

Institutionen för systemteknik  
Department of Electrical Engineering

**Examensarbete**

**Wastegate Actuator Modeling and Tuning of a PID  
Controller for Boost Pressure Control**

Master's thesis  
performed in **Vehicular Systems**

**Andreas Thomasson**

LITH-ISY-EX--09/4232--SE

Linköping 2009



**Linköpings universitet**  
**TEKNISKA HÖGSKOLAN**



# Wastegate Actuator Modeling and Tuning of a PID Controller for Boost Pressure Control

Master's thesis  
performed in **Vehicular Systems**  
**Dept. of Electrical Engineering**  
at **Linköpings universitet**

**Andreas Thomasson**

LITH-ISY-EX--09/4232--SE

Supervisor: **Ph.D. Student Oskar Leufven**

ISY, Linköpings universitet

**Ph.D. Per Andersson**

Charging Controls PDT Tech Specialist, GM Powertrain Sweden

Examiner: **Associate Professor Lars Eriksson**

ISY, Linköpings universitet

Linköping, 11 March, 2009



	<b>Avdelning, Institution</b> Division, Department  Division of Vehicular Systems Department of Electrical Engineering Linköpings universitet SE-581 83 Linköping, Sweden		<b>Datum</b> Date  2009-03-11
	<b>Språk</b> Language  <input type="checkbox"/> Svenska/Swedish <input checked="" type="checkbox"/> Engelska/English  <input type="checkbox"/> _____	<b>Rapporttyp</b> Report category  <input type="checkbox"/> Licentiatavhandling <input checked="" type="checkbox"/> Examensarbete <input type="checkbox"/> C-uppsats <input type="checkbox"/> D-uppsats <input type="checkbox"/> Övrig rapport <input type="checkbox"/> _____	<b>ISBN</b> _____ <b>ISRN</b> LITH-ISKY-EX--09/4232--SE <b>Serietitel och serienummer ISSN</b> Title of series, numbering _____
<b>URL för elektronisk version</b> <a href="http://www.fs.isy.liu.se">http://www.fs.isy.liu.se</a> <a href="http://urn.kb.se/resolve?urn=urn:nbn:se:liu:diva-17207">http://urn.kb.se/resolve?urn=urn:nbn:se:liu:diva-17207</a>			
<b>Titel</b> Wastegateaktuatormodellering och tuning av PID-regulator för laddtryck <b>Title</b> Wastegate Actuator Modeling and Tuning of a PID Controller for Boost Pressure Control  <b>Författare</b> Andreas Thomasson <b>Author</b>			
<b>Sammanfattning</b> Abstract  <p>In some turbochargers, boost pressure is reduced by opening the wastegate valve. In a modern turbo charged car, the most common way for opening the wastegate is with a pneumatic actuator and an air control solenoid, controlled by the ECU. In the control systems studied the ECU utilizes a static feedforward and a PID controller, for the purpose of making the boost pressure follow its reference value. With no systematic method for tuning the controller, this can be time consuming, and a set of well defined experiments to determine PID parameters are desired.</p> <p>When test time in a real engine is limited or expensive, it is advantageous to work in a simulation environment before doing live tests. A model for the wastegate actuator and air control solenoid is developed in the thesis. This is used to simulate controller performance before any tests in a real car is performed.</p> <p>In the thesis a tuning method for the PID controller based on step responses is proposed. The tuning method evaluated is the IMC-choice of controller for a second order system, and it has a single design parameter not given by the experiments. The controller is shown to give desired behavior when the static feedforward is correct or has small error.</p>			
<b>Nyckelord</b> <b>Keywords</b> PID, Wastegate, Boost Control, Turbo Charging			



# Abstract

In some turbochargers, boost pressure is reduced by opening the wastegate valve. In a modern turbo charged car, the most common way for opening the wastegate is with a pneumatic actuator and an air control solenoid, controlled by the ECU. In the control systems studied the ECU utilizes a static feedforward and a PID controller, for the purpose of making the boost pressure follow its reference value. With no systematic method for tuning the controller, this can be time consuming, and a set of well defined experiments to determine PID parameters are desired.

When test time in a real engine is limited or expensive, it is advantageous to work in a simulation environment before doing live tests. A model for the wastegate actuator and air control solenoid is developed in the thesis. This is used to simulate controller performance before any tests in a real car is performed.

In the thesis a tuning method for the PID controller based on step responses is proposed. The tuning method evaluated is the IMC-choice of controller for a second order system, and it has a single design parameter not given by the experiments. The controller is shown to give desired behavior when the static feedforward is correct or has small error.



# Acknowledgments

I like to thank my examiner Lars Eriksson at Vehicular Systems and Per Andersson at GM Powertrain for giving me the opportunity to write this thesis. I would like to thank my supervisor Oskar Leufven for all help in the engine lab and input on the report. I also want to thank Wamidh Kadhim for all the help with the measurements during my visits in Trollhättan. Then I would like to thank Greger Karlströms for the help with the welding and finally my opponent Andreas Myklebust for valuable comments on the report.



# Contents

<b>1</b>	<b>Introduction to boost pressure control</b>	<b>1</b>
1.1	Problem description . . . . .	1
1.2	Limitations . . . . .	2
1.3	Resources . . . . .	2
1.4	Outline . . . . .	3
<b>2</b>	<b>Wastegate model for simulation purpose</b>	<b>5</b>
2.1	Wastegate actuator description . . . . .	5
2.2	Wastegate model identification experiments . . . . .	6
2.2.1	Filtering and alias effects on measured signals . . . . .	7
2.3	Model from actuator pressure to wastegate position . . . . .	8
2.4	Model from duty cycle to actuator pressure . . . . .	9
2.4.1	Models for pressure before and after compressor . . . . .	11
2.4.2	Linear pressure model . . . . .	12
2.4.3	Second degree polynomial pressure model . . . . .	13
2.5	Complete static wastegate model . . . . .	13
2.6	Wastegate dynamic behavior . . . . .	15
2.6.1	Wastegate pressure dynamics . . . . .	15
2.6.2	Wastegate position dynamics . . . . .	15
2.7	Wastegate model simulations with the MVEM . . . . .	18
<b>3</b>	<b>Boost pressure PID controller tuning</b>	<b>19</b>
3.1	Boost controller structure . . . . .	19
3.2	Tuning method for PID controller . . . . .	20
3.2.1	Step responses and process model . . . . .	20
3.2.2	PID parameter identification from process model . . . . .	21
3.3	PID controller modifications . . . . .	22
3.4	PID tuning method summary . . . . .	23
<b>4</b>	<b>Boost pressure control results</b>	<b>25</b>
4.1	Boost controller simulations with the MVEM . . . . .	25
4.2	Boost controller performance in test car . . . . .	25
4.2.1	Further improvements . . . . .	28
<b>5</b>	<b>Future work</b>	<b>31</b>

---

<b>6</b>	<b>Summary and Conclusions</b>	<b>33</b>
6.1	Conclusions . . . . .	33
	<b>Bibliography</b>	<b>35</b>
<b>A</b>	<b>Derivation of parameter expressions and actuator model plots</b>	<b>37</b>
A.1	Derivation of $a$ , $b$ and $c$ in wastegate pressure model 2 . . . . .	37
A.2	Remaining plots from the actuator model chapter . . . . .	38
<b>B</b>	<b>Derivation of PID parameter equations and validation plots</b>	<b>47</b>
B.1	Derivation of expressions for PID parameters . . . . .	47
B.2	Remaining plots from the controller tuning chapter . . . . .	47
<b>C</b>	<b>Nomenclature</b>	<b>51</b>

# Chapter 1

## Introduction to boost pressure control

Turbo charging is today a common way of increasing power output of car engines. By using a smaller turbo charged engine, engine losses are reduced which increases fuel economy while still maintaining the maximum power compared to a larger naturally aspirated engine.

A turbocharger is essentially a compressor and a turbine linked together. Energy from the exhaust gases are led through the turbine providing the energy that drives the compressor, which increases the pressure of the intake air. To control the pressure after the compressor a wastegate valve is used. When the wastegate valve opens, exhaust gases are lead past the turbine thus decreasing the energy provided by the turbine to the compressor, lowering the boost pressure.

Opening and closing of the wastegate valve is controlled with a solenoid valve and a pneumatic actuator. The control signal to the solenoid comes from the car's Electronic Control Unit (ECU). In the case studied in this thesis the boost controller is implemented as a PID controller with a static feedforward term. Having a systematic method for tuning a boost controller will decrease the time needed for calibration. Figure 1.1 shows an overview of the system.

### 1.1 Problem description

The main goal for this thesis is to find a suitable method for tuning a PID controller used for controlling boost pressure. It is desirable if the method can be automated. To make this possible the tuning method should be kept simple and rely on an experiment which is easy to perform, preferably a step response. The method should also rely only on measured signals already available in the system.

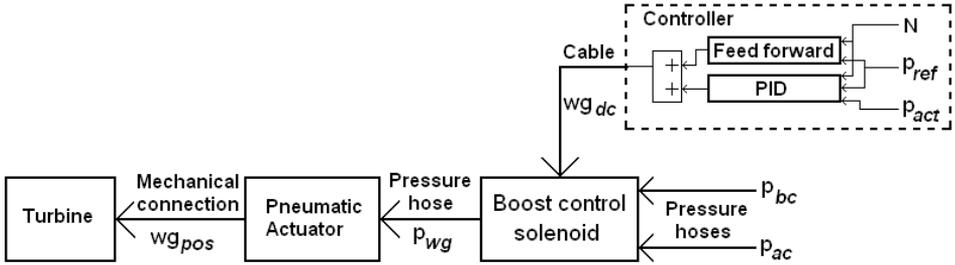


Figure 1.1: An overview of the system studied in this thesis. The wastegate valve on the turbine is mechanically connected to the pneumatic actuator. Actuator pressure is fed by the boost control solenoid, which is controlled by a software boost controller implemented in the ECU. Abbreviations in figure:  $wg_{pos}$  (wastegate position),  $p_{wg}$  (actuator pressure),  $wg_{dc}$  (wastegate duty cycle),  $p_{bc}$  (pressure before compressor),  $p_{ac}$  (pressure after compressor),  $p_{ref}$  (boost pressure reference),  $p_{act}$  (actual pressure),  $N$  (Engine speed). Everything within the broken square are software implementations in the ECU.

## 1.2 Limitations

The controller studied in the thesis is limited to the structure shown in figure 1.1. The static feedforward is assumed to be known and give no or little stationary error. Another limited resource during the thesis work is available time for running tests in the test car.

## 1.3 Resources

Since the time in test car is limited, it is important to be able to do simulations and try different tuning methods before doing tests in the real environment. For engine simulations a Mean Value Engine Model (MVEM) is available. The engine model is implemented in matlab/simulink and the structure of the model is shown in figure 1.2. In the current setup the wastegate opening can only be set to a fixed value. Thus the MVEM needs to be extended with a model for the wastegate actuator and a boost controller to test the tuning method. For more information on the available MVEM see [1, 2, 3, 4].

Tests on real engines have been available in the engine lab at Vehicular Systems and in a test car. Both engines are four cylinder two liter GM engines with turbocharger. Measured data originated from the test car engine have been scaled.

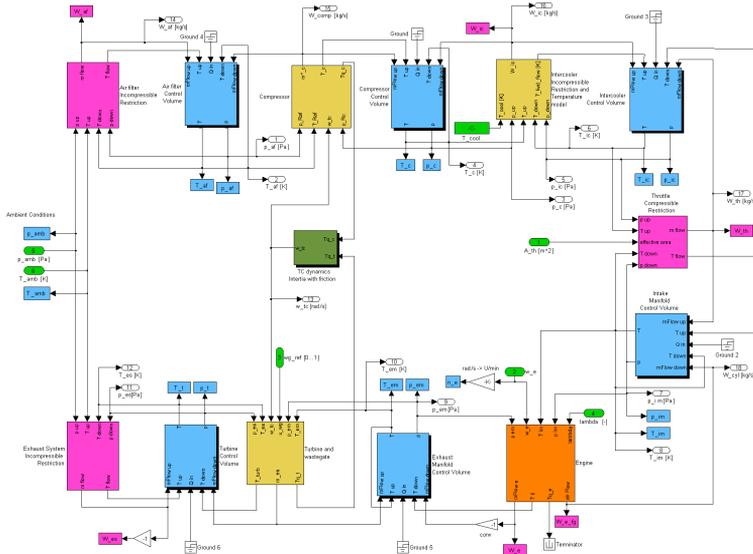


Figure 1.2: The MVEM implemented in simulink. In the current setup the wastegate can only be set to a fixed value. In the thesis a model for the wastegate actuator is developed and used together with the provided MVEM to test the tuning method.

## 1.4 Outline

The report presents an automatic tuning method for boost pressure controllers. To reduce the need for precious test car usage and expensive engine test bed experiments, a model of the wastegate system is developed in Chapter 2. In this case it is sufficient to model the wastegate actuator as the remaining model was already available. The developed model is then used in Chapter 3 and 4 where a systematic PID tuning method is selected and evaluated. During the project several challenges were identified that need more attention, and they are proposed as future work in Chapter 5. Finally, the conclusion and summary is presented in Chapter 6. For nomenclature used throughout the thesis, see appendix C.



# Chapter 2

## Wastegate model for simulation purpose

In this chapter a model for the wastegate actuator is developed. Already implemented in the MVEM is a functionality that has the wastegate opening percentage as an input. The goal for this chapter is therefore to present a model that accurately describes the wastegate position as a result of control signal and engine state. This model is then used in simulation together with the MVEM to test the tuning method presented in chapter 3.

Section 2.2 describes the experiments made to collect data. The modeling work is then split into three main parts. First the wastegate position is modeled as a function of pressure in the actuator which is covered in section 2.3. Secondly, in section 2.4 the actuator pressure is modeled as a function of control signal and pressure before and after the turbine. In section 2.5 the two models are combined to give a complete static model and finally, in section 2.6, dynamic behavior is added to the wastegate position.

### 2.1 Wastegate actuator description

The wastegate flapper valve is mechanically connected to a pneumatic actuator. A sketch of the actuator can be seen in figure 2.1. When the pressure in the actuator rises it will eventually overcome the stiffness of the return spring, opening the wastegate. Actuator pressure is fed by the boost control solenoid which is also connected to the intake system before and after the compressor. The solenoid is controlled by a Pulse Width Modulated (PWM) signal from the Electronic Control Unit (ECU), referred to as wastegate duty cycle ( $wg_{dc}$ ) throughout the report, and the resulting pressure to the actuator ends up between the two pressures  $p_{bc}$  and  $p_{ac}$ .



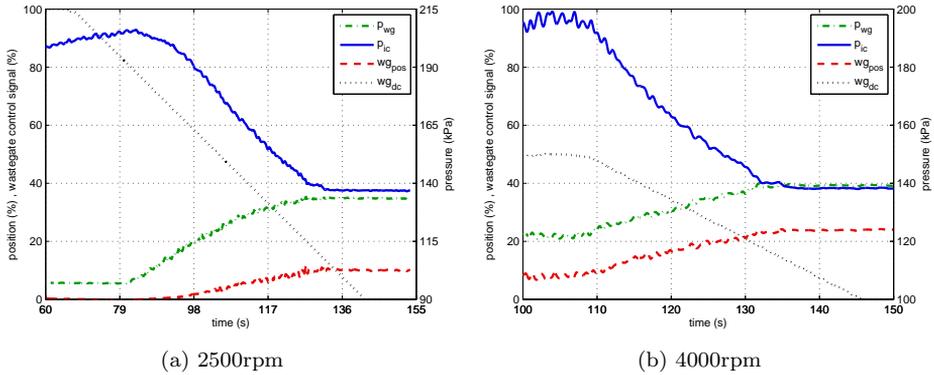


Figure 2.2: Ramp responses for two different engine speeds. The wastegate duty cycle is ramped from the maximum allowed by the control system during normal operation and down to zero. As the duty cycle decreases pressure in the actuator rises. Eventually the actuator pressure overcomes the force of the return spring, the wastegate opens and the boost pressure decreases.

is captured in the measurements. Throughout this section these measurements are treated as static, and even though the behavior is plotted against time it is the dependence on duty cycle which is of main interest. For all these plots the wastegate duty cycle is ramped from the maximum allowed by the control system and down to zero.

### 2.2.1 Filtering and alias effects on measured signals

The original dataset is very noisy and has therefore been filtered. For this task, a low pass butterworth filter with cut off frequency of 2 Hz has been used. This frequency can seem to be low but is justified by the slow ramp and that the purpose of this data is to capture the static properties. The reason the cut off frequency needs to be low lies in the nature of the disturbances. They occur with a frequency which coincides with the opening and closing of the exhaust valves, and for higher engine speeds these disturbances are subjected to alias effects when sampled.

Alias effects appears when the sampling frequency,  $f_s$ , is bellow two times the highest frequency in the sampled signal. For example, a frequency  $f_0 \in [f_s/2, f_s]$  will when sampled appear at a frequency of  $f = f_s - f_0$ . For the measurements in this chapter the signals are sampled at a frequency of 80 Hz. Figure 2.3 shows at which frequency in the sampled signal the real frequency's will appear on for this sample frequency. For more information of sampled signals and alias effects, see [8].

In the case of a four cylinder, four stroke engine, the opening and closing of the exhaust valves causing the disturbances appear at a frequency of engine speed (in rpm) divided by thirty (four cylinders and one opening every second revolution results in a frequency of four divided by two times engine speed divided by sixty).

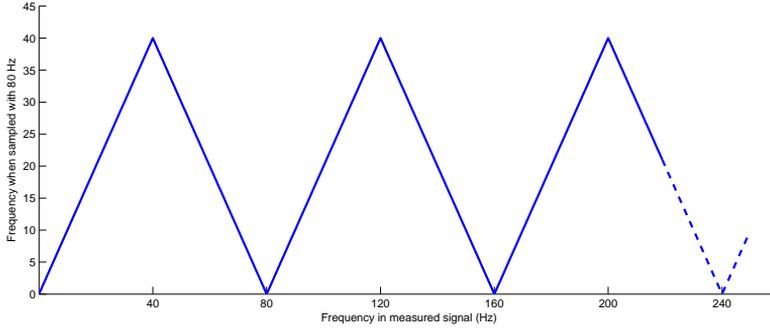


Figure 2.3: The frequency as it appear in the sampled signal as a function of the real frequency when sampled with 80 Hz. For an engine speed of 2500rpm the frequency of the disturbance is  $\approx 83.3$  Hz which appears as 3.3 Hz in the sampled signal.

For an engine speed of 2400 rpm this becomes 80 Hz. With a sample frequency of 80 Hz this will appear as a stationary error and for engine speeds close to multiples of 2400 rpm as a low frequency disturbance. For the measurement done at 2500 rpm the disturbance appears as a 3.3 Hz oscillation, hence the low cut off frequency.

## 2.3 Model from actuator pressure to wastegate position

The wastegate position is mainly a result of the pressure in the actuator. The actuator pressure generates a force in the actuator of  $F = p_{wg}A$ , where  $A$  corresponds the actuators "effective area". This is in its turn balanced by the actuator spring force. Spring forces can usually be modeled by the linear expression  $F = -k_s x$ , where  $x$  is the displacement from an equilibrium position. The actuator is mounted so that the spring is slightly compressed even when the wastegate is fully closed which means that a minimum force,  $F_0$ , is needed for the wastegate to open at all. If no other forces are involved the wastegate equilibrium position will be where the total of these forces is zero (unless the position saturates), which will imply that  $p_{wg}A = k_s x + F_0$  or  $x = A/k_s \cdot (p_{wg} - F_0/A)$ . Ignoring any other forces acting on the wastegate, lumping together  $A/k_s$  into a single constant  $k$  and assigning  $p_0 = F_0/A$ , as this will be the actuator pressure where the wastegate opens, results in equation 2.1. A calculation of  $k$  and  $p_0$  in equation 2.1 with the least squares method from measured data gives the parameters in table 2.1.

$$wg_{pos} = k(p_{wg} - p_0) \quad (2.1)$$

Figure 2.4 compares the suggested model with measured data from two ramp responses, more ramp responses can be found in Appendix A. In this model  $k$

N	2000	2500	3000	3500	4000	4500	5000
k	0.0866	0.3079	0.4598	0.8032	0.8829	0.8273	0.9380
$p_0$	106.8	106.2	108.8	113.4	112.5	106.1	109.7

Table 2.1: Parameter values for the linear function  $wg_{pos} = k(p_{wg} - p_0)$ . The parameters have been fitted with the least squares method from measured ramp responses. This model is only valid for  $wg_{pos} \in [0, 100]$ . Outside this interval the position saturates to respective limit.

and  $p_0$  depend on engine speed. Engine speed should not affect the wastegate behavior directly, but many other engine variables are in one way or another connected to the engine speed. Especially the pressure in the exhaust manifold should contribute with a force on the flapper valve and also the mass flow in the exhaust system. Attempts have been made to get rid of the engine speed dependency by introducing combinations of these two variables, but the result has not been satisfying. By extending equation 2.1 with a linear term in exhaust pressure, and using the least squares method to calculate the parameters, results in a model which almost solely depends on the exhaust pressure. It seems that the linear dependency between exhaust pressure and actuator position is greater than between actuator pressure and position. If the objective were to design an observer for the wastegate position this would be of great use. However since the change in exhaust pressure is an effect of the wastegate position change rather than the cause, it would be useless when designing a model to describe the effect of a change in wastegate duty cycle. The engine speed dependent variables are also very easily implemented in simulink.

The model fit to measured data is not as good for engine speeds below 3000rpm. The wastegate position is not completely linear in pressure and the gain from pressure to position is a bit lower in the region where the wastegate is just about to open. The gain for these engine speeds will be a compromise between the gain for small and large control signals. An increase in  $k$  until the correct wastegate position for zero duty cycle is reached, will cause a too large position to be estimated in the middle region.

## 2.4 Model from duty cycle to actuator pressure

In this section a model for the actuator pressure as a function of the duty cycle and the two pressures feeding the wastegate actuator is developed. The section is split into three subsections, in the first some prerequisites are discussed and the two following presents two different pressure models.

An illustrative picture of the wastegate solenoid is given in figure 2.5. When the wastegate duty cycle signal is high the plunge is pulled towards the right side of the housing, and when it is low it is pulled towards the left. The resulting pressure will be a function of the pressures  $p_{bc}$  and  $p_{ac}$  as well as the wastegate duty cycle. If there is no leakage between the plunge and the housing, the actuator pressure,  $p_{wg}$ , will equal one of the feeding pressures when the duty cycle saturates.

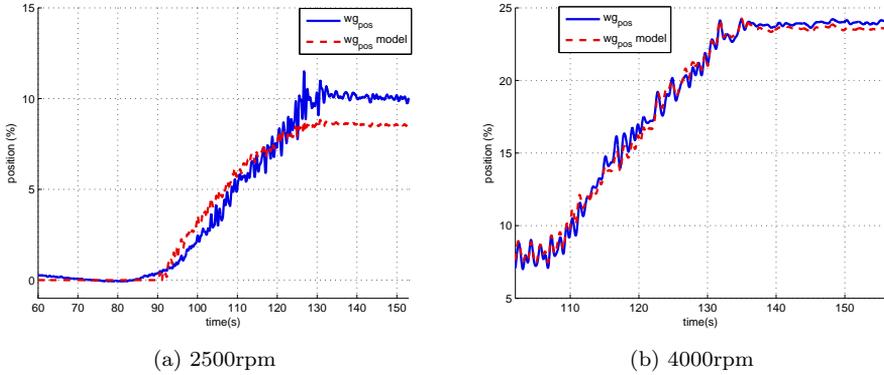


Figure 2.4: Measured wastegate position compared with the estimated model for two different engine speeds. For engine speeds of less than 3000 rpm the estimated position does not fully reach the measured position for  $wg_{dc} = 100$ . Increasing  $k$  until correct end value is reached will cause a too large position to be estimated in the middle region.

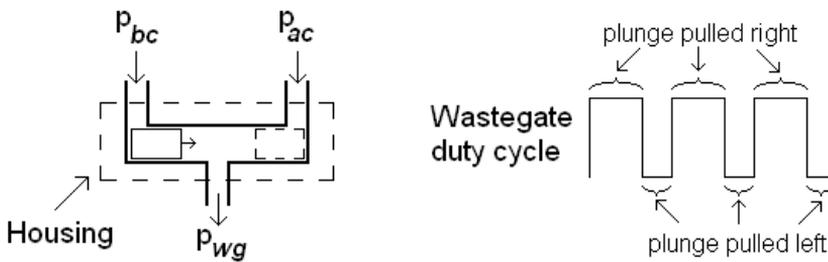


Figure 2.5: Illustrative picture of the wastegate solenoid. Depending on whether the wastegate duty cycle is currently low or high, the plunge is pulled towards either side of the housing.

### 2.4.1 Models for pressure before and after compressor

The first problem which arises when trying to model the actuator pressure is that neither of the pressures that feed the boost control solenoid is measured directly. The pressure sensor in the intake system, which measures the pressure after the intercooler,  $p_{ic}$ , is mounted right before the throttle which is connected to the intercooler with a tube that has the length of half a meter. The intercooler in return has another two meters of tubing back to the compressor, where the pressure  $p_{ac}$  is connected to the wastegate actuator. The other feeding pressure,  $p_{bc}$ , differs from ambient air pressure by the pressure loss over the air filter. Ambient air is not measured during engine operation either but can be taken from the pressure sensor after the intercooler when the engine is turned off. To overcome this problem two assumptions has been made:

- A duty cycle of  $wg_{dc} = 0$  gives  $p_{wg} = p_{ac}$ .
- A duty cycle of  $wg_{dc} = 100$  gives  $p_{wg} = p_{bc}$ .

A model for the two feeding pressures to the actuator from measured pressures is needed. To do this the air mass flow through the air filter and intercooler is modeled as compressible flow restrictions, see equation 2.2. The equation contains the pressure before and after the restriction, making it possible to estimate the pressure before the compressor from ambient pressure and the pressure after the compressor from measured pressure after the intercooler.

$$\dot{m}_{air} = \frac{p_{bef}}{\sqrt{RT_{bef}}} A_{eff} \Psi(\Pi) \quad (2.2a)$$

$$\Pi = \frac{p_{aft}}{p_{bef}} \quad (2.2b)$$

$$\Psi(\Pi) = \begin{cases} \sqrt{\frac{2\gamma}{\gamma-1} \left( \Pi^{\frac{2}{\gamma}} - \Pi^{\frac{\gamma-1}{\gamma}} \right)} & \text{for } \Pi > \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}} \\ \sqrt{\frac{2\gamma}{\gamma-1} \left( \left( \frac{2}{\gamma+1} \right)^{\frac{2}{\gamma-1}} - \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}} \right)} & \text{otherwise} \end{cases} \quad (2.2c)$$

From the three measurements where  $wg_{dc} = 100$  has been possible, the parameter  $A_{eff}$  for the air filter can be estimated. When  $A_{eff}$  has been determined all variables in the equation are known except for  $p_{bc} = p_{aft}$ , which then can be used to estimate  $p_{bc}$  for the whole measurement series.

In the same way, the end of the ramp responses where  $wg_{dc} = 0$  could be used to estimate  $A_{eff}$  for the intercooler. To do this the temperature after the compressor is needed. This temperature has not been measured but can be estimated from equation 2.3 which is also done in [1]. This requires the compressor efficiency,  $\eta_{comp}$ , which is taken from a compressor map of the compressor used and also the pressure ratio  $\Pi_{comp} = p_{ac}/p_{bc}$ . When  $A_{eff}$  for the intercooler has been determined this can be used to estimate  $p_{ac}$ , which for this case becomes an iterative process. The whole procedure becomes as follows.

Start by using the part of measurements where  $wg_{dc} = 0$  so that  $p_{ac} = p_{wg}$ , which is measured signal. Equation 2.3 is now used to estimate  $T_{ac}$ .

$$T_{ac} = T_{bc} \left( 1 + \frac{\frac{\gamma-1}{\Pi_{comp}^{\frac{\gamma}{\gamma-1}}} - 1}{\eta_{comp}} \right) \quad (2.3)$$

The calculated  $T_{ac}$  used in equation 2.2 with  $p_{bef} = p_{ac}$  and  $p_{aft} = p_{ic}$  to calculate  $A_{eff}$  for these data points. The calculated  $A_{eff}$  differ less then 5% between the data points and the average value is used in the following iterative process, which gives  $p_{ac}$  for the whole measurements.

1. Make the initial guess  $p_{ac} = p_{ic} - 5$  ( $kPa$ ).
2. Use equation 2.3 with the initial guess to calculate  $T_{ac}$ .
3. Use equation 2.2 to calculate  $p_{ac} \Psi \left( \frac{p_{ic}}{p_{ac}} \right)$ .
4. Since function  $f(p_{ac}) = p_{ac} \Psi \left( \frac{p_{ic}}{p_{ac}} \right)$  is strictly increasing and thus invertible,  $p_{ac}$  can be calculated as  $f^{-1} \left( p_{ac} \Psi \left( \frac{p_{ic}}{p_{ac}} \right) \right)$  which is done numerically.
5. If the difference between  $p_{ac}$  and the initial guess is smaller then a given number  $\epsilon$  then  $p_{ac}$  has been found. Otherwise use the calculated  $p_{ac}$  as a new initial guess and restart from 2.

With  $\epsilon = 0.01$   $kPa$  the process converge after about 3-5 iterations. This process is repeated for each sample in the measurement series.

## 2.4.2 Linear pressure model

The first pressure model investigated is a linear interpolation between the two pressures before and after compressor. The duty cycle spans the interval  $[0,100]$  but changes in actuator pressure is present in a slightly smaller interval,  $[u_{min}, u_{max}]$ . Outside this interval an actuator pressure that is equal to either the pressure before or after the compressor is modeled. Equation 2.4 defines the model in mathematical terms.

$$p_{wg} = \begin{cases} p_{ac} & \text{if } wg_{dc} \leq u_{min} \\ p_{ac} \frac{u_{max} - wg_{dc}}{u_{max} - u_{min}} + p_{bc} \frac{wg_{dc} - u_{min}}{u_{max} - u_{min}} & \text{if } u_{min} < wg_{dc} < u_{max} \\ p_{bc} & \text{if } wg_{dc} \geq u_{max} \end{cases} \quad (2.4)$$

The parameters  $u_{min}$  and  $u_{max}$  are chosen for best fit to measured data in the sense that the mean square error is minimized. The resulting parameter values are  $u_{min} = 8.2\%$  and  $u_{max} = 74.0\%$  which give a mean square error of  $11.8$   $kPa^2$ . Plots of measured and estimated actuator pressure can be found in figure 2.6. By looking at figure 2.6 it is clear that the linear model does not provide a good fit to measured data. The actuator pressure given by the linear model is to high for large control signals and to low for small control signals. A model better suited for describing the actuator pressure is proposed in the next section.

### 2.4.3 Second degree polynomial pressure model

As a result of the poor performance of the model given in section 2.4.2, this section investigates a slightly more complex model. When doing linear interpolation, like in section 2.4.2,  $u_{min}$  and  $u_{max}$  could also be determined through a visual inspection. It is easy to determine from the plots for which control signals the pressure in the actuator saturates. However, doing this and using equation 2.4, leads to actuator pressures that are too high in the active region. Instead a second order polynomial in duty cycle is proposed. The polynomial is given in equation 2.5 and the coefficients a, b and c are chosen according to the following criteria:

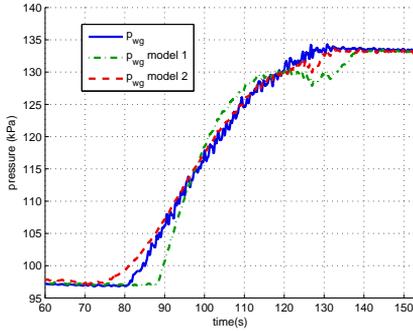
- $wg_{dc} = u_{min} \implies p_{wg} = p_{ac}$
- $wg_{dc} = u_{max} \implies p_{wg} = p_{bc}$
- $wg_{dc} = \frac{u_{max} + u_{min}}{2} \implies p_{wg} = \alpha \cdot p_{ac} + (1 - \alpha) \cdot p_{bc}$

$$p_{wg} = \begin{cases} p_{ac} & \text{if } wg_{dc} \leq u_{min} \\ a \cdot wg_{dc}^2 + b \cdot wg_{dc} + c & \text{if } u_{min} \leq wg_{dc} \leq u_{max} \\ p_{bc} & \text{if } wg_{dc} \geq u_{max} \end{cases} \quad (2.5)$$

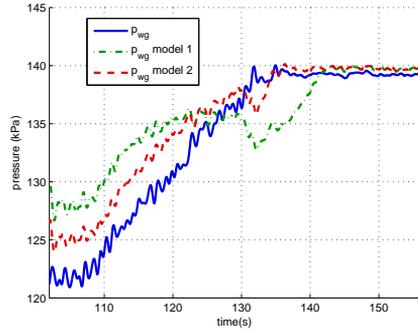
In the third criterion above,  $\alpha$  is the relation of the two pressures feeding the boost control solenoid when the duty cycle is in the middle of its active region. The coefficients in the polynomial become functions of  $p_{bc}$ ,  $p_{ac}$  and  $\alpha$ , their expression and derivations can be found in appendix A. Figure 2.6 shows plots for the two pressure models for two different ramp responses, more plots can be found in appendix A. The mean square error for this model is 4.3 compared to 11.8 for the linear model and does a much better job in describing the pressure, especially for small and large control signals. This model is therefore used in the next section together with the position model and is also implemented in the MVEM. It can be noted that both models exhibit a pressure drop just before maximum actuator pressure is reached. The measured actuator pressure reaches almost maximum pressure when the duty cycle is a few per cent from  $u_{min}$ . From this point, until the duty cycle reaches  $u_{min}$ , actuator pressure increase is very small and this is not captured by the second degree model.

## 2.5 Complete static wastegate model

Combining the two models in section 2.3 and 2.4.3, a complete model for the wastegate position as a function of duty cycle and the two pressures fed to the boost control solenoid is achieved. Figure 2.7 shows a plot with the measured and the calculated positions from the combined pressure and position models. As a result of the position model being less accurate for lower engine speed, for reasons given in 2.3, so is also the combined model. Plots for several other engine speeds can be found in appendix A.

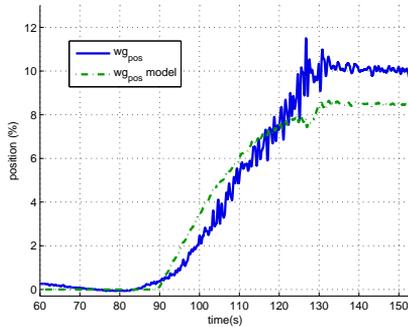


(a) 2500rpm

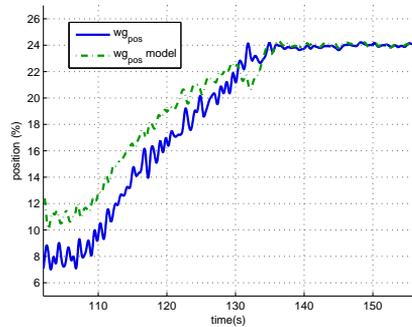


(b) 4000rpm

Figure 2.6: The two pressure models and measured actuator pressure during ramp in wastegate duty cycle. For both engine speeds the quadratic model gives a better fit to measured data. Notable for both models is the pressure drop just before maximum actuator pressure is reached. This is present for all engine speeds which can be seen in figure A.3. Estimated parameters for the second model are  $u_{min} = 12.5$ ,  $u_{max} = 94.5$  and  $\alpha = 0.25$  which gives a mean square error of 4.3.



(a) 2500rpm



(b) 4000rpm

Figure 2.7: Plots for the static model from duty cycle to wastegate position and the measured position. As a result of the position model being less accurate for lower engine speeds, so is also the combined model. An error which also originates from the pressure model is the drop in position just before maximum position is reached.

## 2.6 Wastegate dynamic behavior

When doing step responses in wastegate duty cycle an overshoot both in wastegate pressure and wastegate position is present. The overshoot in pressure can be explained and described by the static model. An increase in wastegate duty cycle will lower the pressure in the wastegate actuator, which closes wastegate. When the wastegate closes, more air will flow through the turbine thus increasing the energy to the compressor. The pressure after the compressor, which is connected to the boost control solenoid, will increase and thereby increase the wastegate pressure once again. This explains the overshoot in pressure and partly the overshoot in position based on the static model. However the measured overshoot in wastegate position is greater than that predicted by the static model, which motivates the introduction of wastegate position dynamics.

### 2.6.1 Wastegate pressure dynamics

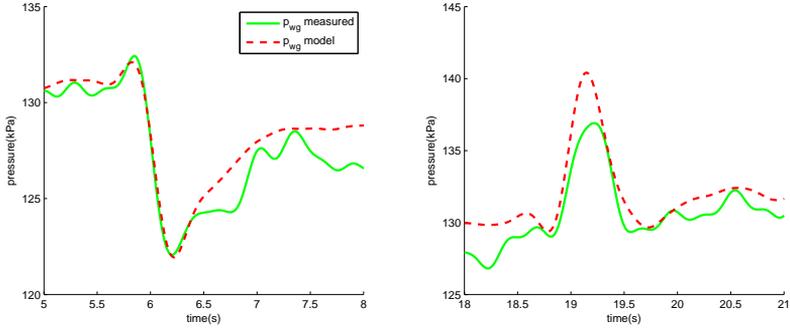
The pressure overshoot in the static model is slightly larger than the measured overshoot and there is a small time delay. A closer look at the step responses shows a delay of about 0.1 s between the rise in measured pressure and estimated pressure from the static model. A step response for measured wastegate pressure and estimated wastegate pressure with the added time delay can be found in figure 2.8. The higher overshoot for the model compared to the measured pressure will be compensated for in the next section. Since the output from the wastegate actuator model is only the wastegate position, there is no restriction in lumping together all dynamics in the position model. The resulting pressure behavior for the remaining engine speeds can be found in figure A.5 and figure A.6.

### 2.6.2 Wastegate position dynamics

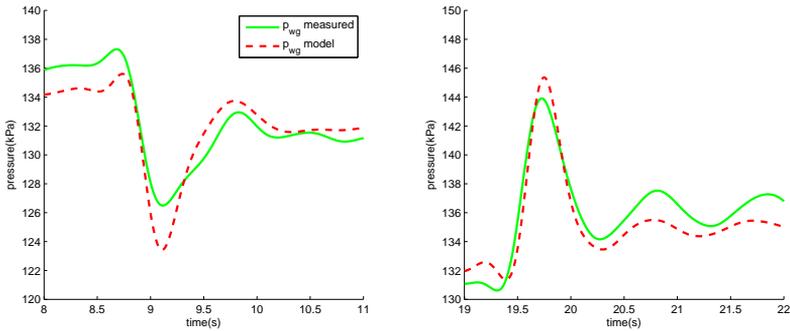
When looking at the step responses for the wastegate position it becomes apparent that the dynamic model should have a large overshoot without oscillations to match the measured data. One solution to this is to use the dynamic model given in equation 2.6 with a double pole in  $s = -1/T$  and a zero in  $s = -1/(\beta T)$ . The model used is a linear model which for  $\beta > 1$  gives the desired overshoot in wastegate position.

$$Y(s) = \frac{\beta T s + 1}{(T s + 1)^2} X(s) \quad (2.6)$$

The motivation for a second order system can be explained by nature of the system seen in figure 2.1. The system consists of a small mass affected by the input force resulting from the pressure, balanced by the spring force. If no other forces were involved this would be a system with a double pole and no zeros. The zero are harder to give a physical interpretation to, but is likely a result of the forces acting on the flapper valve changes as the valve position shift. Comparisons between the static and dynamic model during steps in wastegate duty cycle can be found in figure 2.9. Behaviors for the remaining engine speeds are found in figure A.7 and A.8.

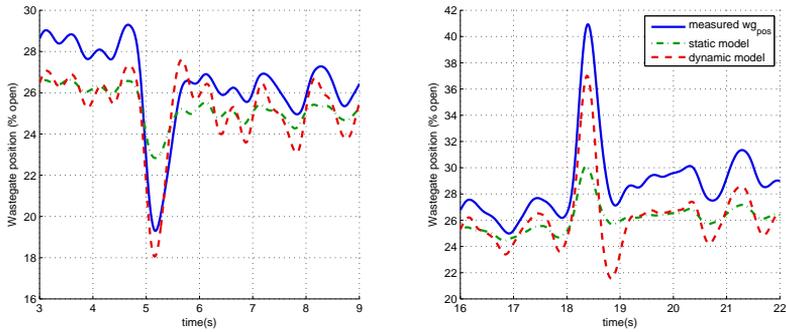


(a) 2500rpm

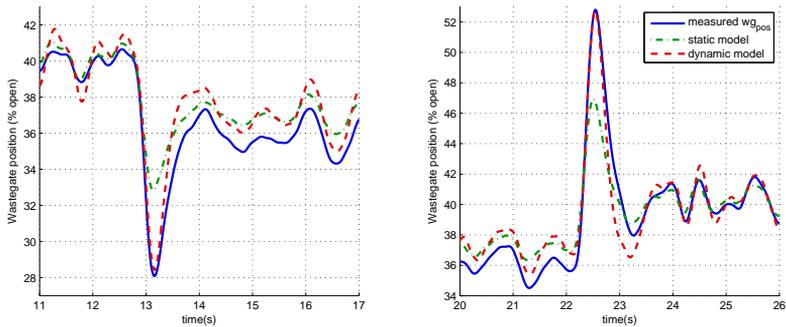


(b) 4000rpm

Figure 2.8: Measured wastegate pressure and calculated wastegate pressure with the static model during a step in wastegate duty cycle. Rise times for measured and calculated pressures are similar but the static pressure model gives a higher overshoot. This is compensated for in the position model where all the dynamics for the actuator model has been lumped together.



(a) 3000rpm



(b) 4500rpm

Figure 2.9: Wastegate behavior for step responses in duty cycle. With the static model the overshoot in wastegate position can't be accurately described by the model. By including dynamics in the wastegate position this error can be greatly reduced.

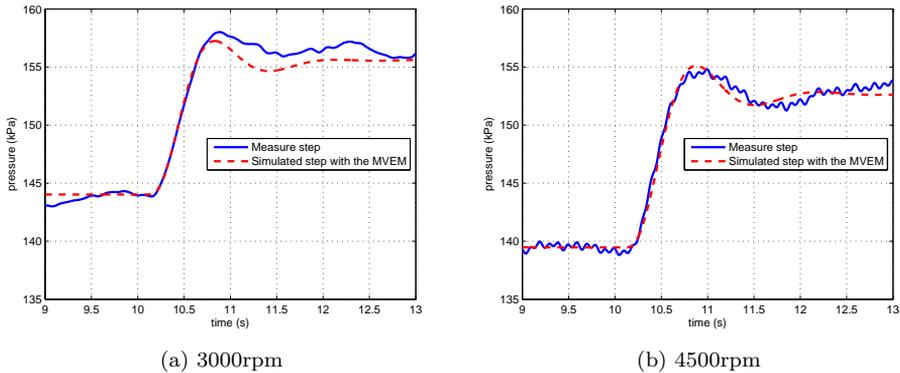


Figure 2.10: A comparison between measured and simulated step responses in wastegate duty cycle. For both engine speeds the pressure behavior is similar to the measured step. For 3000 rpm there is a small bias error but this will neither effect the tuning method nor controller performance. For 4500 rpm the rise time is good, but the dip which comes after the pressure has peaked is a little bit to fast.

## 2.7 Wastegate model simulations with the MVEM

The actuator model was implemented in the available MVEM. Since the model was not tuned for the engine presently installed in the engine lab, a few adjustments to the turbine model had to be made. The output torque of the turbine was scaled so that the maximum boost pressure of the model matched the lab engine. Since this has a negative effect on the turbine acceleration, the turbine inertia was adjusted to give similar response time in pressure. After this adjustment and the implementation of the wastegate actuator model, step responses in wastegate duty cycle were performed and a comparison between measured data and simulated data with the MVEM can found in figure 2.10.

The static gain is slightly lower for the model and there is a few kPa bias error in pressure. In figure 2.10 the bias error has been removed (the simulated pressure is plotted with a maximum offset of 4 kPa), this is done to better be able to compare the pressure transients. A bias error will not affect the controller behavior, and the lower gain for the model will be accounted for in the tuning method. The simulation model captures the overshoot and has a similar rise time as the lab engine, however the pressure peak comes about 50 ms earlier in the simulation. The model gives a similar pressure behavior, which is the important property when evaluating the tuning rules and controller performance in the next section.

# Chapter 3

## Boost pressure PID controller tuning

In this chapter a tuning method for the PID controller is developed. The controller structure is described in section 3.1. Section 3.2 begins with describing the experiments needed to identify the process model which later in the section gives the PID parameters. Some modifications to the standard PID controller have to be made and they are presented in section 3.3.

### 3.1 Boost controller structure

The controller structure was predetermined before the thesis work started. The structure consists of a static feedforward and a PID controller. The feedforward part of the controller takes a pressure reference and the current engine speed as argument. Based on this it gives a duty cycle, which for static condition results in a boost pressure close to the reference value. The PID controllers task is to minimize the response time during steps in desired boost pressure and to eliminate stationary error, without introducing oscillations. An overview of the controller structure can be found in figure 3.1. The PID controller is implemented in parallel form and the ideal controller is described by equation 3.1a, or in the laplace domain by equation 3.1b.

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt} \quad (3.1a)$$

$$U(s) = \left( K_p + K_i \frac{1}{s} + K_d s \right) E(s) \quad (3.1b)$$

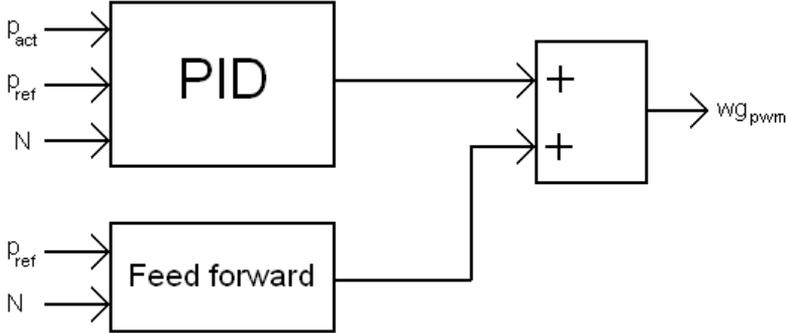


Figure 3.1: Controller structure for the boost controller. The duty cycle consists of a feedforward term added with the output from a PID controller.

## 3.2 Tuning method for PID controller

There is a vast number of tuning methods for PID controllers described in the literature and for all cases you need some information about the controlled process. Small efforts in finding a decent tuning method had already been made. Attempts to use proportional feedback to find the ultimate frequency and then applying Ziegler-Nichols tuning rules, as described in [5], results in PID parameters that are too large. Many tuning methods are based on a process model and from the parameters in the model the controller parameters are given according to some rule. An experiment, which can be used to identify properties of a process model, is a step response in duty cycle. From the experiment, properties like static gain, dominating time constants and characteristic behaviors like tendency to oscillation and overshoot can be obtained [7]. Also since the tuning method preferably would rely on this test it was chosen as a starting point in finding a suitable tuning method.

### 3.2.1 Step responses and process model

Step response in duty cycle is an experiment which in this case is relatively easy to perform. Ideally this would have been done in an engine cell but as this was not available at the time of measurement they were done in a car on a test track. The engine speed was kept constant during the measurements by using a braking trailer, which is a trailer that brakes the car when a certain speed is reached. The throttle plate was kept fully opened, the duty cycle to the wastegate actuator was set manually. A step response within the active region (see section 2.4.3) was made for several engine speeds. Step responses together with the process model described later in this section can be found in figure 3.2.

Looking at the step response it is clear that the pressure transient has an overshoot for higher engine speeds. It is also possible to see a tendency to oscillations in the aftermath of the step. Trying to keep the model simple, the process model

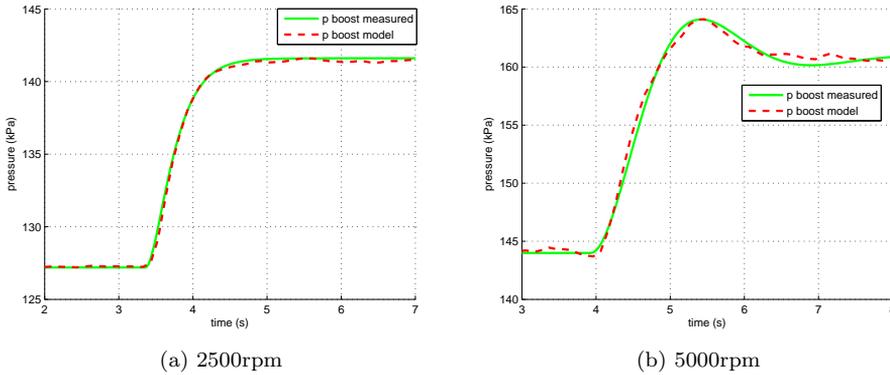


Figure 3.2: Step responses together with the adapted process model for two different engine speeds. The adapted model shows very good fit to measured data. For 5000rpm the pressure peak is slightly sharper compared to the adapted model, but the difference is very small.

given in equation 3.2, where  $\zeta \leq 1$ , is proposed. The model is presented as a transfer function in equation 3.2a, and the step response expression is found in equation 3.2b. This model has a resonance frequency close to  $1/T$  and an overshoot for small  $\zeta$ . For  $\zeta$  closer to one the behavior has similar characteristics as a first order system, which is the case for the step responses for lower engine speeds. The model should therefore, with different parameters, be able to explain the system behavior for both lower and higher engine speeds.

When identifying the linear model the zero level for pressure and duty cycle is set to, respectively, the pressure and duty cycle before the step. The parameter  $K$  is given by the static gain of the model divided by the step size. After  $K$  is determined,  $\zeta$  can be chosen to give correct overshoot. When  $K$  and  $\zeta$  are fixed,  $T$  is adjusted to get the peak of the overshoot at the correct time. The identified model together with measured data can be found in figure 3.2 and more plots for different engine speeds can be found in figure B.1.

$$G(s) = \frac{K}{T^2 s^2 + 2T\zeta s + 1} \quad | \quad \zeta \leq 1 \quad (3.2a)$$

$$\begin{cases} y(t) = K \left( 1 - \frac{e^{-\zeta t}}{\sqrt{1-\zeta^2}} \sin\left(\frac{\sqrt{1-\zeta^2}}{T} t + \phi\right) \right) \\ \phi = \arccos(\zeta) \end{cases} \quad (3.2b)$$

### 3.2.2 PID parameter identification from process model

After identifying the process model a large number of tuning methods are available. An attractive control method which, under certain conditions, can be used

to identify PI- and PID-parameters is Internal Model Control (IMC). IMC is described briefly in [5] and with a little more detailed in [6]. The idea behind the method is to have an internal model in the controller which is fed by the same control signal as the real system. Only the difference between the simulated and actual output is fed back to the controller. With a small amount of calculus the expressions for the resulting controller can be obtained, which for a second order system becomes an ideal PID controller. Expressions for the PID-parameters given by this method is given in equation 3.3 and a derivation can be found in appendix B.1.

$$K_p = \frac{2T\zeta}{\lambda K} \quad K_i = \frac{1}{\lambda K} \quad K_d = \frac{T^2}{\lambda K} \quad (3.3)$$

The relation between the P-, I- and D-parts is given by the process model, and the tuning parameter  $\lambda$  adjusts the controller gain.  $\lambda$  is also the time constant for the closed loop system without the feedforward. In the simulation environment, which was adapted to the engine in the engine lab, a parameter value of  $\lambda = 1.3$  produced good step responses. When tested in the test car this had to be adjusted to  $\lambda = 1.7$  to avoid oscillative behavior. The difference between the two engines is that the gain from controller output to boost pressure ( $K$  in equation 3.2) is much lower for the car engine.

### 3.3 PID controller modifications

Implementing the standard PID controller in a real system is almost never possible. The measured pressure is subject to high frequency measurement noise which makes a direct implementation of the derivative very unstable and the integrator term needs to be protected from windup.

A simple suggestion on how to modify the derivative part to be less sensitive to high frequency disturbances is presented in [5]. This solution is basically a discrete approximation of a first order low pass filter. The algorithm for calculating filtered derivative is given in equation 3.4. In the equation  $d_n$  is the filtered derivative,  $T_S$  is the sample time,  $N$  is the time constant for the first order filter and  $e_n$  is the control error  $r_n - y_n$ . The subscript  $n$  refers to the  $n$ :th sample. The parameter  $N$  should be chosen large enough for undesired disturbances to be minimized but cannot be so large that it significantly slows down the derivative and thus render it useless.

$$d_n = \frac{N}{N + T_S} d_{n-1} + \frac{1}{N + T_S} (e_n - e_{n-1}) \quad (3.4)$$

Furthermore, letting the derivative act directly on the control error can result in large transients in control error during sudden changes in reference value. This can be improved by modifying the reference signal for the derivative part. Instead of feeding the control error  $e = r - y$ , the reference is modified by a constant  $\beta$  so that the equation becomes  $e_d = \beta r - y$ , where  $\beta$  is chosen between 0 and 1. According to [9],  $\beta$  is normally chosen equal to zero which means switching  $e_n$  in equation 3.4 to  $-y_n$ . For simplicity  $\beta = 0$  was chosen.

## 3.4 PID tuning method summary

The tuning method presented in this chapter is based on a step response in wastegate duty cycle. The experiment is easy to perform and only needs boost pressure and engine speed to be measured. Based on the measurements a second degree model is identified. The model parameters are easily calculated one at a time and together with a single tuning parameter they give the PID controller parameters. The performance of the controller is presented in chapter 4.



# Chapter 4

## Boost pressure control results

In this chapter the performance of the controller tuning method presented in the previous chapter is evaluated. Before the tuning method is tested in a test car it is tested in simulation. This is done in section 4.1. In section 4.2 the experiments made in a test car are presented that shows the boost pressure behavior with the tuned controller. The chapter finishes with a discussion of some of the effects that came up during testing and a suggestion on further improvement.

### 4.1 Boost controller simulations with the MVEM

Before testing in a real car, the tuning method is evaluated in a simulation environment with the actuator model developed in chapter 2. Step responses are done in simulation and a process model is adapted according to section 3.2.1. Simulated step responses from the MVEM and for the process model can be found in figure 4.1.

After identifying the process model, the PID-parameters are given by equation 3.3. A PID controller was implemented according to the description in section 3.3 and step responses in reference pressure were simulated. Simulation results are found in figure 4.2. In simulation the overshoot for 2500 rpm is about 7 kPa and for 5000 rpm about 3 kPa. Ideally both would be 5 kPa, but this is a simulation and a 2 kPa deviation is quite small. What's important is that the behavior seems correct, there are no oscillations and a small overshoot.

### 4.2 Boost controller performance in test car

After the simulations have been performed, the controller performance is evaluated in a real car. All tests in this section are done with the same car as the step responses in section 3.2.1, but this time the car was mounted in a chassis dynamometer.

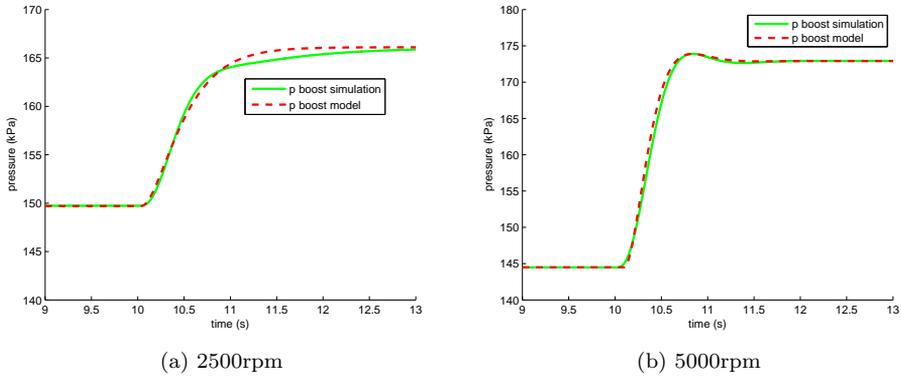


Figure 4.1: Simulated step responses with the MVEM and the actuator model from the previous chapter. Also displayed is an adapted process model on the form given in equation 3.2. The process model manages to describe the simulated step responses very well. The simulation with the MVEM shows a slightly slower pressure increase at the end of the step compared to the process model.

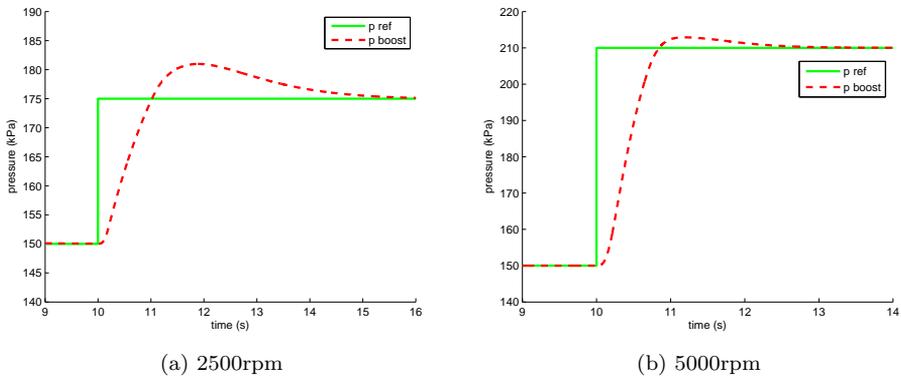


Figure 4.2: Simulated step responses with a step in reference pressure. For the two engine speeds the overshoot is, 7 kPa and 3 kPa respectively, which is not far from the desired overshoot of 5kPa.

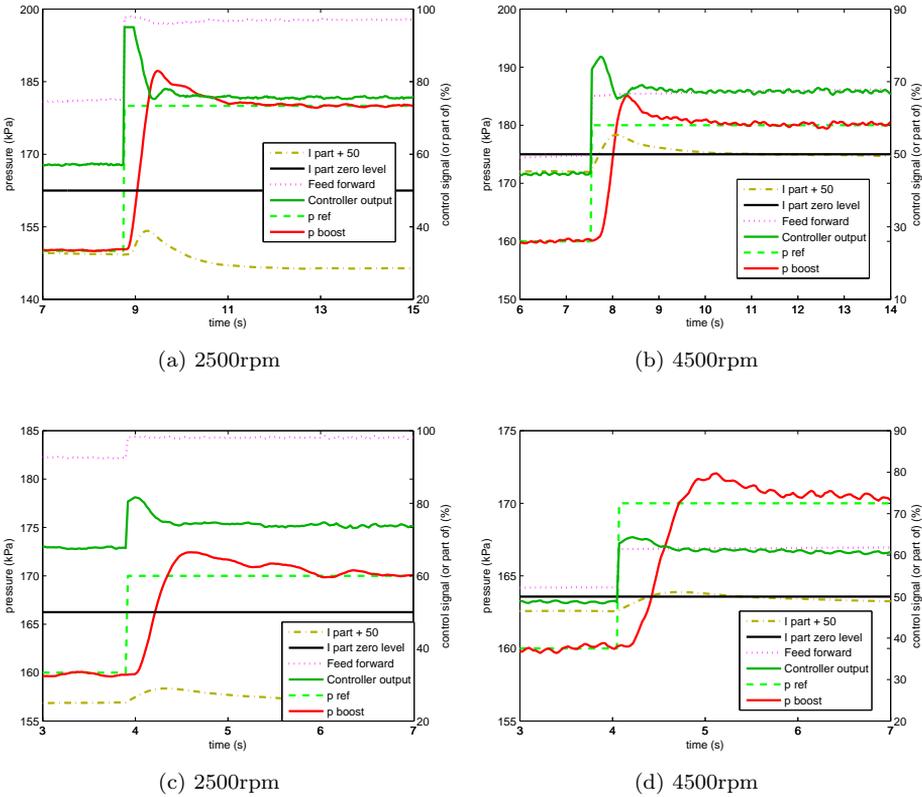


Figure 4.3: Step responses in pressure reference. The step response has a small overshoot of about 5kPa and no significant oscillations, which is the desired behavior.

The first test is a step responses in reference pressure. Ideally there would be a small overshoot (about 5kPa) and no oscillations in the controlled boost pressure. This behavior is achieved when the I-part has the same value before and after the step response. Figure 4.3 shows steps for two different engine speeds and step sizes. Plots for other engine speeds are found in figure B.2. This was however not always the case, in some of the measurements the feedforward term did not produce the correct stationary control signal.

During the evaluation of the step responses, problem occurred when the I-part of the controller had to change its stationary value. This is a result of an error in the feedforward term of the controller. If the step in the feedforward term is smaller or larger than it should be, the response will not be as desired. If the I-part needs to increase its stationary value to reach its desired boost pressure, the pressure transient will be an undershoot. On the other hand, if the I-part needs to decrease its value, the pressure overshoot will be too large. Step responses where this occur

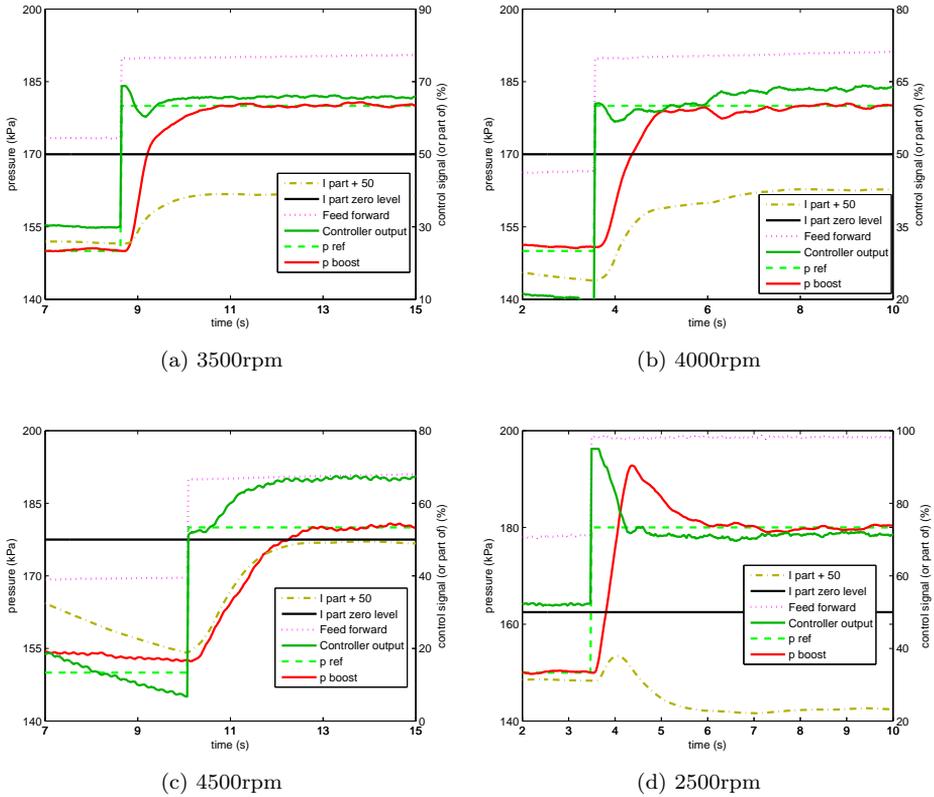


Figure 4.4: Step responses with incorrect step in feedforward. When the I-part needs to increase the pressure undershoots and when it needs to decrease the overshoot is too large.

can be found in figure 4.4. The limit where these behaviors appear, based on measurement data, seems to be around 7-8% in duty cycle, which corresponds to around 25% of the total static change in duty cycle for the steps.

The next test was to do a step in accelerator pedal position. The results for two engine speeds are presented in figure 4.5. For 2500 rpm the overshoot in pressure is at maximum 15 kPa, which is a little bit too much to be acceptable. This is however also a case where the feedforward term is too large after the step.

#### 4.2.1 Further improvements

To get a fast pressure step response, the duty cycle should saturate when the pressure error is large. With the parameters given by the suggested tuning method, this is not the case for all tests. This could be done by increasing the P-part of the controller for large pressure errors. By using the suggested value for  $K_p$  when

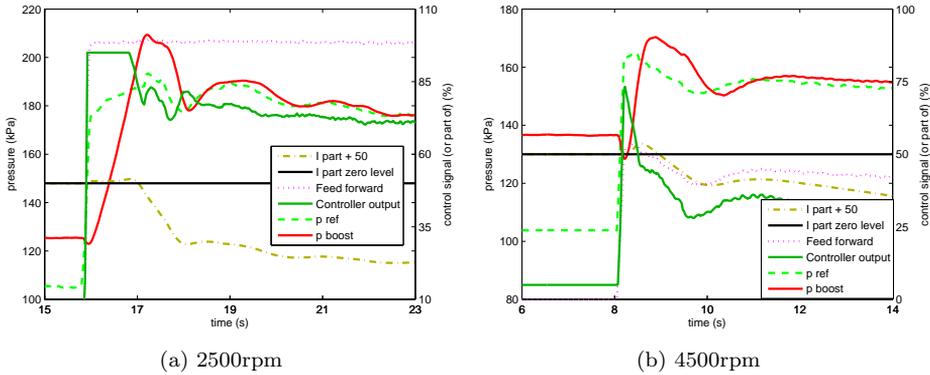


Figure 4.5: Step responses in accelerator pedal position for two engine speeds. In both cases the overshoot is a bit large, and in both cases the feedforward is too large after the step.

the pressure error is smaller than a certain value, and gradually increasing  $K_p$  as the error grows larger, saturation of the duty cycle can be achieved. This would close the wastegate further, forcing more air through the turbine which increases the power to the compressor, speeding up the step response in boost pressure.

A suggestion for how to incorporate this into the developed tuning method is to use  $K_p = K_{p0}$ , where  $K_{p0}$  is the value given by the tuning method, when the error in boost pressure is smaller than a given limit,  $e_1$ . Then letting  $K_p$  increase to  $\beta K_{p0}$  as the error grows from  $e_1$  to another error limit,  $e_2$ . For appropriate values of the error limits  $e_1$  and  $e_2$ , and the parameter  $\beta$ , this behavior would be achieved.



# Chapter 5

## Future work

Interesting issues for future work which has come up during the thesis work are presented in this section.

### **More advanced wastegate model**

The current wastegate model has two engine speed dependent variables. A physical model would rather be dependent on air mass flow and exhaust pressure. Investigating how these affect wastegate position and getting rid of the engine speed dependence would be of interest.

### **More advanced controllers**

This thesis has been limited to PID controller with static feedforward. A more advanced state space model together with state feedback and/or dynamics in the feedforward term would be an interesting topic for further investigation.

### **Generation of reference values**

When performing steps in pedal position the pressure responses sometimes have oscillations as a result of the reference value oscillation. A clear definition or better investigation on what should be done by each part of the control system would be valuable.

### **Combined wastegate and throttle control**

When doing step responses in the lab engine, oscillations originating from oscillations in throttle plate angle were recorded. Investigating how to combine control of throttle plate angle and wastegate to receive smooth transients that are free of oscillations would be an interesting area to investigate.



# Chapter 6

## Summary and Conclusions

This thesis presents a model for the wastegate actuator and a tuning method for a PID controller used for boost pressure control. In chapter 2 a model for the wastegate solenoid and the pressure actuator, which is connected to the wastegate flapper valve, is presented. Chapter 3 presents a tuning method for choosing PID parameters and test results based on measurements in a chassis dynamometer is evaluated.

The model of the actuator is split into two submodels. In section 2.3 the wastegate position is modeled from measured actuator pressure, as a linear function in actuator pressure with engine speed dependent constants. Section 2.4 covers the model for the actuator pressure from feeding pressures and control signal to the wastegate solenoid. These models are combined to give a complete static wastegate model in section 2.5. The model is then completed with dynamic behavior. In chapter 3, a tuning method for the PID controller that controls the boost pressure is presented. The method is based on an IMC-controller for a second degree system which can be interpreted as a PID controller. In section 4.2 the controller performance is evaluated based on measurements from a car mounted in a chassis dynamometer. The pressure behavior is shown to become as desired for no or little error in the feedforward term. When there are large errors in the feedforward the pressure either makes an undershoot or a too large overshoot.

### 6.1 Conclusions

The wastegate position model shows good agreement with measured data, but the model has not been validated against a separate data set. The actuator model together with the MVEM gives similar response in boost pressure to changes in duty cycle, as the real engine.

Using step responses in duty cycle is an easy way of retrieving PID parameters for the boost controller. Beside the information gained from step responses a single tuning parameter is needed. The controller produces desired pressure behavior under the condition that the static feedforward has no or little error (less than about 7-8%).



# Bibliography

- [1] Per Andersson. *Air Charge Estimation in Turbocharged Spark Ignition Engines*. PhD thesis, Linköpings Universitet, December 2005.
- [2] Lars Eriksson. Modeling and control of turbocharged SI and DI engines. *Oil & Gas Science and Technology - Rev. IFP*, 62(4):523–538, 2007.
- [3] Lars Eriksson and Lars Nielsen. *Modeling and Control of Engines and Drivelines*. Vehicular Systems, ISY, Linköping Institute of Technology, 2008. Course Material in Vehicular Systems.
- [4] Lars Eriksson, Lars Nielsen, Jan Brugård, Johan Bergström, Fredrik Pettersson, and Per Andersson. Modeling of a turbocharged SI engine. *Annual Reviews in Control*, 26(1):129–137, October 2002.
- [5] Institutonen för systemteknik. *Industriell Reglerteknik Kurskompendium*. Bokakademin Linköping, 2008. Course Material in Industrial Control Systems.
- [6] Lennart Ljung and Torkel Glad. *Reglerteori*. Studentlitteratur, 2 edition, 2003. ISBN 91-44-03003-7.
- [7] Lennart Ljung and Torkel Glad. *Modellbygge och simulering*. Studentlitteratur, 2 edition, 2004. ISBN 91-44-02443-6.
- [8] Sune Söderkvist. *Tidsdiskreta Signaler & System*. Tryckeriet Erik Larsson AB, 1994.
- [9] Karl J Åström and Tore Hägglund. *Advanced PID Control*. ISA-The Instrumentation, Systems, and Automation Society, 2006. ISBN 1-55617-942-1.



# Appendix A

## Derivation of parameter expressions and actuator model plots

### A.1 Derivation of $a$ , $b$ and $c$ in wastegate pressure model 2

Given the criteria in 2.4.3,  $a$ ,  $b$  and  $c$  should solve the system of equations A.1, A.2 and A.3.

$$au_{max}^2 + bu_{max} + c = p_{bc} \quad (\text{A.1})$$

$$au_{min}^2 + bu_{min} + c = p_{ac} \quad (\text{A.2})$$

$$a\left(\frac{u_{max} + u_{min}}{2}\right)^2 + b\left(\frac{u_{max} + u_{min}}{2}\right) + c = \alpha p_{ac} + (1 - \alpha)p_{bc} \quad (\text{A.3})$$

Using equation A.1 and A.2 gives:

$$a(-u_{max}^2 + u_{min}^2) + b(-u_{max} + u_{min}) = p_{ac} - p_{bc} \quad (\text{A.4})$$

Using equation A.1 and A.3 gives:

$$a\left(-\frac{3u_{max}^2}{4} + \frac{u_{max}u_{min}}{2} + \frac{u_{min}^2}{4}\right) + b\left(-\frac{u_{max}}{2} + \frac{u_{min}}{2}\right) = \alpha p_{ac} - \alpha p_{bc} \quad (\text{A.5})$$

Combining equation A.4 and A.5 and solving for  $a$ :

$$a\frac{1}{2}(u_{max} - u_{min})^2 = (1 - 2\alpha)(p_{ac} - p_{bc}) \Leftrightarrow a = \frac{2(1 - 2\alpha)(p_{ac} - p_{bc})}{(u_{max} - u_{min})^2} \quad (\text{A.6})$$

Combining equation A.4 and A.6 and solving for b:

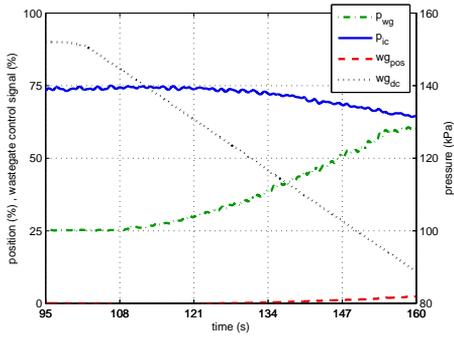
$$\begin{aligned}
 (-u_{max}^2 + u_{min}^2) \frac{2(1-2\alpha)(p_{ac} - p_{bc})}{(u_{max} - u_{min})^2} + b(-u_{max} + u_{min}) &= p_{ac} - p_{bc} \Leftrightarrow \\
 b &= \left(1 + \frac{2(1-2\alpha)(u_{max}^2 - u_{min}^2)}{u_{max} - u_{min}}\right) \frac{p_{ac} - p_{bc}}{(-u_{max} + u_{min})} \quad (\text{A.7})
 \end{aligned}$$

Finally using equation A.3, A.6 and A.7:

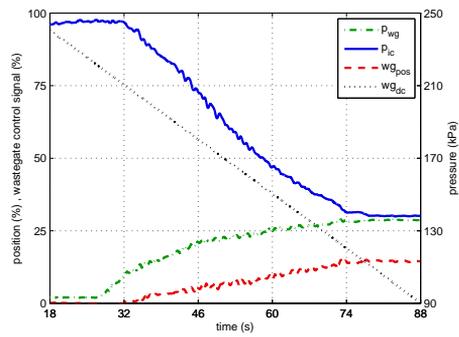
$$\begin{aligned}
 c = p_{bc} - \frac{2u_{max}^2(1-2\alpha)(p_{ac} - p_{bc})}{(u_{max} - u_{min})^2} - \left(1 + \frac{2(1-2\alpha)(u_{max}^2 - u_{min}^2)}{(u_{max} - u_{min})^2}\right) \dots \\
 \dots \frac{u_{max}(p_{ac} - p_{bc})}{(-u_{max} + u_{min})} \quad (\text{A.8})
 \end{aligned}$$

## A.2 Remaining plots from the actuator model chapter

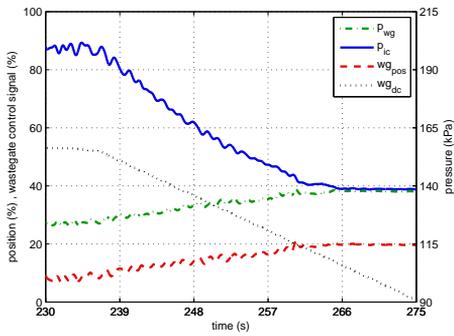
In this section, the remaining plots not presented in chapter 2 can be found.



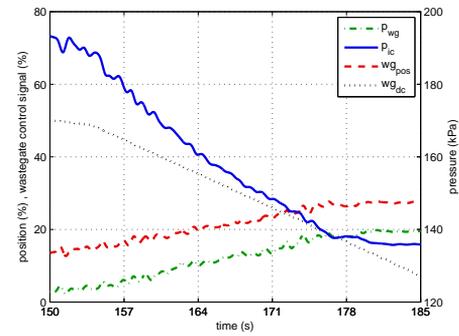
(a) 2000rpm



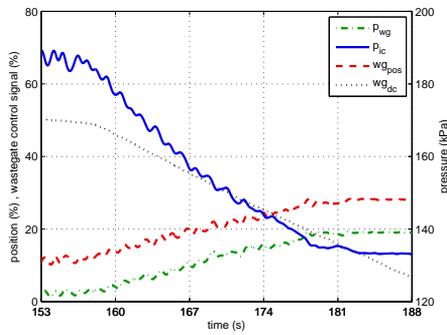
(b) 3000rpm



(c) 3500rpm

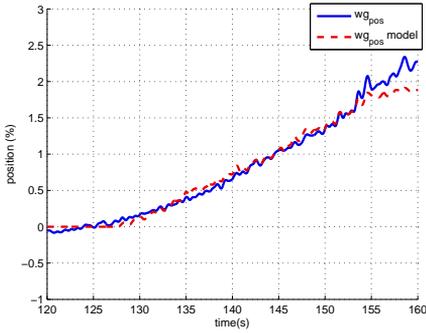


(d) 4500rpm

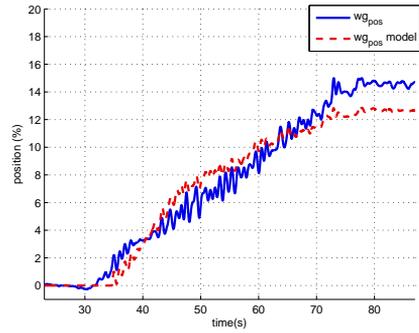


(e) 5000rpm

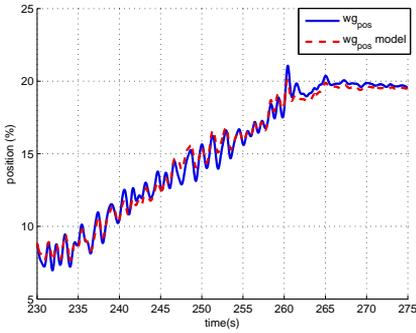
Figure A.1: Ramp responses for the remaining engine speeds not presented in section 2.2. The wastegate duty cycle is ramped from the maximum allowed by the control system during normal operation and down to zero. As the duty cycle decreases pressure in the actuator rises. Eventually the actuator pressure overcomes the force of the return spring, the wastegate opens and the boost pressure decreases.



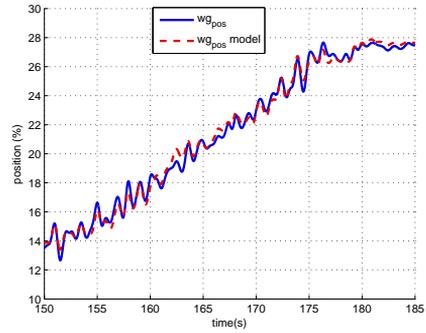
(a) 2000rpm



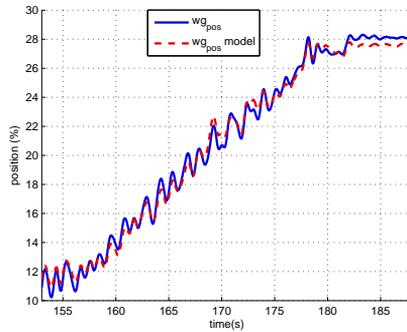
(b) 3000rpm



(c) 3500rpm

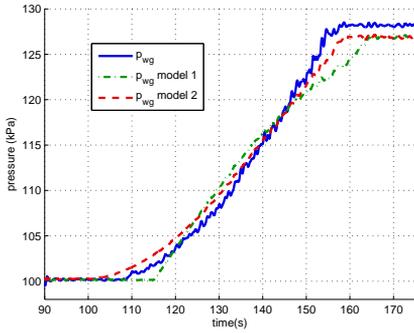


(d) 4500rpm

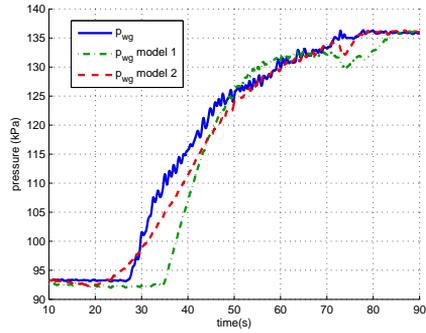


(e) 5000rpm

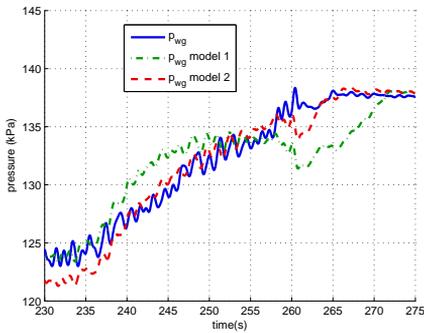
Figure A.2: Measured and estimated wastegate position based on measured actuator pressure, for engine speeds not presented in section 2.3. For engine speeds of less than 3000rpm the estimated position does not fully reach the measured position for  $wg_{pwm} = 100$ . For higher engine speeds the estimated position shows very good agreement to measured data.



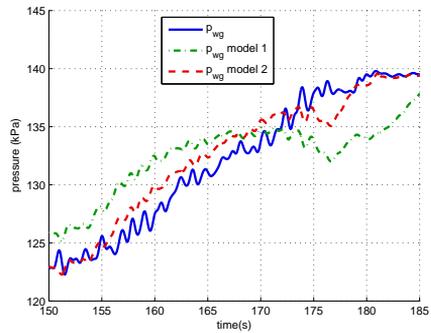
(a) 2000rpm



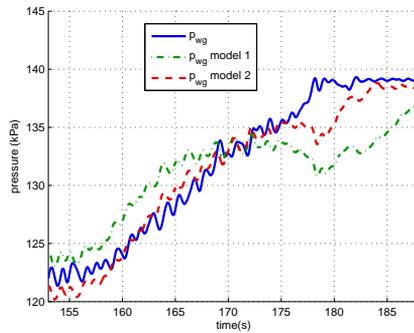
(b) 3000rpm



(c) 3500rpm

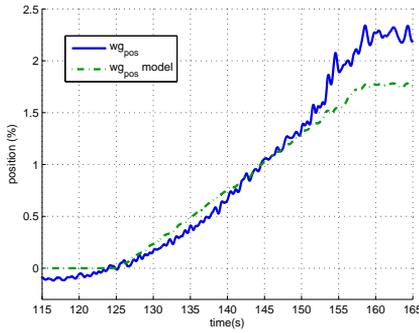


(d) 4500rpm

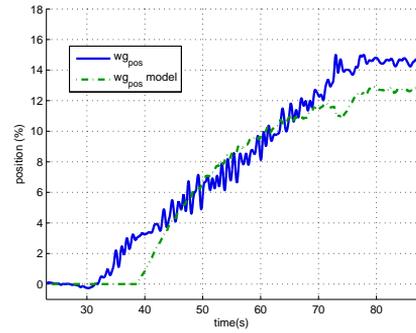


(e) 5000rpm

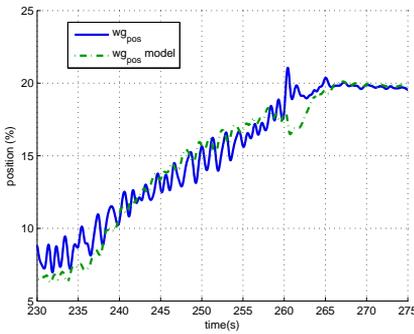
Figure A.3: Measured and estimated actuator pressure for engine speeds not presented in section 2.4. For both models there is a pressure drop slightly before maximum actuator pressure is reached. The second degree model is a big improvement for estimating actuator pressure compared to the linear model.



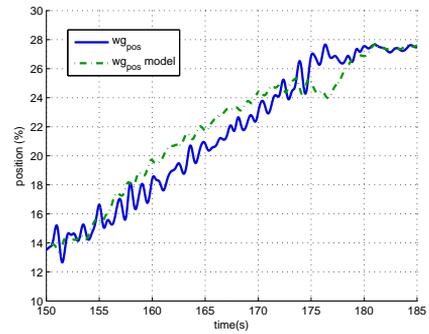
(a) 2000rpm



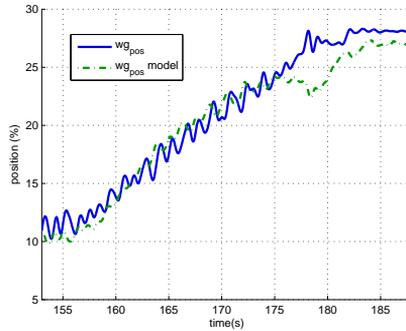
(b) 3000rpm



(c) 3500rpm

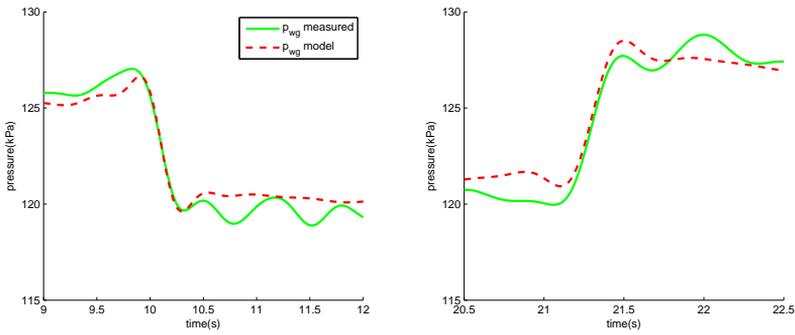


(d) 4500rpm

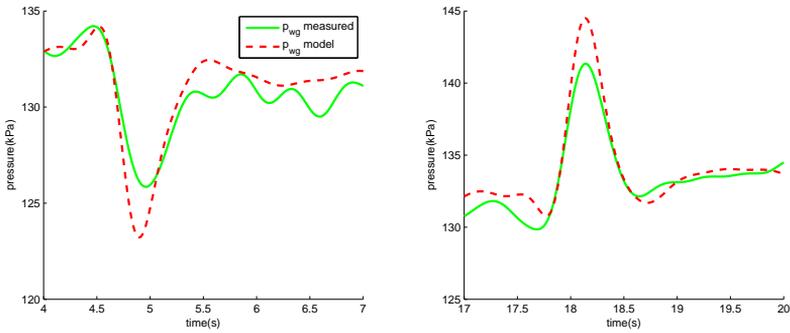


(e) 5000rpm

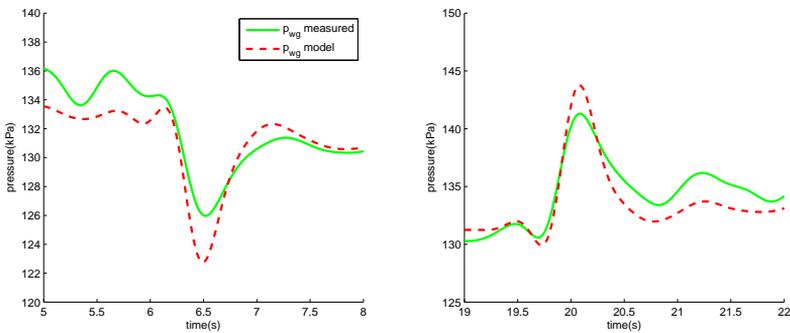
Figure A.4: Measured and estimated wastegate position based on estimated actuator pressure, for engine speeds not presented in section 2.5. For all engine speeds, except for the 3000rpm measurement, the estimated position is close to the measured value until slightly before maximum position is reached. Slightly before maximum position is reached there is a dip in position which originates from the pressure model.



(a) 2000rpm

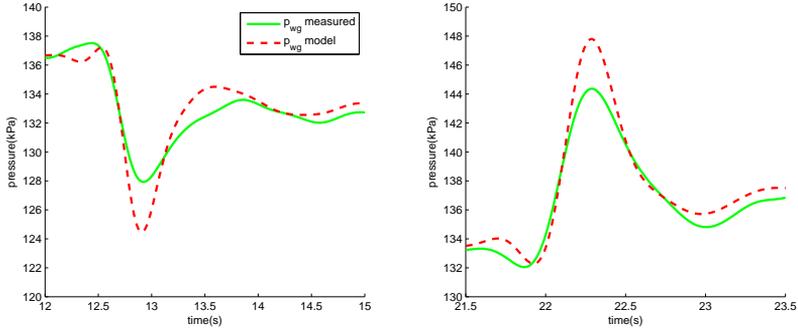


(b) 3000rpm

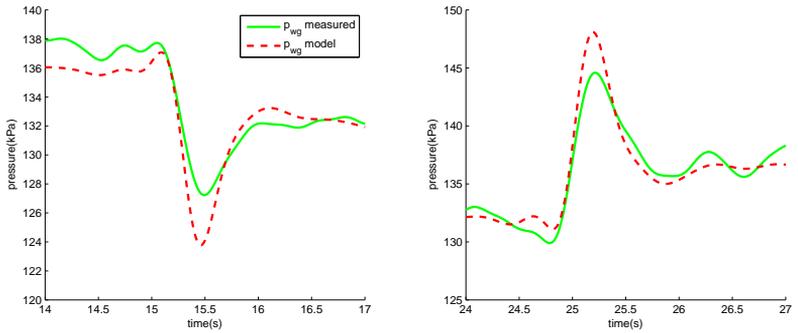


(c) 3500rpm

Figure A.5: Measured and estimated wastegate pressure during a step in wastegate duty cycle not presented in section 2.6.1. The overshoot in pressure is higher for the estimated pressure compared to the measured pressure. This is compensated for in the position model where all the dynamics for the actuator model has been lumped together.

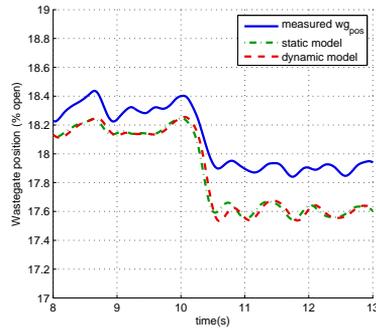
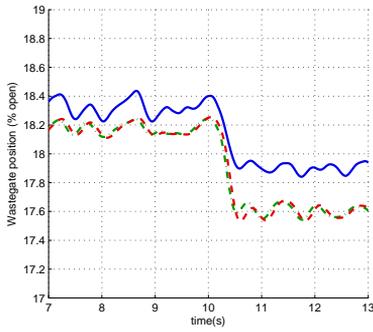


(a) 4500rpm

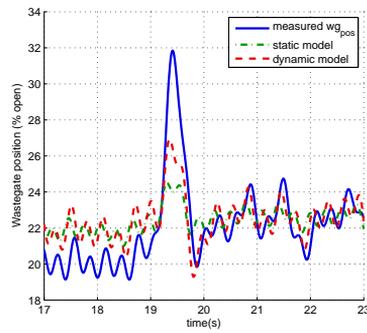
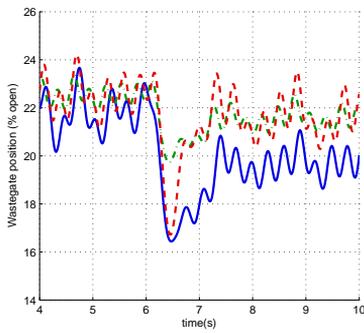


(b) 5000rpm

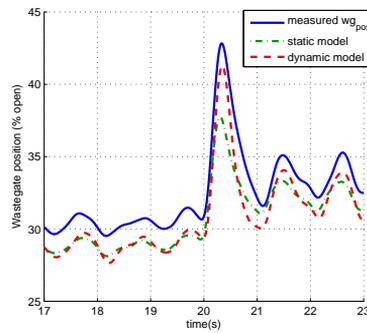
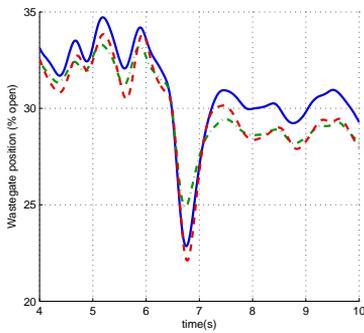
Figure A.6: Measured and estimated wastegate pressure during a step in wastegate duty cycle not presented in section 2.6.1. The overshoot in pressure is higher for the estimated pressure compared to the measured pressure. This is compensated for in the position model where all the dynamics for the actuator model has been lumped together.



(a) 2000rpm

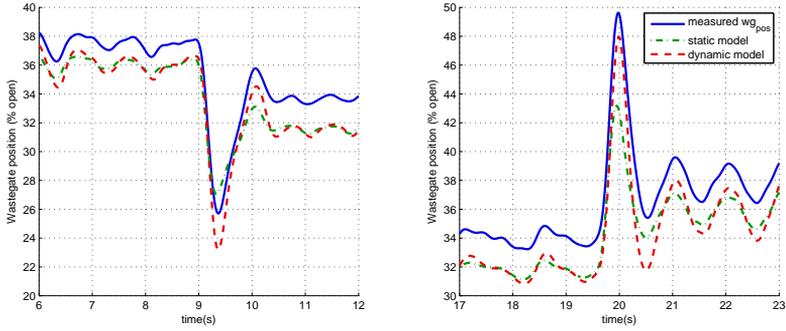


(b) 2500rpm

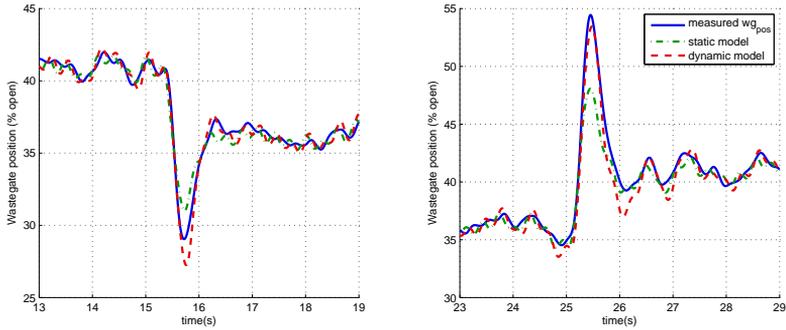


(c) 3500rpm

Figure A.7: Measured and estimated wastegate position during step response in duty cycle not presented in section 2.6.2. With the dynamic, Black Box model, the position estimation comes a lot closer to the measured step response.



(a) 4000rpm



(b) 5000rpm

Figure A.8: Measured and estimated wastegate position during step response in duty cycle not presented in section 2.6.2. With the dynamic, Black Box model, the position estimation comes a lot closer to the measured step response.

# Appendix B

## Derivation of PID parameter equations and validation plots

### B.1 Derivation of expressions for PID parameters

As described in [5],  $Q(s)$  is chosen as:

$$Q(s) = G(s)^{-1} \frac{1}{\lambda s + 1} \quad (\text{B.1})$$

Where  $G(s)$  is the model given in equation 3.2. The controller becomes:

$$F(s) = \frac{Q(s)}{1 - Q(s)G(s)} \quad (\text{B.2})$$

Inserting the expression for  $Q$  above gives:

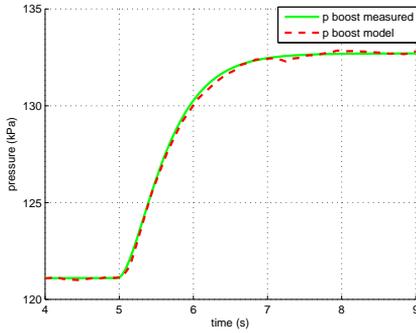
$$F(s) = \frac{\frac{1}{\lambda s + 1} G(s)^{-1}}{1 - \frac{1}{\lambda s + 1}} = \frac{G(s)^{-1}}{\lambda s} \quad (\text{B.3})$$

Exchanging  $G(s)$  for the expression given in equation 3.2 gives:

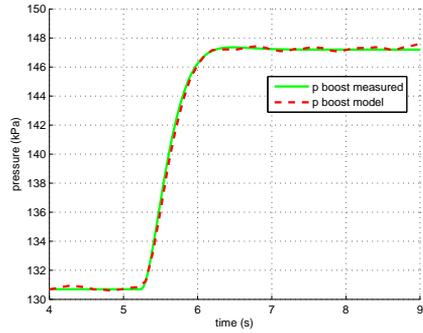
$$F(s) = \frac{T^2 s^2 + 2\zeta T s + 1}{\lambda K s} = \underbrace{\frac{2\zeta T}{\lambda K}}_{K_p} + \underbrace{\frac{1}{\lambda K}}_{K_i} \cdot \frac{1}{s} + \underbrace{\frac{T^2}{\lambda K}}_{K_d} \cdot s \quad (\text{B.4})$$

### B.2 Remaining plots from the controller tuning chapter

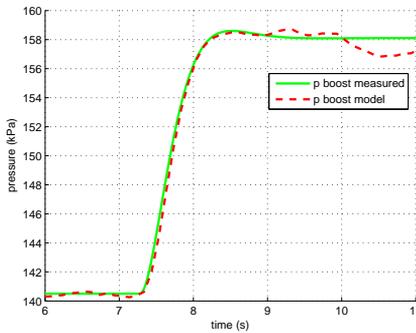
In this section, the remaining plots not presented in chapter 3 can be found.



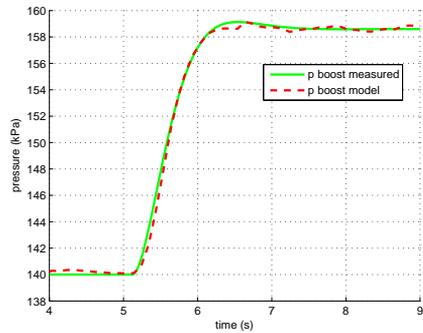
(a) 2000rpm



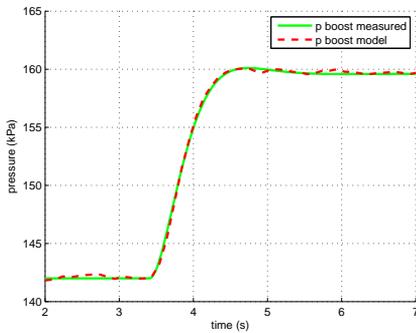
(b) 3000rpm



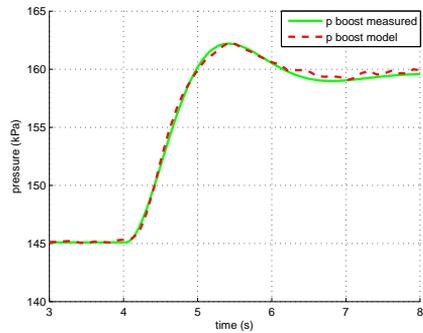
(c) 3500rpm



(d) 4000rpm

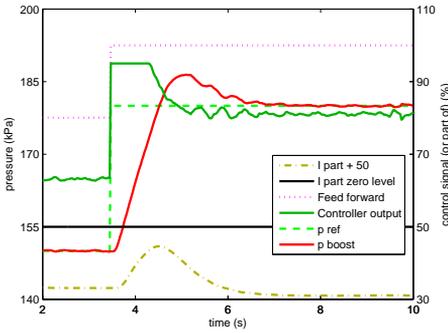


(e) 4500rpm

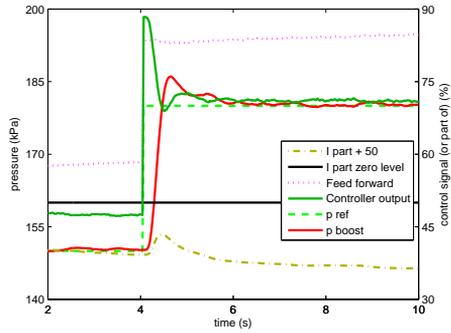


(f) 5500rpm

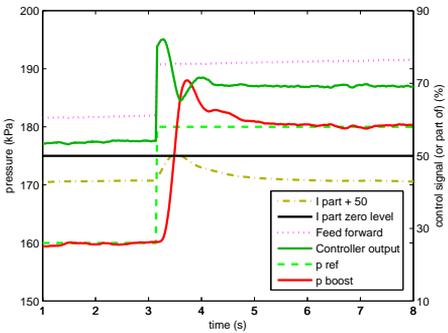
Figure B.1: Plots of step responses in duty cycle and adapted model for engine speeds not presented in section 3.2.1. The adapted model shows very good fit to measured data. For 5500rpm the pressure peak is slightly sharper compared to the adapted model, but the difference is very small.



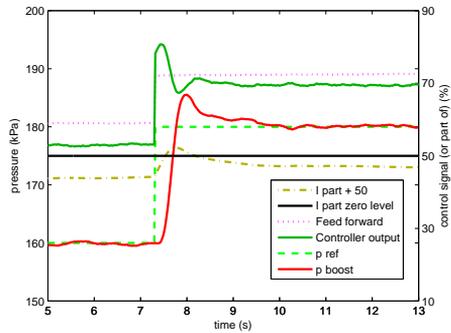
(a) 2000rpm



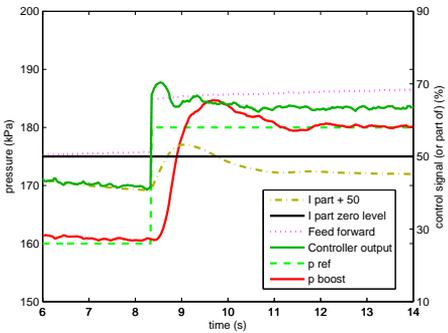
(b) 3000rpm



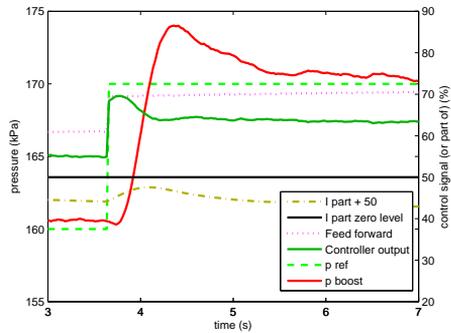
(c) 3500rpm



(d) 4000rpm



(e) 5000rpm



(f) 3500rpm

Figure B.2: Plots of step responses in reference pressure not presented in section 4.2. In almost all cases the pressure behavior becomes as desired, a small overshoot of about 5kPa and no oscillations. Only for 3500rpm the overshoot is slightly to high, about 8kPa.

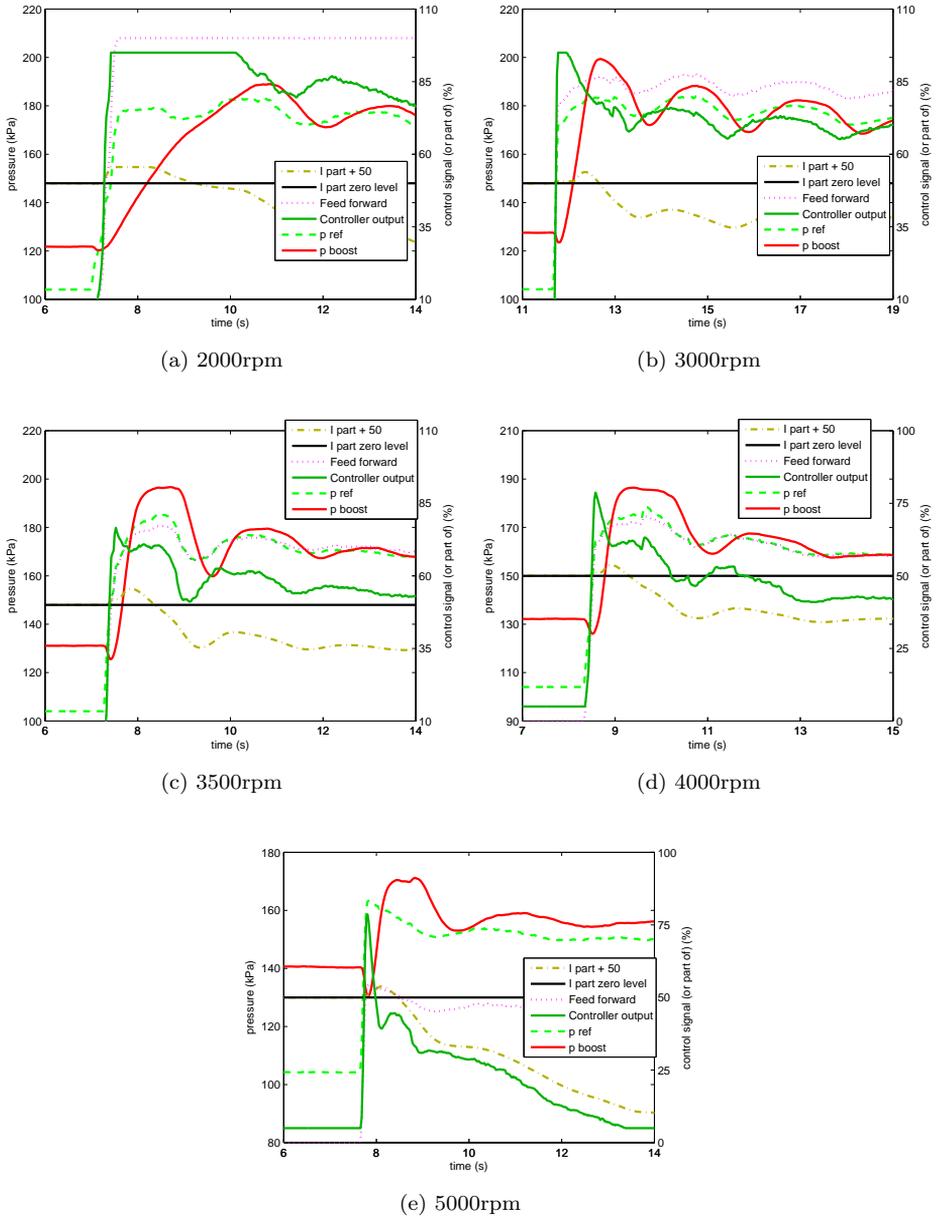


Figure B.3: Plots of step responses in gas pedal position not presented in section 4.2. In all cases the overshoot is a bit large, and in all cases the feedforward is too large after the step. For 3000rpm the the pressure oscillates for several seconds, largely due to the throttle behavior when the overshoot in boost pressure becomes to high.

# Appendix C

## Nomenclature

Listed here are the variables and their subscripts used in the thesis.

Variable	Parameter	Unit	Subscript	Name
$p$	Pressure	$kPa$	$amb$	Ambient
$T$	Temperature	$K$	$bc$	Before compressor
$wg$	Wastegate	–	$ac$	After compressor
$f$	Frequency	Hz	$ic$	Intercooler
$\Pi$	Pressure ratio	–	$im$	Intake manifold
$A$	Area	$m^2$	$bt$	Before turbine
$R$	Specific gas constant	$\frac{J}{KgK}$	$at$	After turbine
$\gamma$	Ratio of specific heats	–	$us$	Upstream
$m$	Mass	$Kg$	$ds$	Downstream
$t$	Time	$s$	$wg$	Wastegate actuator
$N$	Engine speed	$rpm$	$dc$	duty cycle
$e$	Error	–	$pos$	Position
$F$	Force	$N$	$act$	Actual
			$ref$	Reference
			$comp$	Compressor
			$eff$	Effective