial guess with a trajectory as close as possible to the real optimal line. A good initial guess should also require less iterations by the solver. Three methods for obtaining an initial guess are tested.

The first method uses data measured directly from Racer. The second method uses a vehicle trajectory measured from Racer as a reference and creates its own initial guess by using a driver model. The third method also relies on a driver model but here the driver model follows the road center line to evaluate the importance of a good initial guess.

5.5.1 Racer

By using an initial guess from Racer we immediately obtain a solution that is close to the optimal one. The main idea is to measure the car telemetry while a driver is trying to achieve the fastest lap possible. The driven line is then hopefully close to the optimal one.

In order to use measured data from Racer, the data must be transformed into the coordinate system presented in Section 2.2. Each sample of the car telemetry must be associated with correct value of s. This is achieved by finding the nearest point on the road center line for each sample. The nearest point is found by dividing s into pieces of 0.001 m, resulting in a number of points at the road center line. Then we calculate the distance from the car position to all these points and find the smallest one. The method is illustrated in Figure 5.6. This is a brute force method that may require a few minutes to perform. However, it is only done once for each new track.

Due to errors in the model, the initial guess will not be entirely consistent with the car model used for optimizing the racing line. In these cases, it is common to solve a so called Phase-I problem, see Nocedal (1999). After Phase-I, the solution should be feasible and is hopefully close to the original one.

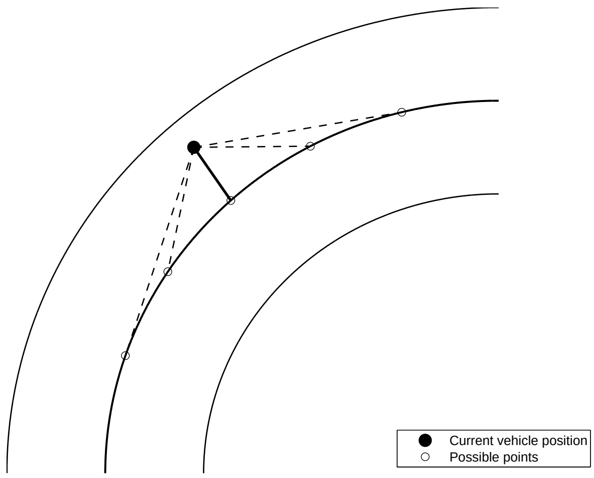


Figure 5.6. The distance is calculated from the car current position to all points at the center line. The shortest distance is chosen. This is repeated for all vehicle positions.

5.5.2 Driver model

To decouple the initial guess from Racer and make it entirely consistent with our vehicle model, a driver model will be used. The driver model will drive the car, simulated by our vehicle model, while following the trajectory and longitudinal velocity obtained from Racer. We should then be able to use the resulting control history and trajectory directly as an initial guess.

The driver must be able to follow a predetermined trajectory and speed. We will treat these as two separate problems. The speed is controlled by a simple PI controller. The reference velocity from Racer is downscaled to help the steering controller. It is required because of errors in model and the limited performance of the driver model. The steering control is based on a driver model developed in Casanova (2000), where the accuracy turned out to be sufficiently good for this application. The controller uses a combination of the current position and preview information to calculate the steering angle.

5.5.3 Initial guess without Racer

To test the importance of a good initial guess, a different approach is tested. The driver model is used but the reference trajectory is simply the road center line and the desired velocity is constant or just a rough estimate made by hand. If we are able to use an arbitrary initial guess there is no need to spend time driving in Racer.

5.6 Driving techniques

To verify the results, they will be compared to known driving techniques used by professional drivers. In this section, some of the concepts are discussed.

5.6.1 Apex

The apex is defined as the inside tangential point when driving through a corner, see Figure 5.7. When the apex is located at the middle of the corner it is called a center apex. An apex located before the center of the corner is called an early apex. If the apex is located after the center of the corner it is called a late apex. There are some guidelines where an apex should occur in a corner to be considered an optimal line. According to (Oneshift, 2006) and (Drivers Domain UK, 2007), the late apex is supposed to be the best strategy when the corner is followed by a long straight. This is motivated by the possibility to increase the throttle earlier and therefore reach a higher speed at the exit of the corner. This is also considered as a safer racing line to drive when the track is hidden after a corner. The center apex is used to pass a corner as fast as possible without considering the track after and before the corner. The early apex is generally not a good line to choose.

5.6.2 Trail braking

The easiest strategy for an inexperienced driver is to use a technique that avoids steering while applying the brakes. Before a corner the driver brakes hard while driving in a straight line until the desired cornering speed is reached, the driver will then start to turn, this is called straight-line braking.

Another method is trail braking, which have some advantages compared to straight-line braking, see Beckman (2001b). The driver is initially braking hard and when entering the corner he will gradually ease of the brakes, while increasing the steering input. This results in a trajectory of decreasing radius. The braking continues until the apex is reached. By braking while turning it is possible to use the longitudinal weight transfer to initiate the car rotation, since it gives increased load at the front wheels resulting in better grip and therefore increasing the ability to induce a rotation. At the same time, the reduced load at the rear wheels will decrease their ability to brake the rotation. This behavior was studied in Velenis et al. (2007a,b).

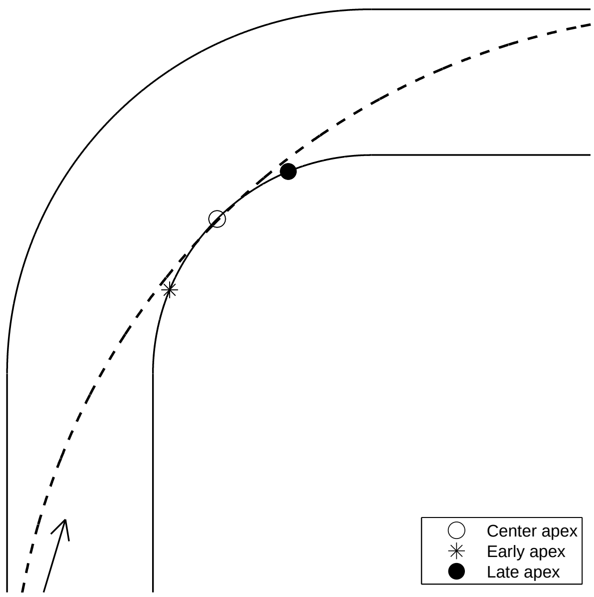


Figure 5.7. The center, early and late apex.

5.6.3 Pendulum turn

The pendulum turn is in fact a method more typical for rally driving and not track racing. In Casanova (2000), there were some indications that the pendulum turn is beneficial even during racing. Therefore, this method is included in our evaluations as well. The main characteristic of the pendulum turn method is that the driver is steering in the wrong direction before entering a corner. By steering away from the corner while braking, the car starts to rotate away from the corner. The driver will then start turning toward the corner while releasing the brakes and give a short throttle input. This short "throttle blip" results in a weight transfer to the rear. The transferred weight gives increased grip at the rear wheels which now can help to initiate the rotation required to pass the corner. The pendulum turn is studied in Velenis et al. (2007a,b).

Chapter 6

Results

All the results from optimizing the racing line are presented together with an evaluation of the additional techniques presented in Chapter 5. The optimal racing line is also presented as driving instructions in Racer. The driver can follow a line drawn at the tarmac. An example is shown in Figure 6.1.

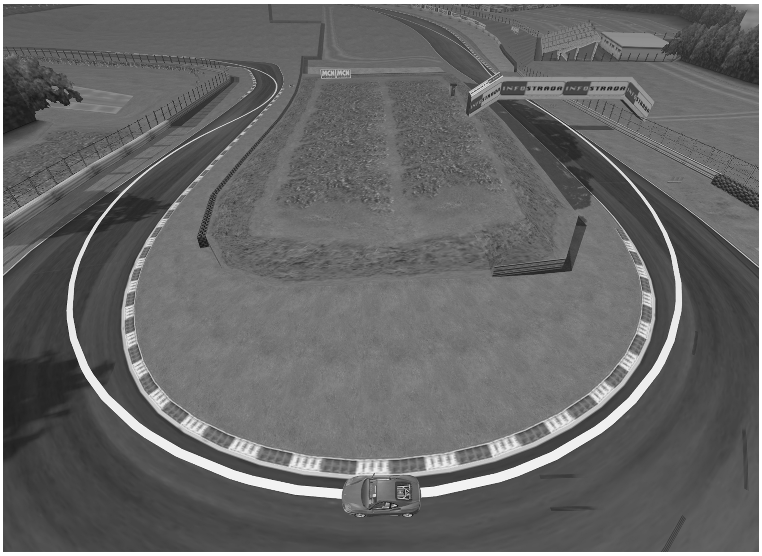


Figure 6.1. The optimal racing line is painted in white at the tarmac.

6.1 Additional Techniques

In this section results from the additional techniques are presented and their usability is evaluated.

6.1.1 Initial guess

When the data measured from Racer is used directly as an initial guess some problems were experienced. Because of the model errors, a Phase-I stage was needed to make the guess feasible. The solution was a racing line too different from the original to be useful, especially when longer road segments were covered. The vehicle dynamics model in Racer is not completely known. The result is modeling errors that probably are too large for this technique to be usable, but by using a driver model this problem can be avoided.

Using a driver model with data measured from racer as a reference path proved to be useful. The driver model was set to follow the path and velocity driven in Racer. The velocity was downscaled to about 80% since the driver model was unable to follow the specified trajectory at full speed. This gave a maximum error of 0.4 m sideways. The largest error in speed was 0.5 m/s. The accuracy of the driver model can be increased by downscaling the reference velocity even further. The drawback is a possible increase in the number of iterations required to find a solution. The trajectory obtained from Racer is shown in Figures 6.2 and 6.3.

At Brands Hatch sections can be found where the initial guess obviously not is a good line. However, the obtained solution still appears to be correct. An example is clearly seen in the fourteenth corner of Brands Hatch in Figure 6.4. This indicates that the initial guess for this particular problem does not have to be perfect to find the optimal solution. To examine the sensitivity more thoroughly, the optimal line is solved for Sviestad by using three different initial guesses. The driver model is supposed to follow the road center, the left road boundary or the right road boundary. All these three initial guesses resulted in identical solutions. This indicates that if the obtained solution only is a local minimum, it should not have been caused by the choice of initial guess. There are however some indications that the number of iterations required to solve the problem increases when the initial guess is too far away.

The difference in longitudinal velocity between the initial guess and final solution is sometimes large as seen in Figures 6.5 and 6.6.

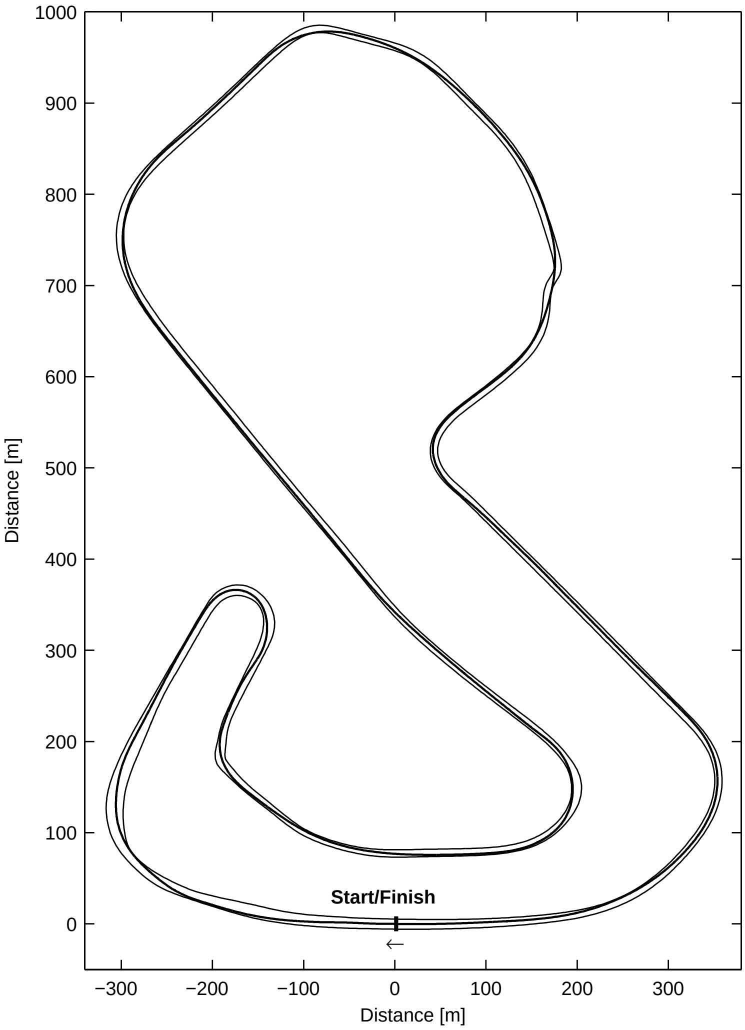


Figure 6.2. The initial guess from Racer when driving at Brands Hatch.

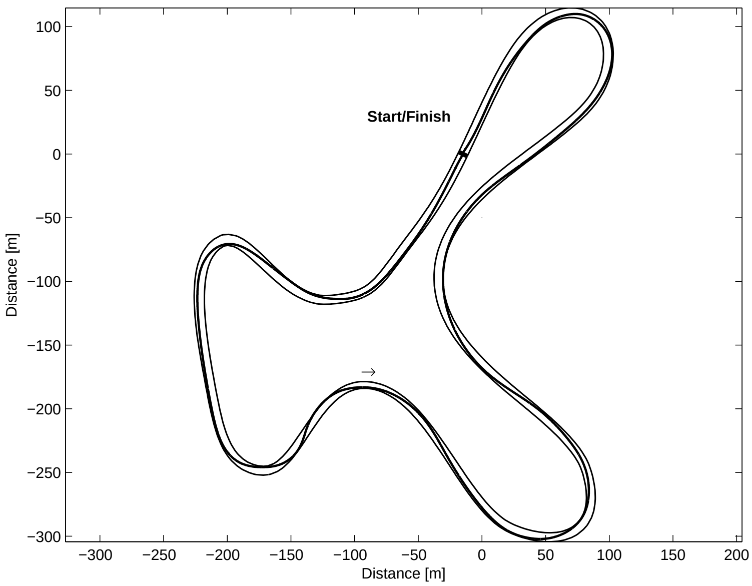


Figure 6.3. The initial guess from Racer when driving at Fernstone.

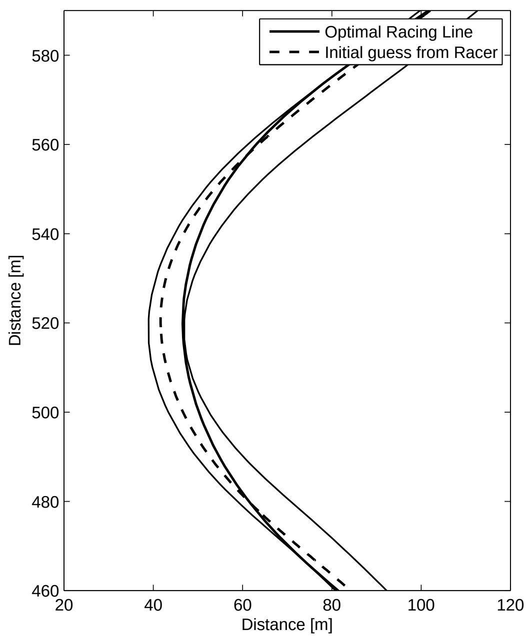


Figure 6.4. Differences between the optimal line and initial guess

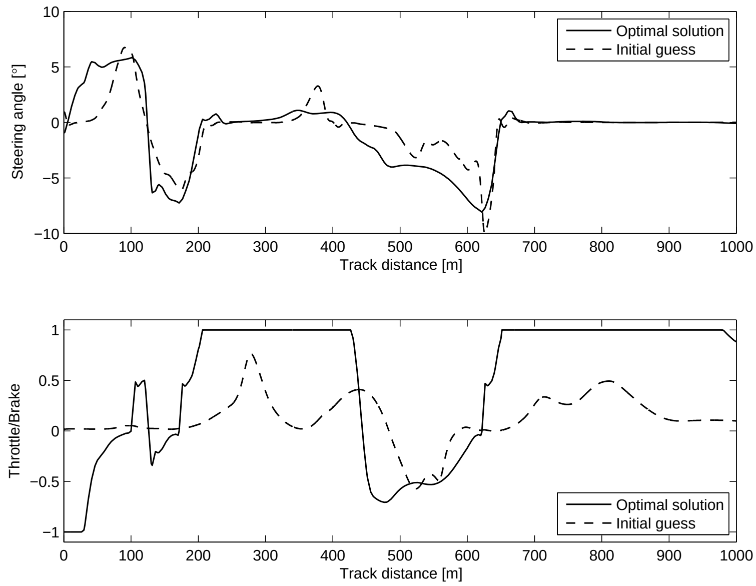


Figure 6.5. Control inputs when comparing initial guess and final solution.

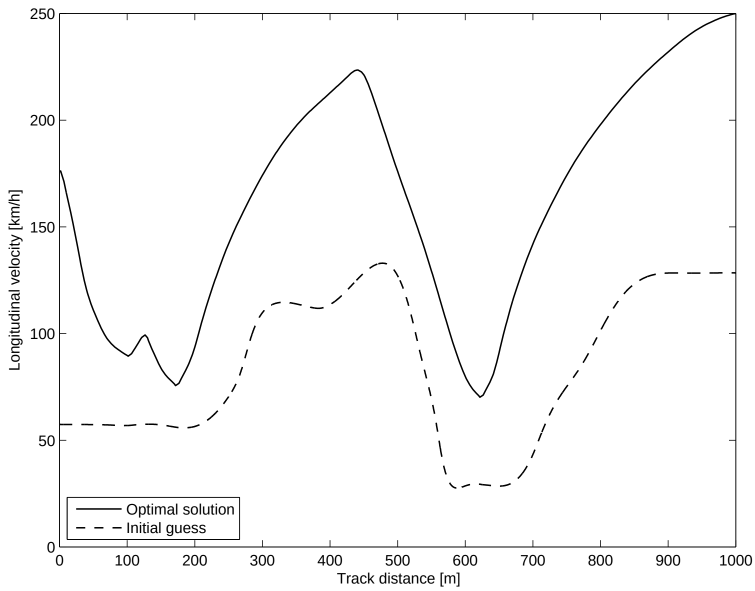


Figure 6.6. Vehicle velocity when comparing initial guess and final solution.

6.1.2 Decoupled road segments

A few simple experiments will give an indication when two solutions with different initial states converge to an identical solution. By varying the starting point the result in Figure 6.7(a) is obtained. It can be seen that the same racing line is obtained after only a few corners. Varying the initial velocity gives similar result in Figure 6.7(b), where the different velocities quickly becomes equal. A similar behavior is observed when different terminal velocities and end points of a segment is used. Figures 6.7(c) and 6.7(d), shows the result when using different end points and velocities. The racing lines do not diverge until the last corner before the end. In some cases the solutions are not exactly equal, which is discussed in Section 6.1.3.

When applying this to an entire track, Brands Hatch is used as an example. The track is divided into five segments according to Figure 6.8. The longitudinal velocity for each segment is shown in Figure 6.9 and the distance from the road center line is shown in Figure 6.10. The segments are chosen to be as long as possible and with sufficient overlap. The overlap is needed in order to find a common point in both segments that is part of the optimal lap. A point is a part of the optimal solution when it is decoupled from the start and end point of the current segment. This occurs when the vehicle state is no longer affected by the segment initial and final states.

To know when two points are decoupled, the following rule of thumb was used:

- Vehicle speed has reached both a local maximum and minimum.
- Vehicle position has reached both the left and right road boundary.

If both the above statements are fulfilled, events before and after this interval are most often decoupled from each other. A road section between two of these intervals is therefore probably a part of the optimal solution and any point can be used to join two segments. The rule of thumb turned out to work well. However, at some occasions the segments can still be joined even though the rule of thumb is not fulfilled, indicating that further studies is needed to evaluate its usefulness. To illustrate the rule of thumb the first two segments of Brands Hatch are used. Figure 6.11 shows the road section where two segments can be joined with respect to velocity. Figure 6.12 shows the road section where two segments can be joined with respect to the driven path. By finding the intersection of these two sections, we get the final interval where the segments can be joined.

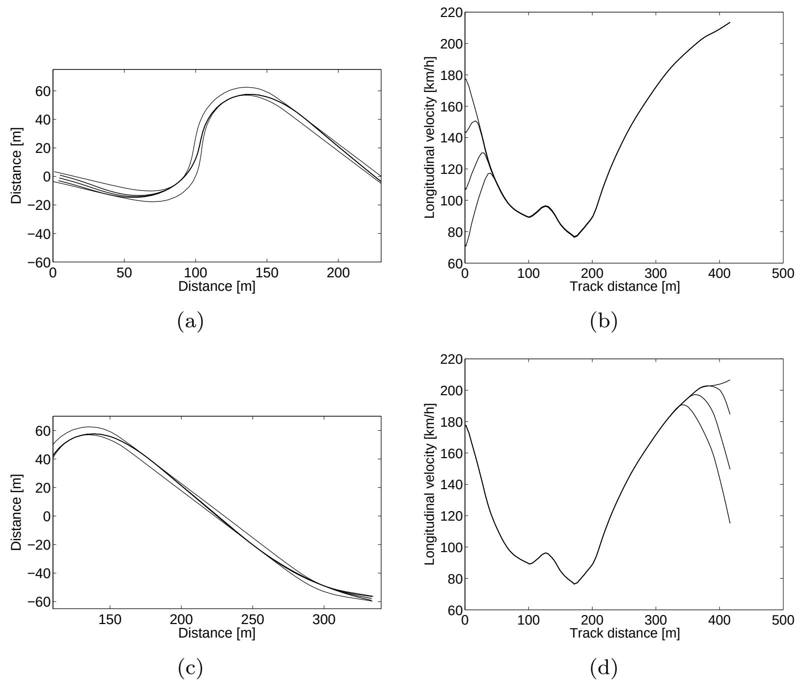


Figure 6.7. The effect of varying the initial position and speed is shown in (a) and (b) respectively. Different final positions and speeds is show in (c) and (d) respectively.

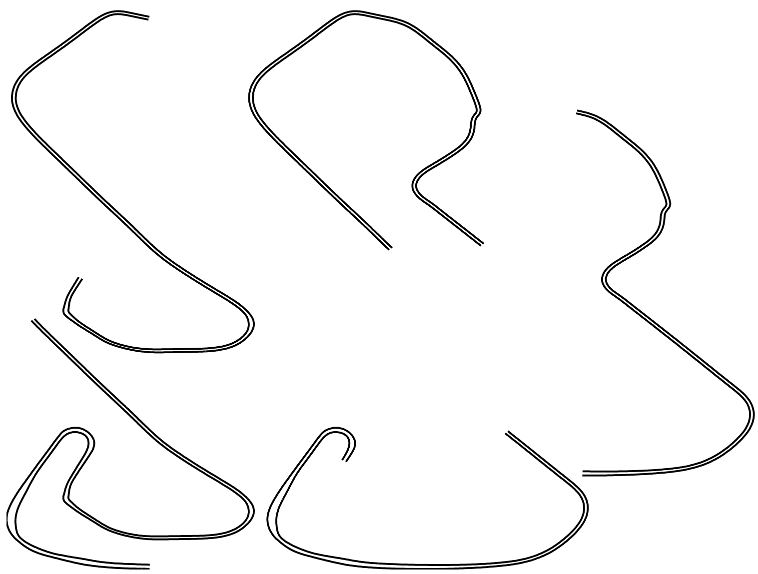


Figure 6.8. Brands Hatch is divided into five overlapping segments.

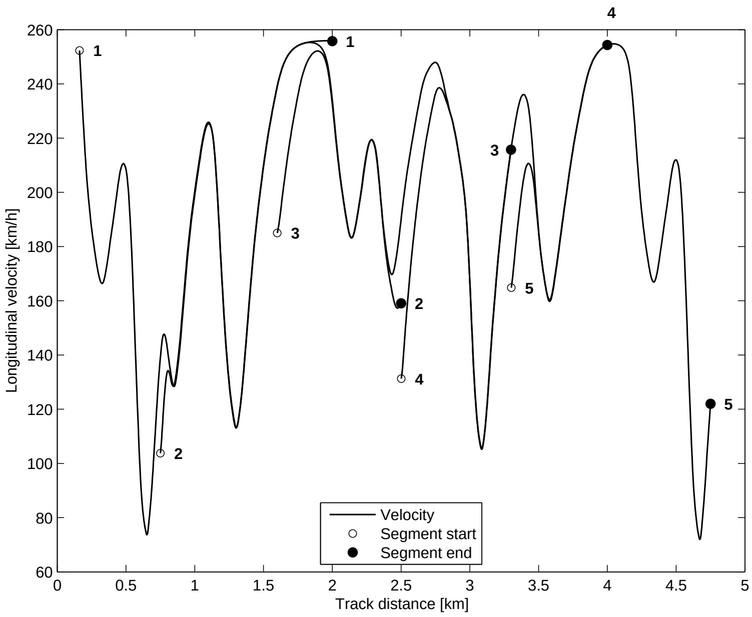


Figure 6.9. The longitudinal velocity when driving at the five segments on Brands Hatch. The fifth segment passes the finishing line and overlaps with the first segment.

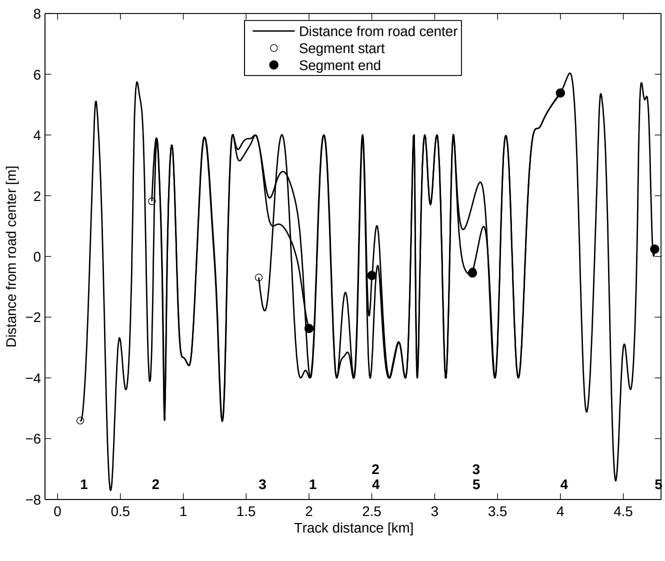


Figure 6.10. Distance from the road center when driving at the five segments on Brands Hatch. The fifth segment passes the finishing line and overlaps with the first segment.

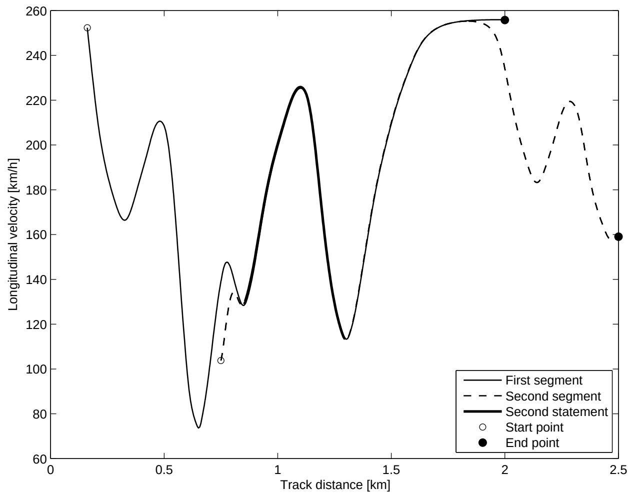


Figure 6.11. The car velocity on two segments, illustrating the first statement of the rule of thumb. The bold line is a road section where the first statement is fulfilled on both sides. Observe that both segments have a maximum and minimum on both sides of the road section.

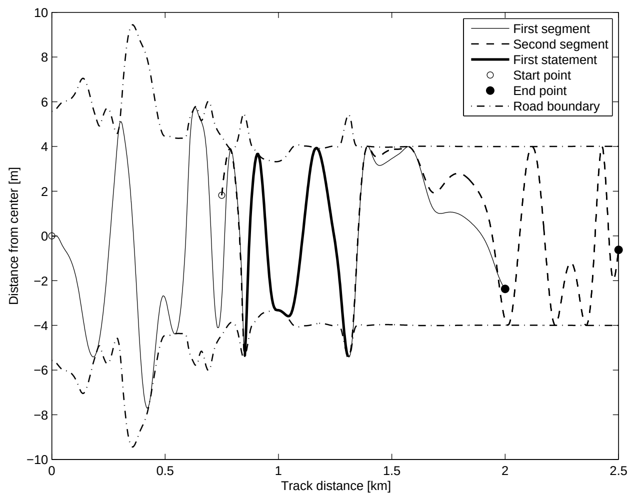


Figure 6.12. The car distance from the road center on two segments, illustrating the second statement of the rule of thumb. The bold line is a road section where the second statement is fulfilled on both sides. Observe that the car touches the left and right boundary before and after the road section in both segments.

6.1.3 Control input penalty

To evaluate the effects of the input penalty we will study a case where penalty is used and compare it against using no penalty at all. The control inputs can be compared in Figure 6.13. It can be seen that the spikes which appear in the case with no penalty are removed using the penalty.

When using the decoupled road segments in Section 5.2, it was discovered that differences between two racing lines could be found in sections where they should have been equal. The differences occurred far away from the starting points where the racing line should have been independent from the initial state of the vehicle. In Figure 6.14, the car distance from the road center line is shown when the car starts at three different positions. The variations are visible after 580 meters which should be too far away to be caused by using different starting points. By reducing the penalties the variations can be made so small that their effect becomes invisible.

The variations is caused by the fact that all points in the control history across the entire track is connected by the cost function, defined in Section 5.1. If the steering derivative is forced to be increased in one section it will most likely decrease somewhere else to avoid an increased value of the cost function. This is depicted in Figure 6.15, where the three cases all have different steering requirements the first 50 meter. Similar results are obtained for the throttle/brake control.

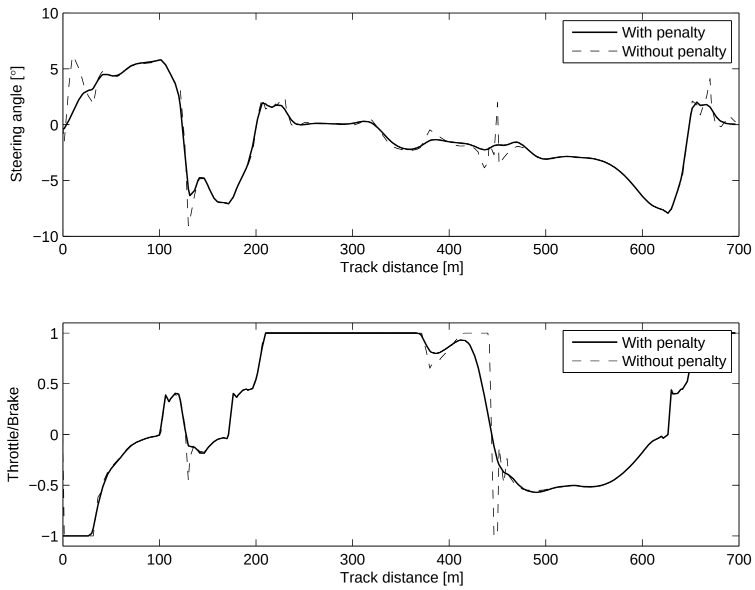


Figure 6.13. Comparing control history with and without penalty.

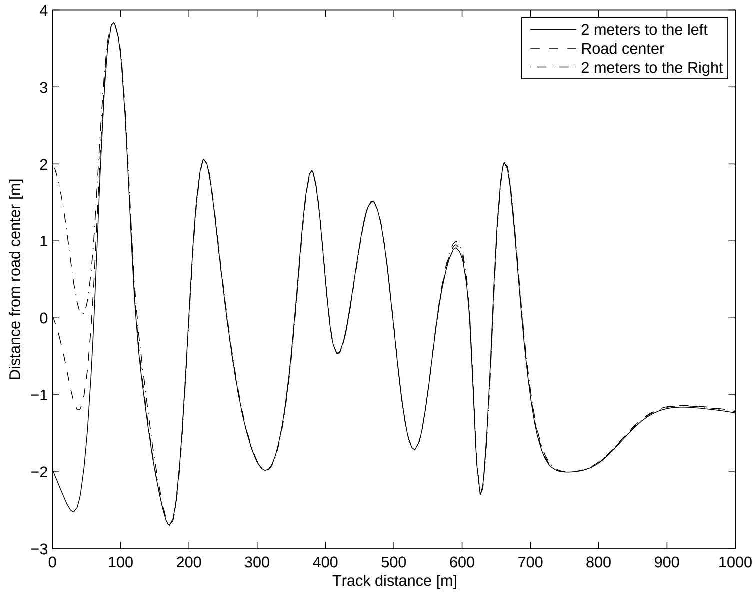


Figure 6.14. Variations of car distance from the road center line with control input penalty.

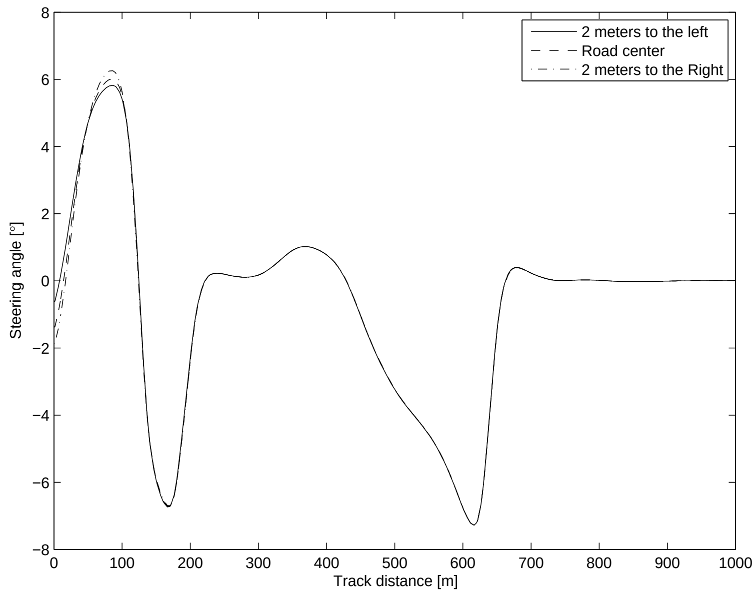


Figure 6.15. The steering angle when starting at different positions.

6.1.4 Grid

Figure 6.16(a), shows how the lap time varies when the grid resolution is changed. This is related to the change in cost function, shown in Figure 6.16(b). The differences between a high and low grid resolution is illustrated in Figure 6.17, where the optimal lines are compared. The impact of having more than 300 points/km is negligible, indicating that this is a good resolution to choose.

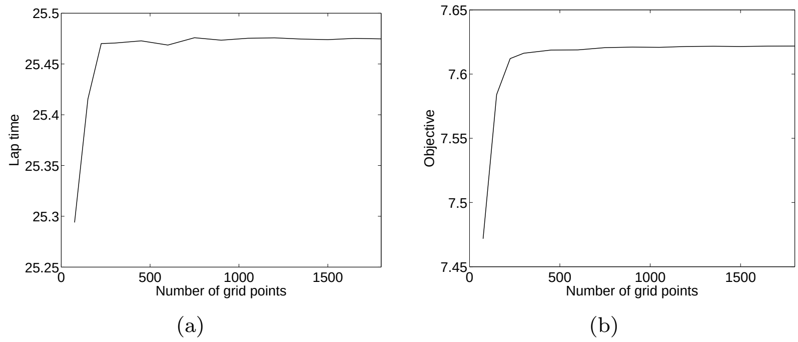


Figure 6.16. The lap times for different grids are shown in (a). The corresponding values of the cost function are shown in (b).

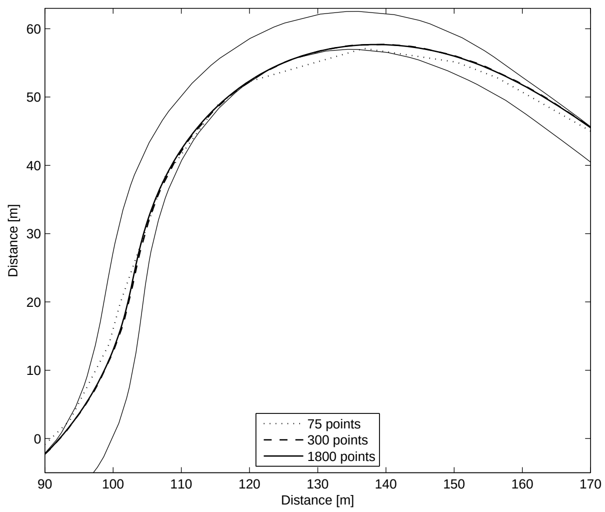


Figure 6.17. Comparing three different grid resolutions.

6.2 Tracks

The resulting optimal racing lines from the different tracks are presented and the result is analyzed by identifying the concepts in Section 5.6.

6.2.1 Brands Hatch

This track is too long to solve directly and requires the use of techniques described in Sections 5.1 and 5.2. The result is composed by five segments, created using the technique deployed in Section 6.1.2. The optimal racing line is shown in Figure 6.18. The control history is shown in Figures 6.19 and 6.20. The lap time is 76.87 seconds. Generally, the optimal line shows a behavior where the car is utilizing the entire track width to achieve large cornering radii.

In curve number two we see a behavior very close trail braking. The control inputs are shown in Figures 6.21 and 6.22. The braking input is gradually decreased before the apex combined with increasing steering input.

There is also a small similarity to a pendulum turn visible as the car steer slightly to the left before the second corner. The throttle/brake control is however not typical for the pendulum turn since the "throttle blip" is not visible. This can also be observed in corner seven and fifteen as a small steering input in the opposite direction of the corner, see Figure 6.23. A similar behavior was observed in (Casanova, 2000) where one hypothesis is that the driver can take advantage of the oscillatory behavior of the car around its yaw axis. Due to the pendulum turn, it is not always the shortest path that is the fastest way to traverse a long straight. This is most apparent at the straights before the seventh and fifteenth corner.

In high speed corners, it is not always necessary to reach the apex at the inside, as shown at turn nine and ten. It seems to be more important to be correctly lined up for the s-curve at turn eleven and twelve, see Figure 6.24. It is also the writer's opinion that this is a good way to pass this combination of curves, since experience from simulations in Racer shows that there is no need to brake until the fourteenth corner, when lined up properly before the s-curve.

There are no clear indications that the late apex is to prefer in a corner preceding a straight. This will be studied further in Section 6.2.4, where the corner is symmetrical and easier to analyze.

There are occasions where the tires are allowed pass the point where optimal grip is obtained. In Figure 6.25, the front right tire is exceeding the limit when entering the second corner of Brands Hatch. In the same corner the left front tire is far from its limit due to the weight transfer. There is probably more grip gained by increasing the usage of the left tire than what is lost by oversaturating the right one. We can also see that the rear tires have lots of additional grip to give, which indicates that the car is understeered. For further discussion, see Section 6.4.

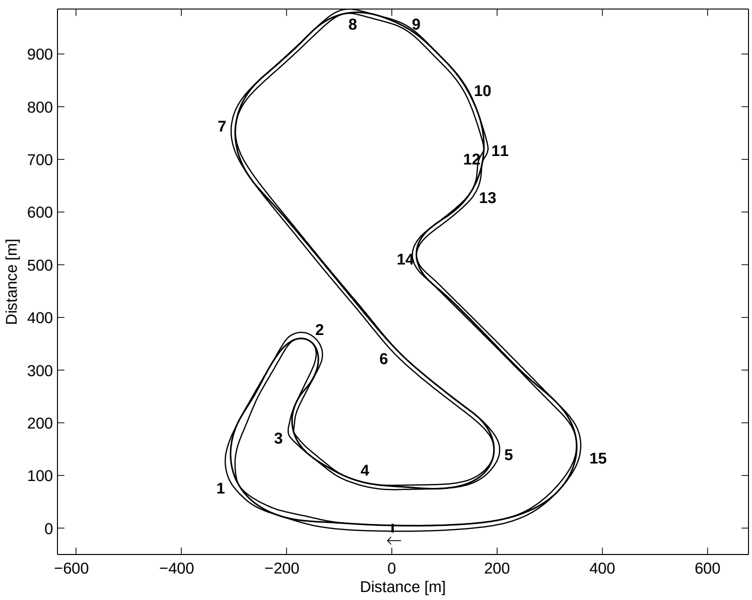


Figure 6.18. The optimal racing line at Brands Hatch. The car is driven clockwise around the track.

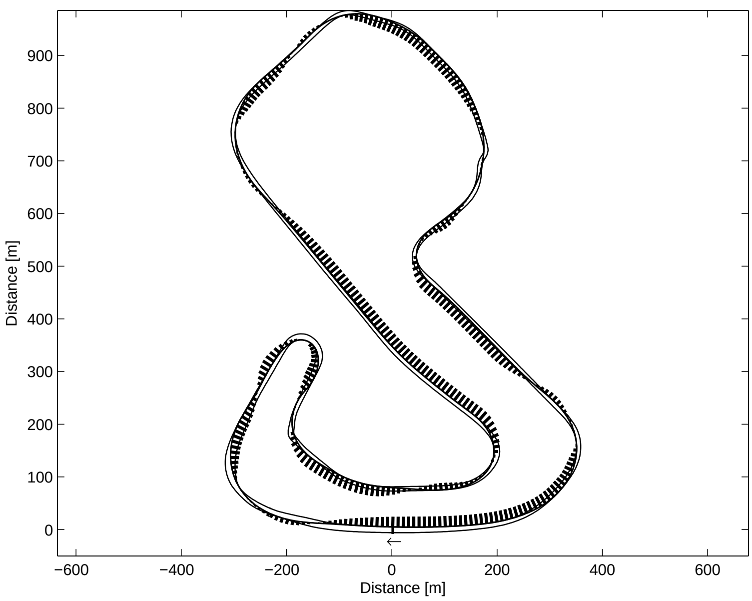


Figure 6.19. The throttle/brake input at Brands Hatch. A marker pointing to the left means that the car is currently braking.

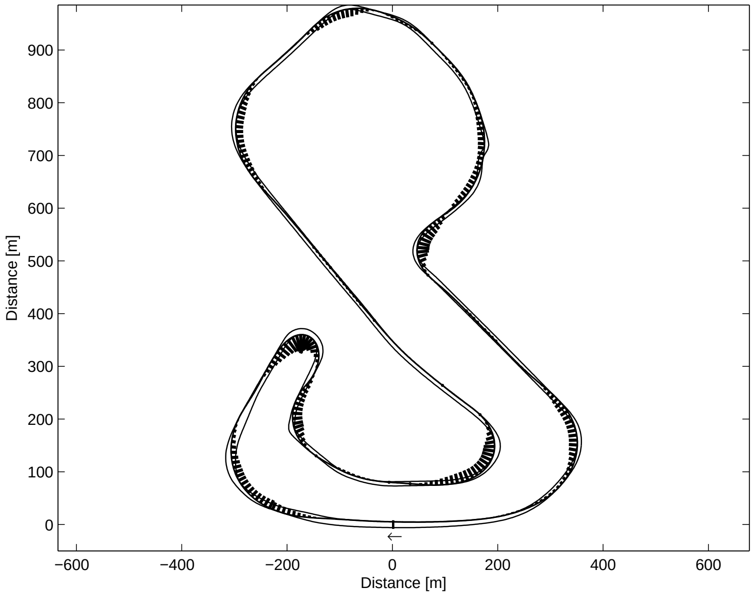


Figure 6.20. The steering input at Brands Hatch. The markers are pointing in the steering direction.

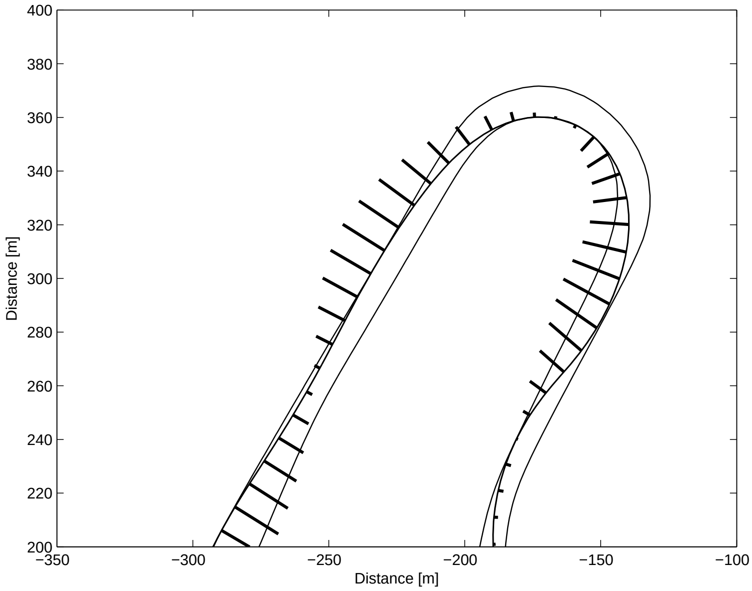


Figure 6.21. The throttle/brake input when trail braking through the second corner. A marker pointing to the left means that the car is currently braking.

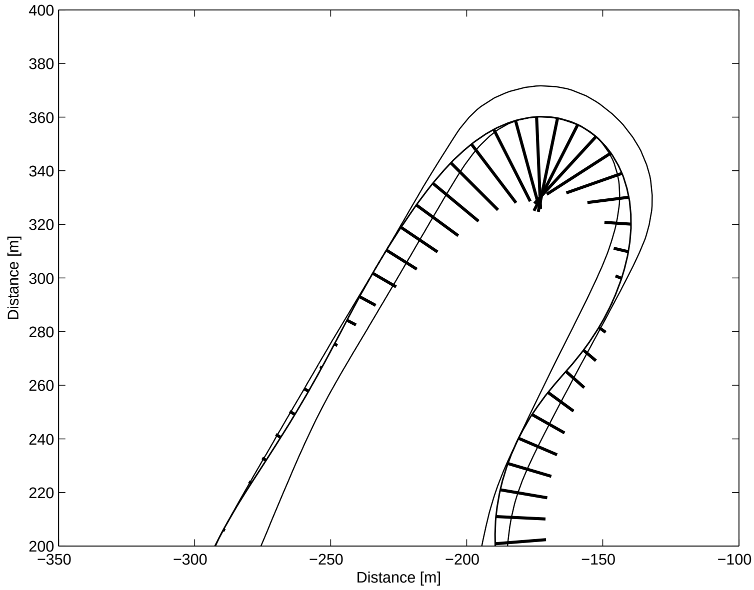


Figure 6.22. The steering input when trail braking through the second corner. The markers are pointing in the steering direction.

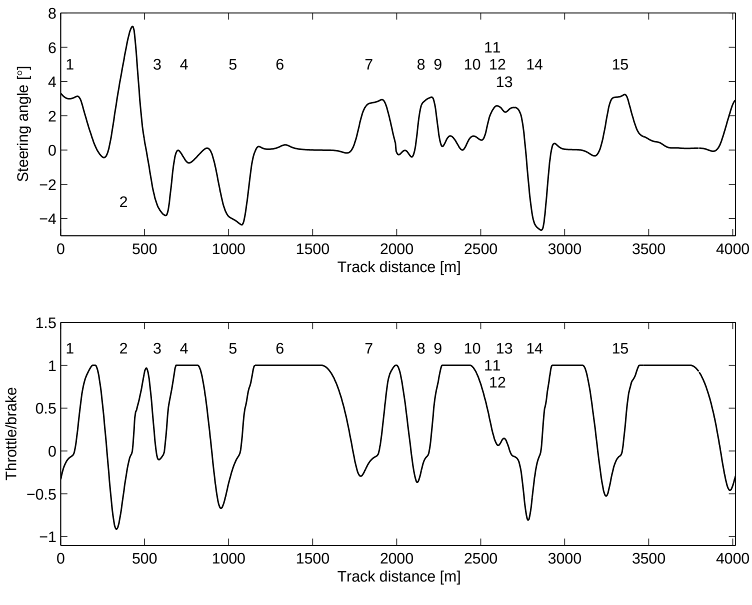


Figure 6.23. The control input history at Brands Hatch. The numbers 1 to 15 are the corner numbers. Observe that the car is initially steering away from the fifteenth corner.

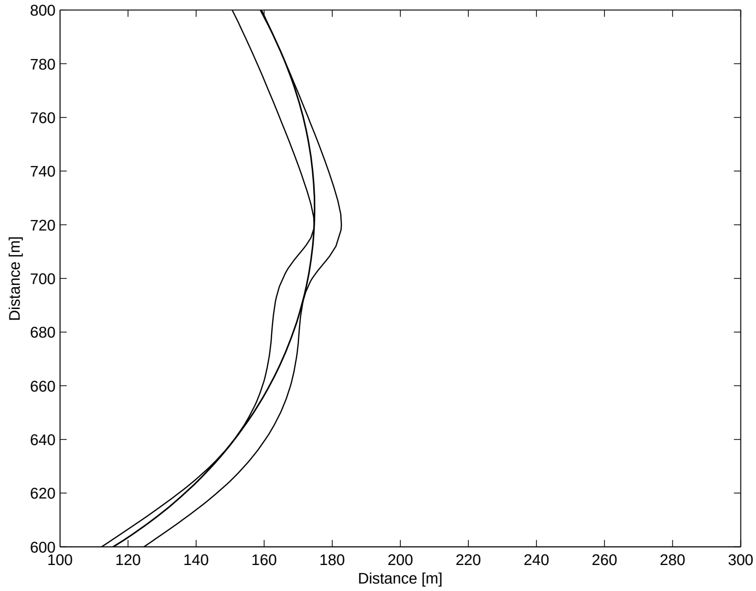


Figure 6.24. The S-curve at Brands Hatch.

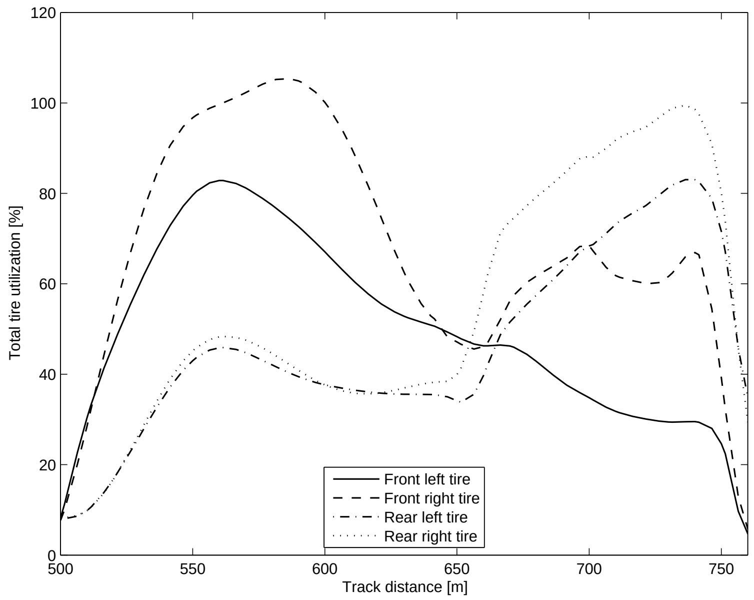


Figure 6.25. Tire utilization when entering the second corner.

6.2.2 Fernstone

The optimal line at Fernstone is shown in Figure 6.26 and the control inputs are shown in Figure 6.27. The optimal lap time is 44.42 seconds. Since this track has no straight, the car is almost never limited by the engine. The tire utilization is therefore higher than at Brands Hatch, see Figure 6.28.

The late apex seems to be important when a corner is quickly followed by another corner in the opposite direction. This can be observed in the first and third corner in Figures 6.29 and 6.30. The center of a corner can be hard to find at a track with non-symmetric corners with variable radius. The late apex will therefore be examined further in Section 6.2.4 for a custom made track.

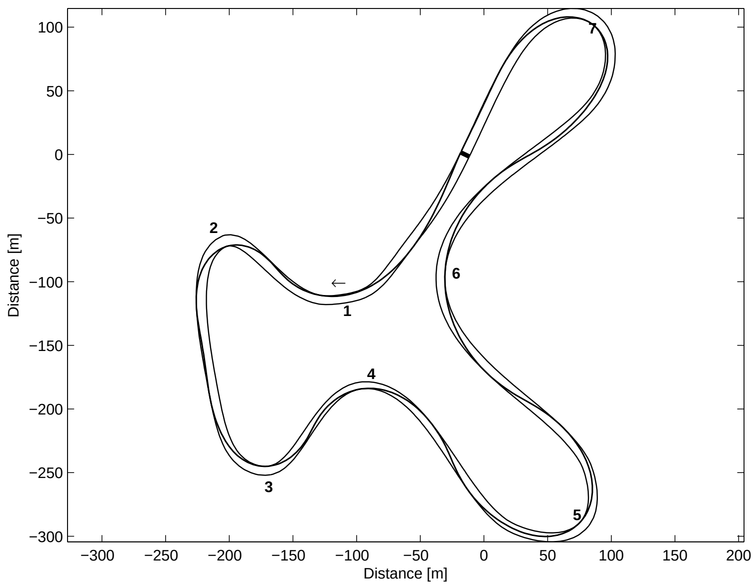


Figure 6.26. The optimal line at Fernstone.

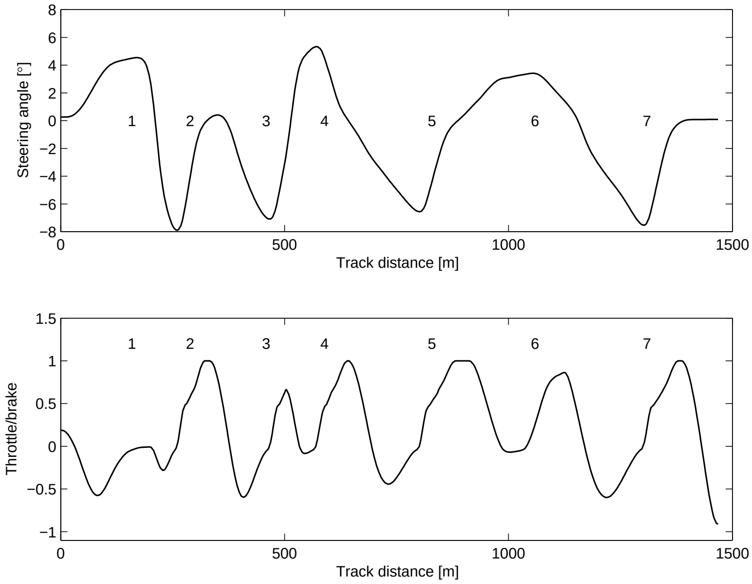


Figure 6.27. The control inputs at Fernstone.

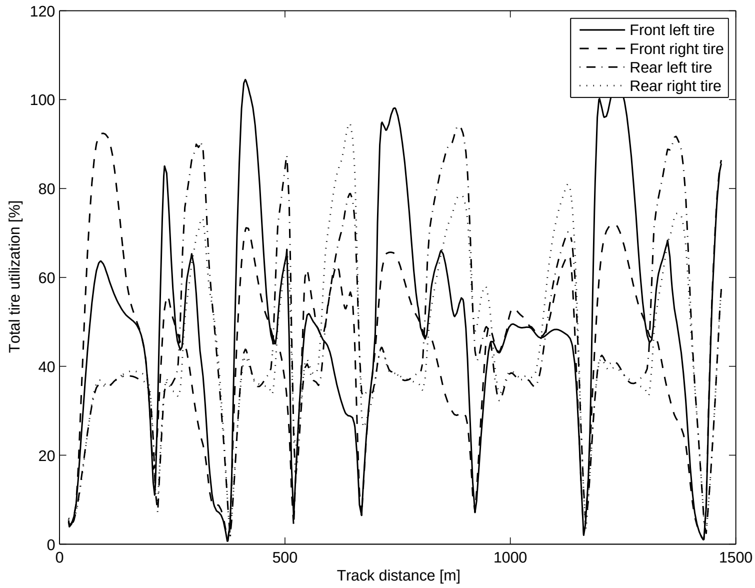


Figure 6.28. The tire utillization at Fernstone.

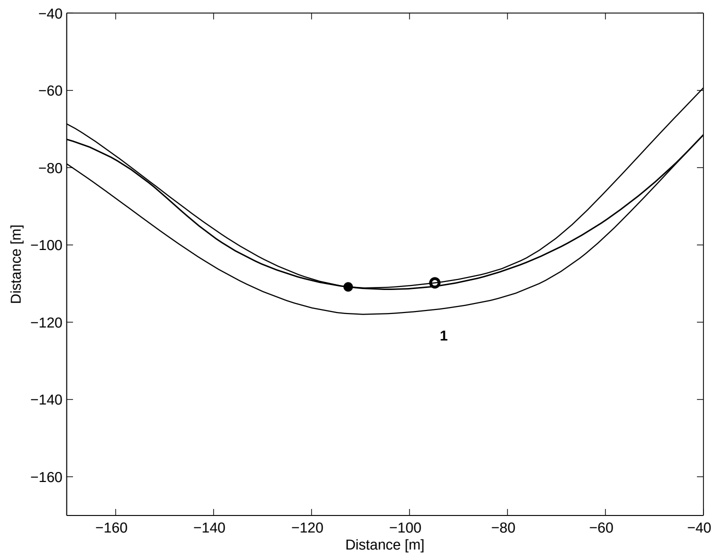


Figure 6.29. First corner of Ferstone where the center apex is marked by a circle and the late apex is marked by a dot.

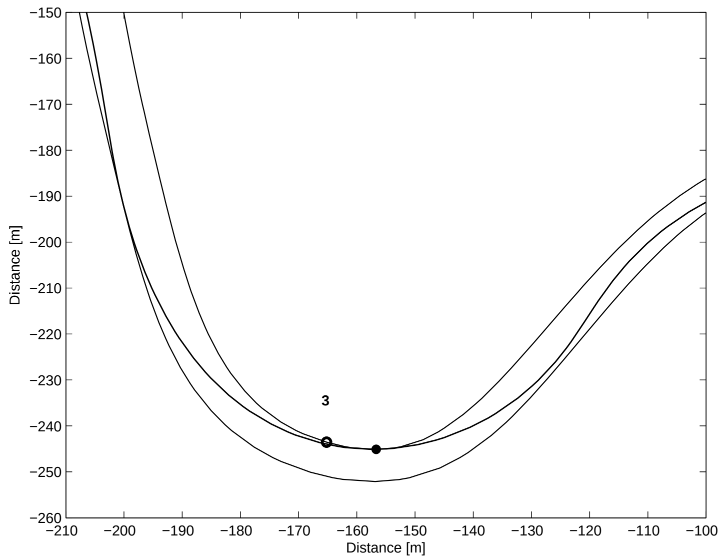


Figure 6.30. Third corner of Ferstone where the center apex is marked by a circle and the late apex is marked by a dot.

6.2.3 Sviestad

In Figure 6.31, the optimal line for Sviestad is shown. The steering and throttle/brake control is presented in Figures 6.32 and 6.33. We can see that trail braking is used in the first and fourth corner. In the third corner, enough grip is available to pass the corner without braking. Directly after the third corner, it is apparently important to reach the left side as fast as possible, to prepare for the last corner.

To examine the benefits of trail braking, the driver will be forced to be more careful. This means that the driver will avoid applying the brakes while turning. The result is shown in Figures 6.34 and 6.35. This driving style is also called straight-line braking and is clearly visible when braking before the last corner. The driven line has been straightened up to allow for a safe deceleration. Straight-line braking gives a lap time of 25.94 seconds compared to 25.0 seconds when trail braking.

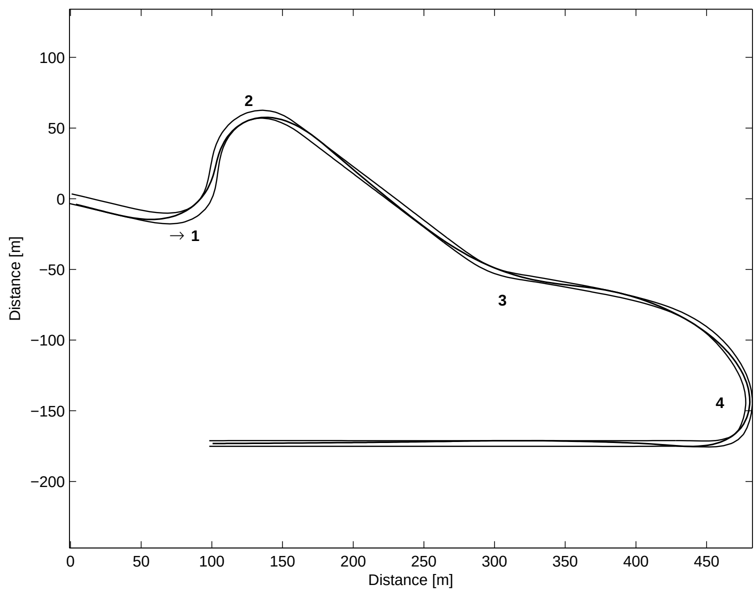


Figure 6.31. The optimal line at Sviestad.

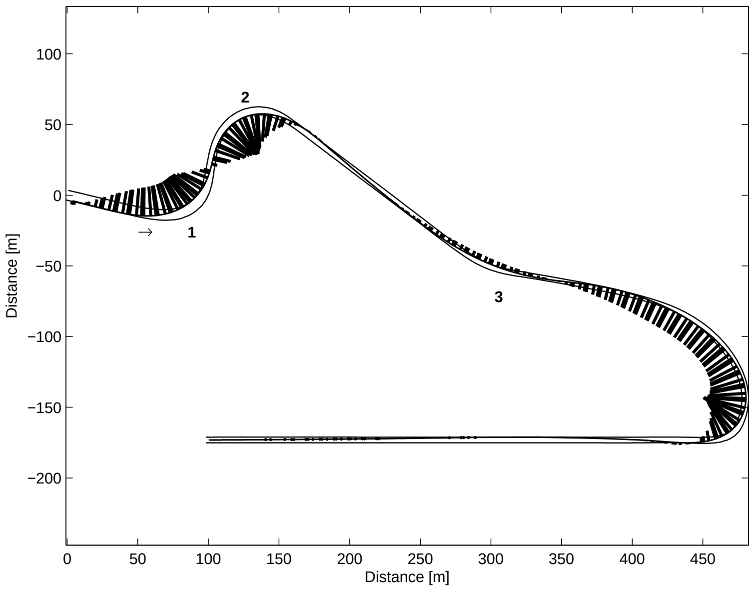


Figure 6.32. The optimal steering control at Sviestad. The markers are pointing in the steering direction.

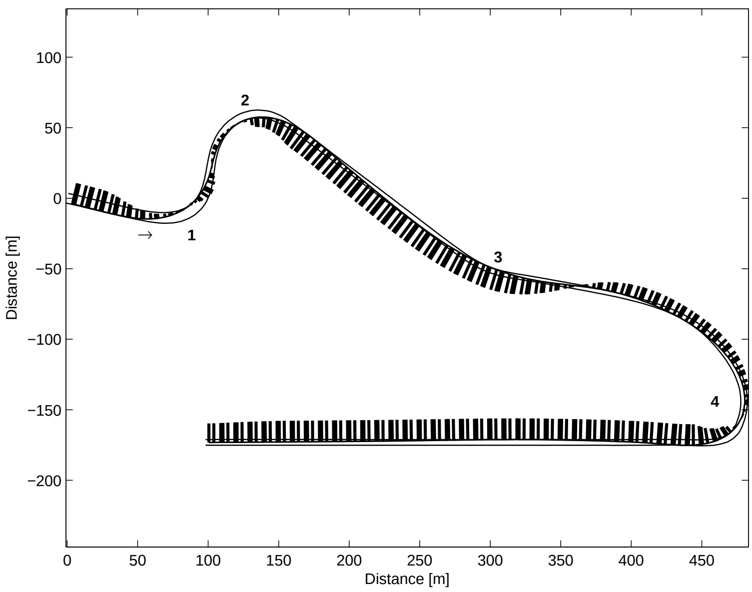


Figure 6.33. The optimal throttle/brake control at Sviestad. A marker pointing to the left means that the car is braking.

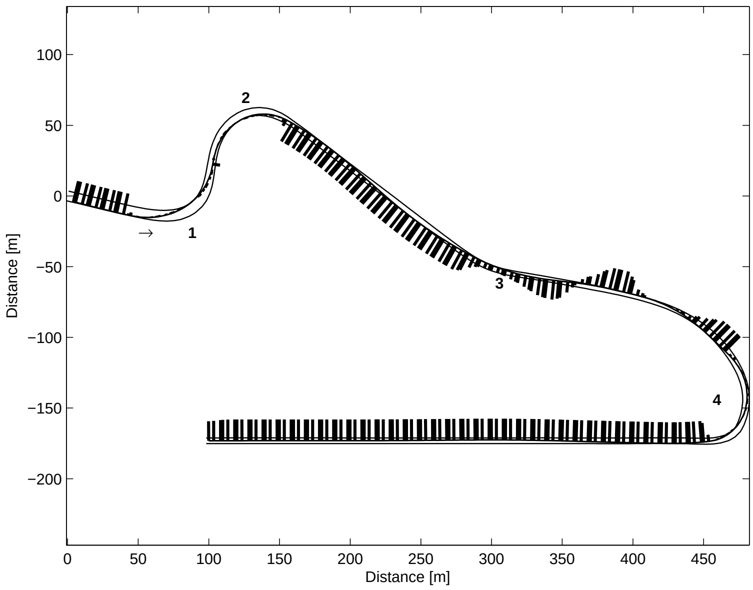


Figure 6.34. The throttle/brake control at Sviestad without trail braking. A marker pointing to the left means that the car is braking.

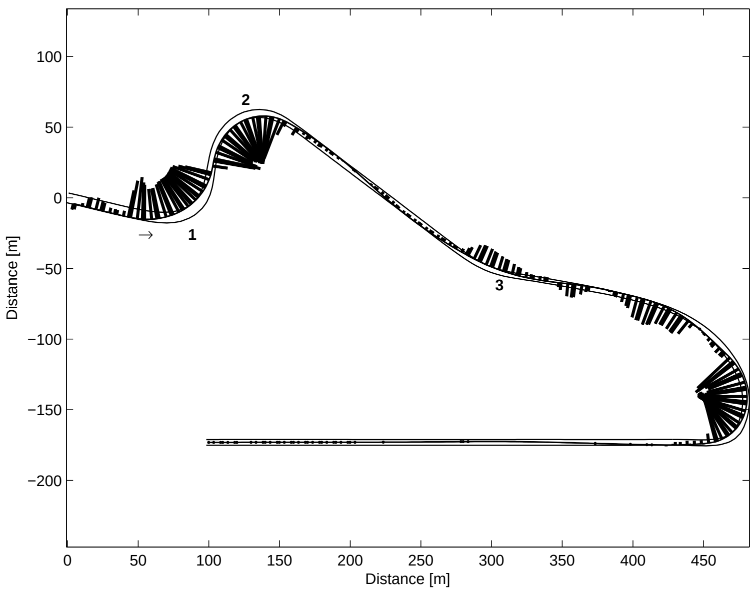


Figure 6.35. The steering control at Sviestad without trail braking. The markers are pointing in the steering direction.

6.2.4 The Hairpin turn

The optimal line in the Hairpin turn is shown in Figure 6.36. Here we can see the result of trail braking. The trajectory through the corner has a parabolic shape instead of a half circle, as in the case when maximizing the cornering radius.
In Figure 6.37, we can see that after hard braking, decreased brake input is combined with increased positive steering input. The positive steering input is in fact started during the hard brake. This is an indication that braking is not enough to exceed the tire limits at this speed. To verify this, the braking capability of the car is changed. Increased maximum brake torque reduces the use of a pendulum turn. Reducing the available brake torque gives the opposite effect.

There is no clear indication that a late apex gives an advantage, regardless how long the succeeding straight is. The result from Fernstone indicates that a late apex is to prefer if something after the corner is forcing the car to take a line with smaller radius, exiting the corner on the inside. In Figure 6.38, the corner exit is obstructed to enforce a late apex. The same behavior was observed in Section 6.2.2, where a corner is directly followed by another corner in the opposite direction.

To complement the result from Sviestad, we will examine the benefits of trail braking by comparing it to a case where the driver is forced to use straight-line braking. The differences can be observed in Figure 6.39. Without trail braking the driver will start turning later and get a larger cornering radius. The control inputs are shown in Figure 6.40. The new throttle/brake control is shown in Figure 6.41(a). Here we see that braking must start earlier without trail braking. In Figure 6.41(b), the steering is initiated later since the driver must wait until he is done braking. Figure 6.42 shows where time is gained when using trail braking. Trail braking is actually slower near the apex due to a lower minimum speed. But time is regained during braking and accelerating.

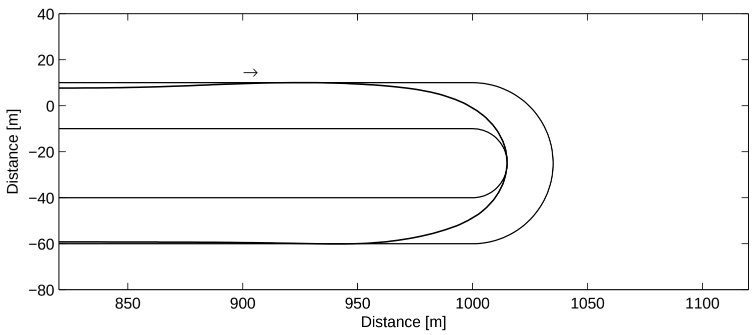


Figure 6.36. The optimal racing line in the Hairpin turn.

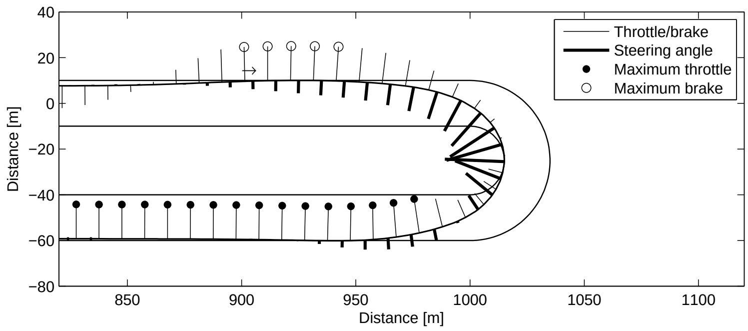


Figure 6.37. Throttle/brake and steering control in the Hairpin turn.

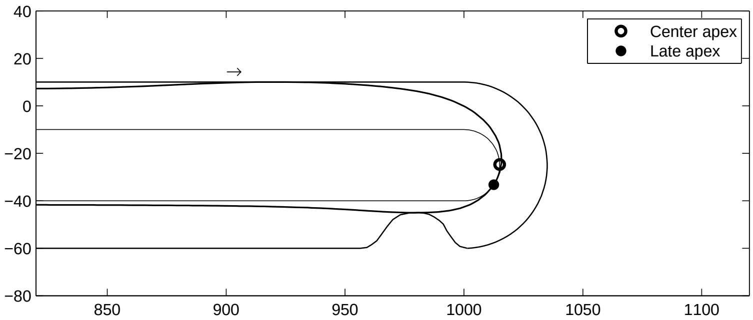


Figure 6.38. Late apex due to obstructed corner exit.

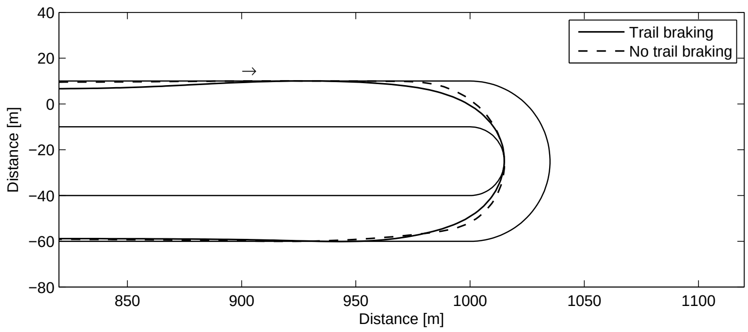


Figure 6.39. Differences in the driven line between trail braking and straight-line braking.

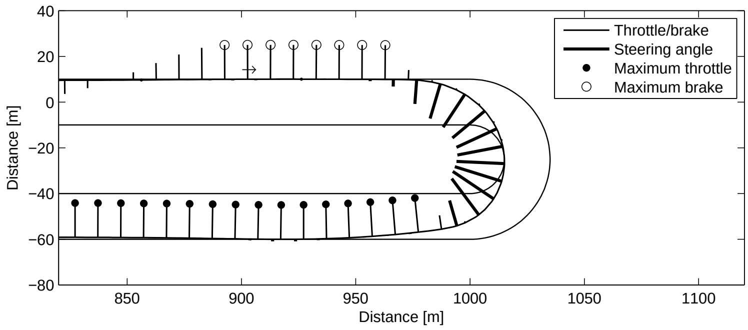


Figure 6.40. Control inputs without trail braking.

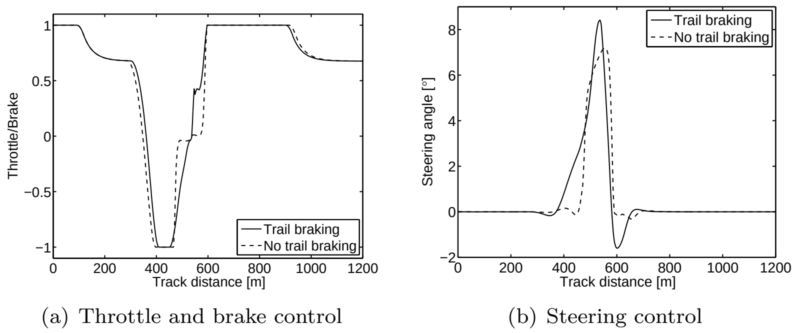


Figure 6.41. Differences in control inputs between trail braking and straight-line braking.

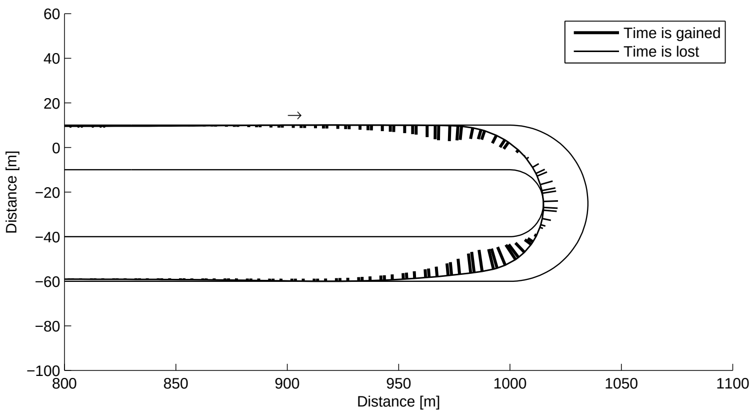


Figure 6.42. This figure shows where time is gained with trail braking.

6.3 Different Car

The optimal line for the Ferrari 360 Modena is shown in Figure 6.43. The line driven by the Modena is generally of larger radius. The largest difference is shown in Figure 6.44, where the start/finish straight is driven at the left side to increase the radius at the first corner. It is possible that the lack of aerodynamic downforce require us to drive more carefully. This car also has situations when oversaturated tires are beneficial. But now this occurs at the rear tires when accelerating after a corner.

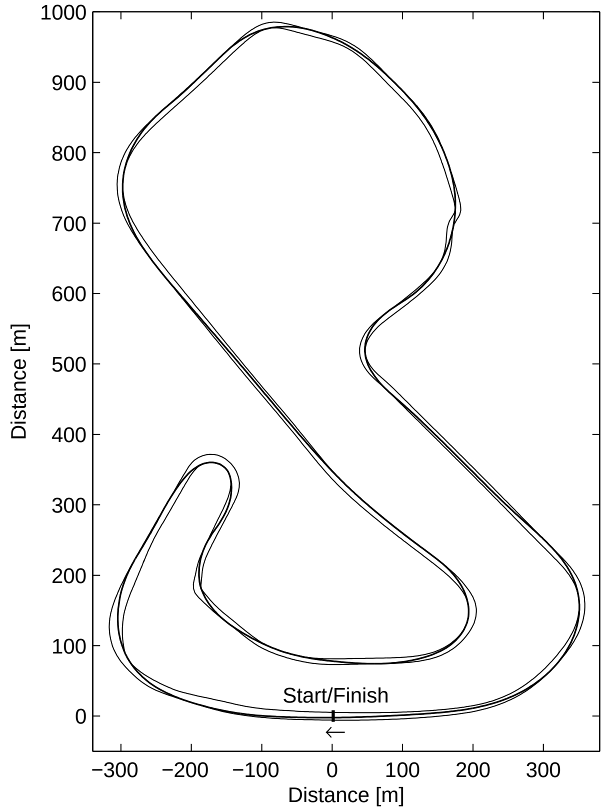


Figure 6.43. The optimal line at Brands Hatch when driving a Ferrari 360 Modena.

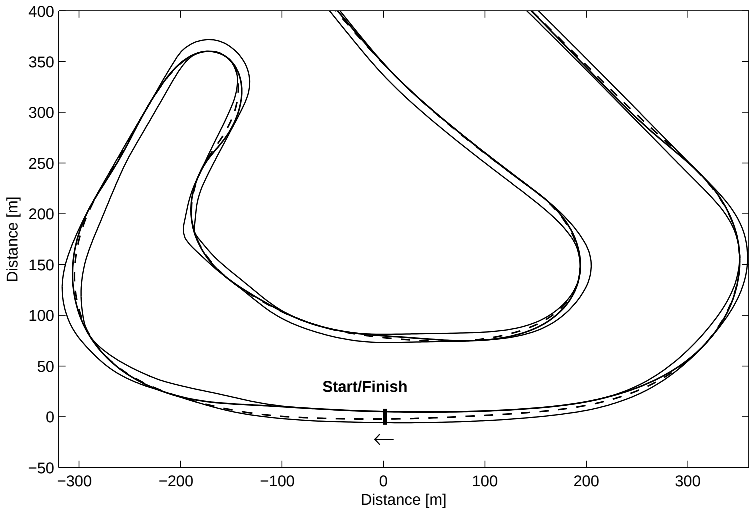


Figure 6.44. The straight after the fifteenth corner is driven on the left side by the Modena to enter the first corner more smoothly.

6.4 Car tuning

The optimal car setup is highly dependent on the track. For example, higher wing angles give higher cornering speed but lower top speed. This means that a track with many corners will benefit from high wing angles while a track with long straights require lower angles due to the increased air resistance. Since the result is track dependent the technique described in Section 5.2 cannot be used. This would give different optimal parameters for different segments. We will therefore only use the first two kilometers of Brands Hatch.

The car to be tuned is the Ferrari 333 SP. In Racer, this car is understeered for the default setting. The OU-properties of the car are tested by driving the car on a circle of constant radius and studying how the steering angle changes when the speed is increased. If an increased speed requires a higher steering angle, the car is understeered, while if the opposite is true, the car is oversteered. The result of the optimized setups is shown in Figure 6.45.

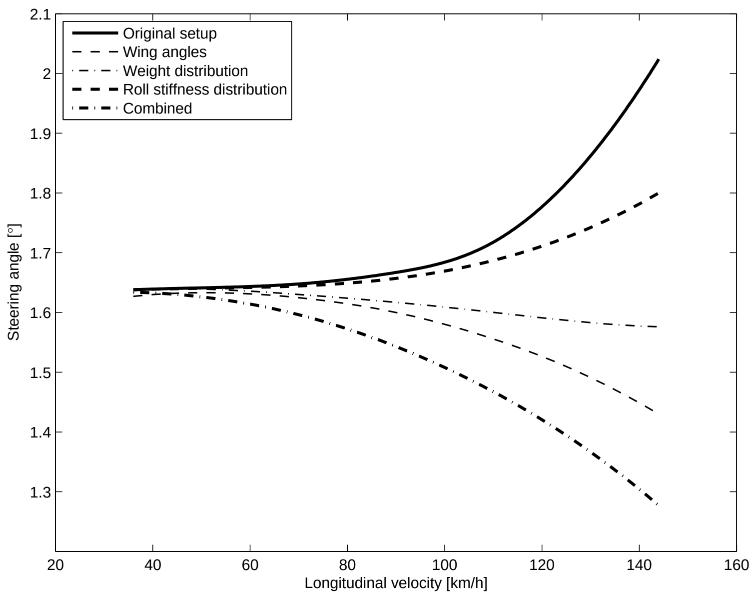


Figure 6.45. The required steering angle to maintain a fixed cornering radius when changing speed.

Wing angle

When optimizing the wing setup, the new angle of the front wing is at its maximum value of 40°. This indicates that straights are not long or fast enough to limit the cars performance by the increased air resistance. The rear wing should have an angle of 34°, showing that it is more important to have a well-balanced car than just pure grip, since increasing this angle would increase understeer. The setup results in a slightly oversteered car.

Weight Distribution

Weight is moved towards the rear wheels and a slightly oversteered car is obtained, which accords with the theory presented in Section 5.3.2.

Roll stiffness distribution

The roll stiffness is moved to the rear wheels as much as possible but the understeer is only slightly reduced. This property has apparently not enough effect on the balance of the car to achieve oversteering.

Brake Bias

The original car setup has a brake bias of 55/45. When the brakes are tuned, a brake bias of 40/60 is obtained. The driving style is completely changed where high slip angles are used and countersteering is required to pass a corner. By moving brake torque towards the rear, we get an oversteered car during braking. Even though this setup gives a faster lap, the optimal racing line would be hard to achieve by a human driver and furthermore the tires would quickly be worn out.

In addition, the brake bias was also optimized when the original car is heavily oversteered. Now the new optimal brake bias becomes 59/39. This means that brake torque is now moved forward to reduce oversteer.

Combined

Wing angles, roll stiffness and weight distribution is optimized simultaneously and gives an even more oversteered car. None of the parameters reaches their limits. The optimized car has reduced the lap time by 12% which is lower than any lap time achieved when the parameters was optimized separately. Test drives in the car simulator with this setup, shows the same order of magnitude in time reduction.

6.5 Implementation

Because of the representation and implementation of the track in AMPL, a lot of memory is required. When optimizing the first kilometer of Brands Hatch, the software requires more than twice the amount of memory than when optimizing the first kilometer of Sviestad. Note that both of these nonlinear programming problems have the same size in terms of the number of variables and constraints, since the number of grid points are equal and the same vehicle model is used. The only difference is the number of splines used in the track model. The current memory usage in Ipopt is over 2 GB when the entire Brands Hatch is loaded. To avoid increasing the hardware requirements, the number of splines needs to be kept at a minimum. A possible solution is discussed in Section 8.2.

The entire vehicle model can be implemented as a state space model to minimize the number of variables required. However, this will result in very complex expressions that significantly increases the time required to compute each iteration. The vehicle model is therefore implemented in a DAE form:

$$\begin{aligned} \dot{x} &= f(x, y, u) \\ y &= g(x, y, u) \end{aligned} \tag{6.1}$$

which leads up to a model where the equations in Section 4.1.1 can be used directly, see Kunkel and Mehrmann (2006) for more information about the DAE form. When the entire lap at Brands Hatch is solved, the nonlinear programming problem consists of about 120000 variables and the same amount of constraints. The maximum time to solve a segment takes about 10-15 minutes and requires at most 100 iterations. The largest segment is two kilometers long.

The constraint responsible for keeping the car on the track was originally written as $d^2 \leq (\frac{w_t - w_{\text{car}}}{2})^2$, where w_{car} is the car width. To obtain linear constraints it was replaced by $d \leq (\frac{w_t - w_{\text{car}}}{2})$ and $d \geq -(\frac{w_t - w_{\text{car}}}{2})$.

Chapter 7

Conclusions

In this chapter the results are discussed.

7.1 Optimal Racing Line

Trail braking is very useful to reduce lap times. In Section 6.2.3, the gain was about one second when driving a section of one kilometer. If this is translated to Brands Hatch it would give about four seconds a lap. In a race with 20 laps this would definitely be larger than the difference between first and last place.

The result shows that a late apex is useful when the exit of the corner cannot occur along the corner outside. Two examples are when the corner is directly followed by another corner in the opposite direction and when the exit of the corner is narrow. There are no indications that time is gained when using the late apex before a long straight. It is possible that a late apex is more useful when driving other types of cars than those used in this thesis. The tire utilization is in most cases only at its maximum during cornering. This comes from the fact that the car is limited both by its engine and brakes at high speeds.

When driving the Ferrari 333 SP, the pendulum behavior is visible at high speeds, when braking before entering a corner. A similar result was obtained in (Velenis and Tsiotras, 2005), where they believe the purpose is to induce the necessary yaw motion in a more smooth way. In Casanova (2000) it is believed that this can be achieved by utilizing the oscillatory behavior of the car. High speed makes the pendulum turn possible since the wings make sure that the tires are not oversaturated, even when braking hard. Increased braking efficiency reduces the pendulum turn since less lateral grip is left during braking.

The largest difference when changing to the 360 Modena is the use of larger cornering radii. This is probably caused by a combination of reduced grip and increased mass.

7.2 Initial guess

The current vehicle model differed too much from the one used in Racer to allow a direct use of the initial guess from Racer. The driver model is a good method to make the initial guess consistent with our model and it still contains most of the information in the original initial guess. Even though the driver model is not perfect and does not recreate the initial guess exactly, it is still usable. In fact, as the simulations in Section 6.1.1 shows, the initial guess does not need to be good at all. The same optimal solution is still obtained regardless of which initial guess are used. There are however some indications that the number of required iterations may increase when the car velocity is too far from the optimal one. In Casanova (2000), the concept of obtaining the initial guess from real car telemetry is studied. This could be replaced in future works by the artificially created initial guess used in this thesis.

7.3 Control penalty

The control penalty are currently necessary but should be avoided in the future. It is clear that the solution is affected in numerous ways. The driver is prevented from being infinitely fast but is sometimes to slow. A possible reason is that the control derivative is calculated with respect to distance and not time. An alternative solution is proposed in Chapter 8. It is possible that the control penalty makes it possible to find a solution even though the initial guess is poor. The control penalty could therefore still be useful as a technique to improve the initial guess. Another side effect is that road segments are never truly decoupled. This affect the accuracy of the technique used in Section 5.2.

7.4 Decoupled road segments

This type of problem turns out to have a natural tendency to easily be divided into smaller subproblems that can be solved independently of each other. The results from these subproblem can be joined together seamlessly by using overlapping segments. The ability to find a solution is significantly improved by dividing the track into shorter segments. The longest segment is half the length of the original track which then is the largest optimization problem that needs to be solved. The result is then lower memory requirements and no limit in track length. Since problem size can be reduced by shortening the segments, the accuracy can be improved by increasing the grid density. If the introduction of more complex vehicle and track models yields new convergence issues, even shorter segments can be chosen in order to solve these issues. Since all segments can be solved independently, the computations can be performed in parallel to reduce the time required to find a solution.

7.5 Car tuning

The car setup can be optimized by using optimal control and treating the properties to be optimized as constant input variables. Several properties can be optimized simultaneously to obtain a better result than if each property is optimized one by one. After the optimization of the model parameters described in this thesis, the optimal car setup is a slightly oversteered car which accords well with the standard setup used in racing. However, when optimizing it is important to remember that the car setup must yield an easy car to drive since at the end it is supposed to be driven by a human driver, and an optimal car setup is of no use if the car is impossible to drive. Since the vehicle model is not perfect, the optimal setup is probably only usable as an initial setup. The final tuning must still be done by test driving the real car.

Chapter 8

Further work

There are numerous ways to improve or build upon the work in this thesis. Some suggestions are presented in this section.

8.1 Model

To increase the accuracy of the result, more complex models can be used. Increased model complexity can however lead to new problems such as difficulties to converge to a solution.

Suspension dynamics

Add springs and dampers to get a more realistic roll and pitch behavior. The complexity can be reduced by avoiding individual springs for each wheel. If roll and pitch behavior is separated, the car can be considered as a rotating mass connected to a torsional spring and damper. The mass will rotate around the vehicle pitch or roll axis. The concept of this approximation is illustrated in Figure 8.1.

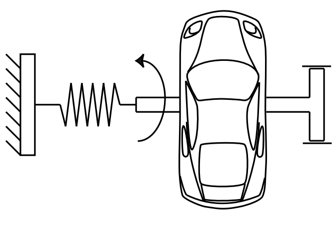


Figure 8.1. Illustrating the idea of modeling dynamic pitch with a spring and damper.

Road Altitude and Banking

If the track is going uphill or downhill it can have a big impact on acceleration and braking performance. The effect could be approximated by adding a force pulling downhill, with a magnitude proportional to the slope angle and vehicle weight. Banked corners will allow higher cornering speeds. The same approximation as above could be applied but the corner radius and vehicle velocity also has an effect.

Dynamic tire modelling

The tire models used so far have been static. Introducing dynamic tire models may give increased accuracy. Possible models are discussed in (Canudas de Wit et al., 2002).

Aerodynamics

The aerodynamic model used in this thesis is rather simple. There are several aspects that could be of interest. The car underbody affects available downforce and the wing performance is affected by the vehicle roll and pitch angles. The wings can also have a lateral effect. The forces resulting from the wings are currently applied directly above front and rear wheel axis, at the height of center of gravity. It would be better to model the wings at their actual positions.

Simultaneous throttle and brake

In this thesis the driver cannot use the brake and throttle at the same time. Sometimes real drivers use the brake and throttle simultaneously when balancing the car through a corner. It is therefore of interest to find out if and when this behavior is optimal.

Gears

To find the optimal gear shifting strategy, it would be useful to introduce real gears. The choice of gear should be treated as an control input along with u_{st} and u_{tb}. This is could be implemented as an integer variable in AMPL.

Tire wear and fuel consumption

In racing, the number of pit stops can totally change the outcome of a race. During Formula 1, two or three pit stops are common depending on your strategy. If you only use two pit stops the car must carry additional fuel and perhaps use harder tires, compared to the three stop strategy. These changes will ultimately affect the driving style since the choice of tires and amount of fuel will affect the car performance. To take this in to account when calculating the optimal lap, you must add models for tire wear and fuel consumption.

8.2 Implementation

Variable grid

When using Optimica, the grid has a constant step size. A variable step size is preferred since it is not always necessary to have a very dense grid. A dense grid is for example not required in the middle of long straights where the throttle is constant and steering angle is zero. A variable step size was successfully used by Casanova (2000).

Fewer constraints

There are some constraints that not always are active. The most important one is $d^2 \leq (\frac{w_t - w_{\text{car}}}{2})^2$, which makes sure that the car stays on the track. When studying any of the optimal racing lines in Section 6.2, it is clear that only at a few points, d is at its boundary. These points occur when the car is reaching the far sides of the road. It is therefore possible that the constraints can be removed in those parts where d is far from it limits.

Reducing spline usage

The way splines are implemented is probably the cause of the huge memory requirements. The splines are implemented by a set of if-statements, which for each grid point selects the correct spline segment, depending on where on the track it is located. When AMPL is sending the problem to Ipopt, the grid points have probably not yet been issued a specific spline segment. The model of Brands Hatch consists of about 650 splines, resulting in an equal number of possible splines associated to each grid point. Since over a thousand grid point is used, it is possible that this is the cause of the large memory requirement.

When the grid has been chosen by Optimica and the AMPL code is created, it is now known for each point to which spline they belong. The if-statements are therefore unnecessary and should be replaced with the correct splines. The resulting code should then only assign one spline to each grid point.

Select startpoint

The current implementation that allows us to choose starting point is rather inefficient. The starting point is selected by locking the segment preceding the starting point to prevent its solution from changing. The segment of track before the starting point does not need to be loaded into AMPL, as done in the current implementation. If only the desired segment is loaded, then AMPL and Ipopt would require less memory. This is especially true when only the last part of the track is of interest, since the entire track is loaded.

Optimal splines

Splines are currently hand made to fit the data as good as possible while minimizing the number of splines required. This could be replaced by an optimal spline fitting algorithm. This algorithm should allow us to choose a maximum absolute error, to achieve the desired accuracy.

Importing track and vehicle

To get closer to a useful system, it must be possible to convert cars and tracks from Racer automatically. One important step is the optimal splines described above. Importing cars should only be a matter of finding the relevant parameters in Racer, read its values and write a file with the correct Modelica syntax. An algorithm to import tracks from flight photos would be useful since this a time-consuming task to do by hand.

Presenting optimal velocity

In addition to displaying the optimal racing line in Racer, it could also be useful to see the current optimal speed. A good way of presenting this would be some kind of heads up display in Racer, perhaps in combination with the speedometer.

Input Penalty

Due to the results in Section 6.1.3, the control input penalties should be avoided, but the noise in the control input should still be reduced. One solution could be to introduce dynamic models for the steering wheel and throttle/brake pedal. The steering wheel could be modeled by a rotating mass affected by the aligning forces of the tire. The new steering input is the torque applied on the steering wheel by the driver. Earlier, the driver was able to instantly choose the wheel angles. The throttle and brake can be modeled using a similar approach. A sliding mass connected to a spring could represent the foot and pedal. The new control input is the force applied to the mass.

Car tuning

Additional parameters to be optimized are for example camber, caster, toe, gear setup, springs and shock absorbers. If the improvements in Section 8.1 are introduced, it can also be useful to find the optimal choice of tires and amount of fuel.

Variable brake distribution

In Section 6.2, it was found that the tires on the inside of a corner was allowed to pass their limits to better utilize the tires on the outside, which currently have better grip. Since braking is involved at these maneuvers, it indicates that a possible way to increase performance would be to transfer brake torque between the left and right side of the car. By introducing this in the vehicle model it would be possible examine if lap times could be improved.

The longitudinal brake bias is normally not optimal with respect to minimizing the braking distance. The purpose is to allow better stability when cornering. If the longitudinal brake bias is adaptive, it would be better suited for both these cases.

Various stability improvements

Absolute value functions should be avoided since they are not differentiable. These can be replaced by using a method described in Betts (2001).

When a rally car was used, it was very hard to find a solution for longer road sections. This type of car has typically high slip angles and high lateral velocities. When studying (2.9), it is clear that cases exist where the equation has singularities. One occurs when $V_x = V_y$ and $\psi_t - psi_v = 45°$. This state is achievable when driving through a tight corner with a rally car.

When the scaling factor was derived in (Casanova, 2000), $V_x \gg V_y$ was assumed. To better support the use of rally cars, the scaling factor should be rewritten to a more general form.

8.3 Theory

Discretization error

To further explain the result in Section 6.1.4, it could be useful to calculate an estimate of the error originating from the discretization.

Decoupled road segments

In this thesis a rule of thumb was presented to find where two segments could be joined. Since no proof was given it would require an mathematical analysis to determine when two points on the track are truly decoupled.

Verifying optimality

A method to verify optimality should be useful. The vehicle model should be validated against a real car and not only to Racer. Telemetry from a real car could then be used to evaluate optimal line obtained in this thesis.

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