Institutionen för systemteknik
Department of Electrical Engineering

Master Thesis

Optimal Speed Profiles for Passenger Cars with Gear Shifting

Master Thesis performed in Vehicular Systems
at Institute of Technology at Linköping University
by

F. Xavier Llamas Comellas

Linköping 2012

Linköpings universitet
TEKNISKA HÖGSKOLAN

Department of Electrical Engineering
Linköpings universitet
SE-581 83 Linköping, Sweden

Linköpings tekniska högskola
Linköpings universitet
581 83 Linköping

Optimal Speed Profiles for Passenger Cars with Gear Shifting

Master Thesis performed in Vehicular Systems
at Institute of Technology at Linköping University
by

F. Xavier Llamas Comellas

Supervisor: **Christofer Sundstörm**
ISY, Linköpings universitet

Examiner: **Lars Eriksson**
ISY, Linköpings universitet

Linköping, 22 May, 2012

Abstract

Fuel consumption in passenger vehicles has become more important due to the significant increase of fuel cost and the awareness of the environmental impact. Thus finding optimal speed profiles and gear shift strategies to cover a distance in order to decrease fuel consumption is an important issue.

The optimization problem is presented and solved by means of dynamic programming and using a quasistatic vehicle model approach. Different models are presented in order to solve the problem with different assumptions. For one model the gear shifts are assumed to be instantaneous, and for another model used the gear shift maneuver requires a certain time. For the second model, the influence of taking into account a realistic way to compute the losses during a gear shift maneuver is also developed and studied. Comparing the results provided by the two models, this method to compute the gear shift losses in the optimization problem has influence in the optimal speed profiles compared to the models that do not take into account losses during the gear shift.

There are two engine models used in this project, a Willans approximation engine model and a map based engine model. The differences between the results from each engine model are explained and illustrated. Moreover, the influence of the trip length is studied, stating that without a time constraint of the driving mission, the time constraint has no influence on the results. This investigation highlights the importance of selecting a suitable discretization. A smart choice of the discretization is explained in order to have the coasting solution achievable in the deceleration phase.

In order to study the results in different road situations, the optimization problem is solved by changing the road topology. It is stated, like in previous research papers, that keeping constant speed to drive through a hill is optimal, if no braking is required to maintain constant speed. Finally some vehicle parameters are changed in order to observe its sensitivity on the optimization results. Results for several vehicle masses and engine sizes are illustrated and compared. Regarding the engine size it is shown that it has a significant influence on the fuel consumption as well as on the optimal speed profile.

Acknowledgments

I would like to thank my examiner, Lars Eriksson, for giving me the opportunity to do this master thesis at vehicular system division. I am very grateful to my supervisor, Christofer Sundstörm, for helping me every time I had a problem, and for his tremendous work reading and suggesting improvements to this report. I would also like to thank both of them and Martin Sivertsson for their valuable feedback during the weekly meetings.

Contents

Chapter 1

Introduction

Driving more efficiently has been an important issue since the fuel cost has increased significantly during the last decades. One way to reduce the fuel consumption, and thereby reduce the CO_2 emissions, is to improve the efficiency of the powertrain by means of technical advances, e.g. hybridization. But there is also another way to reduce the fuel consumption, and this is optimizing the driver behavior to reduce the fuel consumption. Giving advice to the driver, or directly controlling the vehicle speed and gear shifting, can be useful to operate the powertrain in the most efficient operating points, and thus reduce the fuel consumption. Further, the control can manage to use the vehicle energy in a better way, e.g. not dissipating energy while braking or using the kinetic and potential energy in a more efficient way in order to require less energy to propell the vehicle.

1.1 Background

Finding optimal speed profiles that minimize the fuel consumption has been done mainly in two ways. On one hand it is done by analytical methods including Pontryagin maximum principle or Lagrangian methods. There are a couple of papers using analytical methods. One of the first, Schwarzkopf and Leipnik [16], started to mathematically verify some various empirical driving tips for economical driving, e.g. keeping constant speed or constant throttle. Mathematically the optimization problem is solved using the Pontryagin maximum principle. The acceleration results from zero to a cruising speed in a level road conclude that shifting up at lower speeds could be useful to reduce fuel consumption. This is also stated by Saerens et al. [15]. Also maintaining a steady state speed is optimal. Schwarzkopf and Leipnik also show some optimal speed gear shifting plots for a given parameterized car in different road slope situations. In these situations the optimal speed profile is not constant and the throttle is also varying, but the variations in the speed profile are small. Later, Hooker [10] criticized previous works of using a too simplified car model in order to be able to get accurate results.

Chang and Morlok [3] also faced this problem using analytical methods. This paper reviews what is found by Schwarzkopf and Leipnik [16]. The benefits of maintaining the speed is proved using Lagrangian methods. It is also proved that this is true for not only level roads. Maintaining the speed is optimal for curve sections and uphill slope sections, which contradicts what is stated in Schwarzkopf and Leipnik. If a downhill slope is added, the optimal solutions will still be to maintain speed if no braking is required to maintain it. The assumptions made for solving the slope problem are that the speed must satisfy any speed limits and also that vehicle must have enough power to maintain the speed on the ascending slopes.

On the other hand there are the numerical methods, which mostly are used for solving this optimization problem. A common method used by the researchers is dynamic programming. The studies done by numerical methods show many different results depending in which kind of tests are performed. For example a lot of them focus on the acceleration phase. This can be treated as a single phase, just to find the optimal speed profile for accelerating from one speed to another one. Hooker [10] performed a study with 8 different cars, and the optimal speed profile had a high variation from car to car, and none of them could be chosen as representative. Despite of this, the fuel economy varied relatively little over the different cars. Others like Jorge et al. [11] and Saerens et al. [14] proved that it was possible to design an algorithm with dynamic programming in order to find the optimal acceleration profile. The first one proposes a recursive algorithm that finds the optimal solution after a number of iterations. The second one performs sensitivity analysis to compare the optimal path with different alternatives showing the effects of the relative start time and the shape of the engine speed profile. In that paper, Saerens et al. conclude that the total fuel savings might be low in general terms but these are of the same order that the savings achieved with start/stop and regenerative braking. In another study, Saerens et al. [15] tested the acceleration phase comparing the results between assuming an instantaneous gear shift or considering the shift dynamics. They state that the shift dynamics influence is small in the fuel consumption, however no fuel consumption is considered during the gear shift maneuver.

There are other studies that are focused on finding the optimal speed profile for covering a fixed distance. In that case there are the studies of Gausemeier et al. [4] and Luu et al. [12]. Both are quite similar, and suggest not only the fuel consumption as an objective function to be minimized but also adding other terms like time cost. The two studies show how the resulting speed profiles differ with different values of the weight of the factors in the objective function. Gausemeier et al. [4] also suggests a way to reduce the searching points of the algorithm in order to reduce the computation load. Luu et al. [12] takes into consideration the driver comfort in its calculations, and concludes, as Hooker [10], that smoothing the speed profile has little effect on the fuel economy.

Hooker also provides some results in finding an optimal speed profile for covering a distance. He states that the fuel economy is sensitive to how quickly one covers the distance, but less sensitive to the shape of the acceleration path. Hooker also suggests that hard braking was the most efficient way to stop the car. This is stated because with hard breaking there is less idling time and as a result less fuel consumption during idling, but nowadays the cut-off fuel feature exists. Hooker also provided results regarding optimal cruise speed for 8 different vehicles and the optimal speed profiles for driving over hills. The interesting fact is the low optimal cruise speed for some vehicles like the Toyota Corolla, which is $28km/h$.

Finally there is another kind of study (Mensing et al. [13]) that tries to transform an existing drive cycle into an eco-drive cycle. The most interesting results are that even there is an important reduction in fuel consumption; this one does not come mainly for operating the ICE in more efficient operating points. The most important part of the fuel saving comes from a reduction of the energy required to perform the drive cycle. Also keeping the vehicle mean speed lower reduces the resistance forces. The fuel consumption reduction with this eco-drive cycle is about 16% compared to the conventional driving cycle.

To sum up, despite the number of similar studies in this area, gear shifting has not been treated deeply as part of the optimization problem. The authors that studied the influence of gear shifting in a complete trip are Hooker [10], Saerens et al. [15], and Mensing et al. [13], while others used an automatic gearbox with a certain shift strategy [12, 4]. It is not clear how the gear is optimally selected in the first two papers [10, 15], but it seems that there is no penalty factor for gear shift and that this can be done instantaneously. In[15], a certain speed penalty factor is taken into account during gear shifting, however no fuel consumption is considered during the maneuver.

1.2 Problem Formulation

The aim of this Master Thesis is to provide useful information about optimal speed profiles and gear shifting patterns for different driving situations and car parameters. Especially, how to perform the acceleration to cruising speed and then how to stop the car to cover a distance in an optimal way. Furthermore the problem will be solved for different parameter configurations, e.g. changing the road topology and varying trip length. The addition of constraints will be useful to provide information for different driving situations, like setting the time to drive from one point to the other.

In order to validate the results, other investigations will be performed within this Master thesis, e.g. investigate how the optimal constant cruising speed is affected by the trade-off between the air drag in high speed and the increased engine friction in lower gears. Another topic that will be investigated is the influence in optimal driving behaviour of the vehicle using an instantaneously gearshift model compared to a gearshift that requires some time to shift the gear.

Chapter 2

Vehicle Model

The vehicle model is designed following a quasistatic approach, some times also called the inverse approach. This approach uses the vehicle speed and acceleration as inputs, as well as the grade angle of the road, in order to compute the tractive force required at the wheel. A driveline model, presented in Section 2.1, and also an engine model, presented in Section 2.2, are used to translate the required tractive force into fuel consumption. Figure 2.1 shows a diagram about the quasistatic approach used in this study. Further information about the quasistatic approach can be found in e.g. [7].

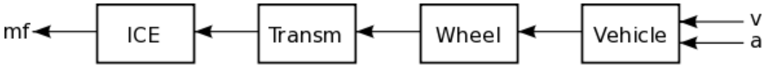

Figure 2.1. Quasistatic approach diagram of the vehicle.

2.1 Driveline Model

As said previously, the force or power required at the wheels to achieve the requested speed, v, has to be computed. In this case the power required in the wheels is computed using equation (2.1), from [7]

$$P_{wheel} = \underbrace{\frac{1}{2}A_f \cdot \rho_a \cdot c_d \cdot v^3}_{\text{air drag}} + \underbrace{c_r \cdot m_v \cdot g \cdot v}_{\text{rolling resistance}} + \underbrace{(m_v + m_r) \cdot a \cdot v}_{\text{inertia}} \tag{2.1}$$

where A_f is the frontal area of the vehicle, ρ_a is the air density, c_d is the aerodynamic coefficient, c_r is the rolling coefficient, m_v is the vehicle mass, and g is the gravity acceleration. The vehicle acceleration is denoted a, and m_r is the rotating mass computed by

$$m_r = \frac{1}{r_w^2} J_w \tag{2.2}$$

where r_w is the wheel radius. The shafts and the clutch are assumed stiff and their inertia is lumped with the wheel inertia, and is denoted J_w.

Level road is assumed in (2.1). If this is not the case, the gravitational term P_g, has to be added to (2.1)

$$P_g = m_v \cdot g \cdot \sin\alpha \cdot v \tag{2.3}$$

where α is the road slope angle.

Once the power in the wheels is computed, this power can be transferred through the drivetrain in order to find the torque, T_e, required of the engine. This transfer takes into account if power is required or provided by the wheels.

$$T_e = \begin{cases} \dfrac{P_{wheel}}{\eta_{gb} \cdot \omega_e} & T_e > 0 \\ \dfrac{P_{wheel} \cdot \eta_{gb}}{\omega_e} & T_e < 0 \end{cases} \tag{2.4}$$

In the expression ω_e is the engine torque and speed respectively, and η_{gb} is the gearbox efficiency. The engine speed is computed as

$$\omega_e = \frac{v \cdot i}{r_w} \tag{2.5}$$

where i is the gear ratio.

2.2 Gasoline Engine Model

Once the engine speed and torque have been found, the fuel consumption is to be computed. There are several ways to model an internal combustion engine, ICE. In this thesis two methods are used to model the engine; the Willans approximation and an engine map.

The engine map model is based on measurements of an engine and thereby it is a model that represents a real engine. On the other hand the Willans aproximation is a very useful aproximation of an engine based on few model equations. Despite this simplicity, the fuel consumption values are accurate and close to the ones provided by the map model. This is stated by comparing Figure 5.1 with Figure 5.2. However, the behaviour of both models is not exactly the same, specially regarding the engine efficiency at lower engine speeds. This different caracteristic can be seen by comparing the efficiency plots presented in Figure 2.2. The two engine models are further described in Sections 2.2.1 and 2.2.2.

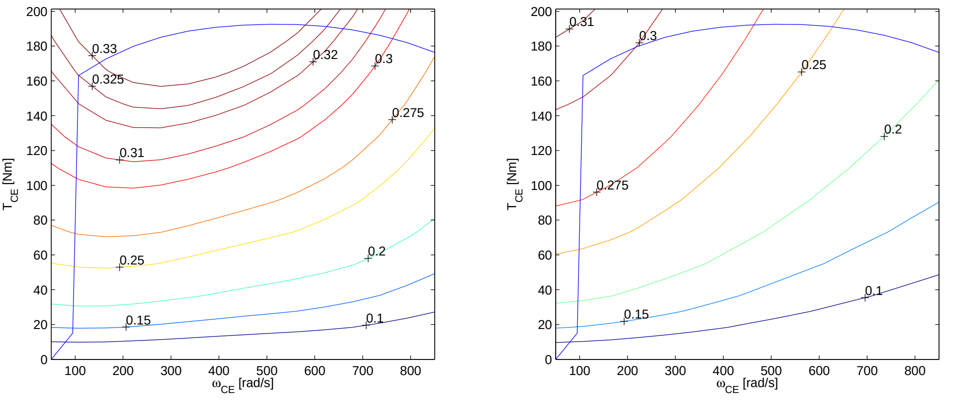

Figure 2.2. Efficiency maps and maximum torque curves for the map based model (left) and the Willans engine model (rigth).

2.2.1 Willans Line Aproximation

The Willans aproximation is based on normalized engine variables, that do not depend on the engine size, like the mean effective pressure, p_{me}, and the fuel mean pressure, p_{mf}. The output power of the engine, represented by the mean effective pressure, is computed by an affine equation [7]

$$p_{me} = e \cdot p_{mf} - p_{me0} \tag{2.6}$$

where the variable p_{me} is computed as

$$p_{me} = \frac{N \cdot \pi \cdot T_e}{V_d} \tag{2.7}$$

where V_d is the engine displacement and N is a parameter that depends of the engine type, $N = 4$ for a four-stroke engine. The parameter e is the indicated engine efficiency, and is modelled to be a constant in this investigation. The variable p_{mf} is the mean effective pressure that a 100% efficiency engine would produce, and is computed as

$$p_{mf} = \frac{H_l \cdot m_f}{V_d} \tag{2.8}$$

where H_l is the fuel lower heating value and m_f is the mass of fuel burnt in every combustion. Finally the parameter p_{me0} represents the friction losses and the pumping losses in the engine, and is modelled as

$$p_{me0} = p_{me0,f} + p_{me0,g} \tag{2.9}$$

where $p_{me0,g}$ represents the pumping loss and is assumed to be constant. The term $p_{me0,f}$ represents the friction losses and is modelled using the ETH friction model from [6].

$$p_{me0,f} = k_1 \cdot (k_2 + k_3 \cdot S^2 \cdot \omega_e^2) \cdot \Pi_{bl} \cdot \sqrt{\frac{k_4}{B}} \tag{2.10}$$

where k_1, k_2, k_3, k_4 are model constants, S is the engine stroke, B is the engine bore, and Π_{bl} is the boost layout of the engine.

Finally, these parameters are used to compute the mass of fuel consumed, given T_e and ω_e from (2.4) and (2.5) respectively [7].

$$m_f = \frac{\omega_e}{e \cdot H_l}(T_e + \frac{p_{me0} \cdot V_d}{4 \cdot \pi} + J_e \cdot \dot{\omega}_e) \cdot \Delta t \tag{2.11}$$

Where J_e is the engine inertia and $\dot{\omega}_e$ is the engine angular acceleration.

2.2.2 Engine Map

The second model used to compute the fuel mass consumed is based on an engine map from the QSS toolbox [5]

$$\dot{m}_f = f(T_e + J_e \cdot \dot{\omega}_e, \omega_e) \tag{2.12}$$

The fuel mass flow in kg/s can be computed using the engine torque plus the inertial effects and speed as inputs. In this case the efficiency and the losses are already taken into account in the data provided. This model can be scaled in order to obtain data for different engine sizes.

As for the Willans model the friction losses are modeled by (2.10), that is function of engine speed, for the map model the friction losses are included in the map value. However, in order to decide when the fuel cut-off feature can be enabled, the engine friction torque is modelled with an affine function that depends on the engine speed. The fuel cut-off is an important feature during the deceleration phase of the problem, the influence of this feature is further explained in Section 5.5.1.

$$T_{e,f} = A + B \cdot \omega_e \tag{2.13}$$

The parameters A and B are selected in order to set the friction curve with similar values that the curve computed by (2.9).

2.3 Vehicle Parameters

Every optimization considers different constraints and assumptions. Despite this, there are many parameters that do not change throughout the different optimizations if the contrary is not specified. Table 2.1 contains some physical constants used, and the vehicle and driveline parameters used for carrying out the different optimizations are given in Table 2.2. Table 2.3 contains the used parameters for the Willans model and Table 2.4 contains the parameters used for the map based model.

Table 2.1. Physical constants used in the optimizations.

Parameter	Symbol	Value	Unit
Gravity acceleration	g	9.81	m/s^2
Air density	ρ_a	1.29	kg/m^3
Gasoline lower heating value	H_l	44.6	MJ/kg

Table 2.2. Vehicle and driveline parameters used in the optimizations.

Parameter	Symbol	Value	Unit
Vehicle mass	m_v	1500	kg
Frontal area	Af	2	m^2
Air drag coeff.	c_d	0.3	-
Rolling res. coeff.	c_r	0.01	-
Wheel radius	r_w	0.3	m
Driveline inertia	J_w	0.6	$kg \cdot m^2$
Gearbox efficiency	η_{gb}	0.98	-
Ratio gear 1	i_1	13.0529	-
Ratio gear 2	i_2	8.1595	-
Ratio gear 3	i_3	5.6651	-
Ratio gear 4	i_4	4.2555	-
Ratio gear 5	i_5	3.2623	-

Table 2.3. Willans model parameters used in the optimizations.

Parameter	Symbol	Value	Unit
Displacement	V_d	$2.3 \cdot 10^{-3}$	m^3
Indicated efficiency	e	0.35	-
Idling speed	ω_{idle}	95	rad/s
Stroke	S	$79.5 \cdot 10^{-3}$	m
Bore	B	$96 \cdot 10^{-3}$	m
Cylinders	-	4	-

Table 2.4. Map model parameters used in the optimizations.

Parameter	Symbol	Value	Unit
Displacement	V_d	$2.3 \cdot 10^{-3}$	m^3
Idling speed	ω_{idle}	95	rad/s

The maximum torque available, $T_{e,max}$, that is a function of ω_e, is determined by interpolating the maximum torque data from the QSS toolbox [5]. This data is scaled in order to match the engine dimensions for both models.

Chapter 3

Dynamic Programming and Discrete Approach

The optimization problem is solved by means of Dynamic Programming (DP). The DP algorithm requires a discrete state-space model. Thus, the problem has to be reformulated into discrete form. The following sections describe the mathematical implementation of the algorithm, as well as an outline of the DP process. More information about this technique can be found in [1, 2].

3.1 State-Space Discretization

For this project the variable to be optimized through the distance is the speed. Thereby, the state required for the DP algorithm is speed. The tracking variable used throughout the algorithm is the distance and not the time, because using distance allows to set the trip length constraint. Depending of the problem constraints and the assumptions made, other states have to be added like gear ratio or time.

The trip distance is discretized in N steps with an step length constant and equal to h. The step number is denoted by k and follows the relation

$$x_k = k \cdot h \tag{3.1}$$

The discretization of the states is made equidistant for each state variable e.g. speed or gear engaged, using a step width (δ) as thin as possible in order to get more accurate results e.g.

$$V = \{v_o, v_o + \delta, v_o + 2\delta, \ldots, v_f\} \tag{3.2}$$

where V is speed state.

The discretization of the states used in Chapter 5, if there is not specified another value, is $\delta = 0.1m/s$ and $h = 5m$.

3.2 Cost Function

The optimization objective is to minimize the fuel consumption. Thereby the fuel used to travel a certain distance between two known states in discrete formulation is

$$J = \sum_{k=0}^{N-1} \zeta_k(x_k, x_{k+1}, v_k, v_{k+1}, \alpha_k) \tag{3.3}$$

where ζ_k is the fuel consumed in step k, x_k is the distance in step k, v_k is the speed in step k, and α_k is the road slope angle in step k and is assumed to be constant during the interval.

For each step, the cost function ζ_k, is computed using (2.11) or (2.12) depending on the engine model used. The variables required by (2.11) or (2.12) are computed using (2.4), and (2.5). Constant speed is assumed during the interval, and the speed is computed as

$$v = \frac{v_k + v_{k+1}}{2} \tag{3.4}$$

and the acceleration as

$$a = \frac{v_{k+1} - v_k}{t} \tag{3.5}$$

where

$$t = \frac{x_{k+1} - x_k}{\bar{v}} \tag{3.6}$$

3.3 DP Algorithm

The algorithm objective is the minimization of the cost function,

$$\min \sum_{k=0}^{N-1} \zeta_k(x_k, x_{k+1}, v_k, v_{k+1}, \alpha_k) \tag{3.7}$$

with a given number of constraints depending on each problem. Defining S_k as the possible states in stage k, $\zeta_k^{i,j}$ is the arc cost to go from $x^i \in S_k$ to $x^j \in S_{k+1}$. The DP algorithm proceeds as follows:

1. Assign a final cost $J_N(x_N) = \zeta_N(x_N)$

2. Set $k = N - 1$
3. For all the points in the state-space grid, find the optimal cost to go

$$J_k(x^i) = \min_{x^j \in S_{k+1}} \left\{ \zeta_k^{i,j} + J_{k+1}(x^j) \right\}, \forall x^i \in S_k. \tag{3.8}$$

4. Repeat (3) for $k = N - 2, N - 3, \ldots, 0$.
5. The optimal cost is J_o. The optimal control is obtained by following the path stored in the solution arrays with a given initial state.

Chapter 4

Model Design

As mentioned previously, the aim of this study is to obtain optimal speed profiles for cars in different driving situations. The different models based on dynamic programming designed for obtaining these speed profiles are explained in this chapter. The results provided by running optimizations with the models of this chapter are presented in Chapter 5.

4.1 Instantaneous Gear Shift Model

The basis of this model is the assumption that no time is required in order to perform the gear shifting maneuver. Thus, the model can be designed without adding the engaged gear as a state variable of the system, so the only state variable is the vehicle speed. For every arc calculation, the algorithm computes the cost for every gear ratio and stores the gear that achieves the lowest fuel consumption value.

The same constraints and assumptions mentioned in the previous paragraph, used to design the model with instantaneously gear shift using the Willans engine model, are used in order to design the model with the map based engine model. However, the solution obtained from that model is oscilating where it should keep a constant cruising speed. Moreover with these oscillations, the system gains energy by assuming a linear form to a fuel consumption model that is not linear and thus distorts the results. One way to solve this problem is by using the energy formulation presented in [8]. However, the solution implemented in this project for solving that problem is to change (3.4) that computes the mean speed in the interval, for an expression that takes into consideration the non linearity of the speed profile during the interval. The equation used is expressed as follows

$$\bar{v} = \begin{cases} v_k + r \cdot (v_{k+1} - v_k) & v_k \leqslant v_{k+1} \\ v_{k+1} + q \cdot (v_k - v_{k+1}) & v_k > v_{k+1} \end{cases} \tag{4.1}$$

where r and q are constant coefficients. Assuming a concave speed profile when $v_k \leqslant v_{k+1}$, r is a parameter that sets the mean speed value higher compared to the value computed by (3.4). The parameter q works in the same sense, but assuming a convex speed profile when $v_k > v_{k+1}$ and thereby it sets the mean speed value slightly below the value computed by (3.4). With this improvement in the map model, the oscillations are prevented.

4.2 Instantaneous Gear Shift Model with Time Constraint

Adding a time constraint to the model from Section 4.1 is initially done by including time as a new tracking variable. Moreover the state-space discretization has two variables, speed and delta distance. Delta distance is a variable that keeps track of the deviation in distance computed to the distance traveled if the speed is equal to the average speed at all times. However, the price to pay for adding a new state variable is a high computational time, and thereby achieving acurate results with a refined grid will require a lot of computational time. A different way to solve the problem is to add time as a term in the cost function [9]. This is the solution used in this project and the cost function is

$$J = M + \beta T \tag{4.2}$$

where M is the amount of fuel, T is the time, and β is a scalar penalty parameter that weighs the importance of time in the expression. Hence, a solution to set the trip time is to find a suitable β parameter for a given problem. Following the criteria in [9], a constant speed solution is assumed that travels the problem distance, L, in the desired time.

$$\bar{v} = L/T \tag{4.3}$$

Thus, β is found by finding the β value that minimizes (4.2). Analytically, and using (2.11) and (2.1), this leads to

$$\beta = \frac{\bar{v}^2 \cdot i}{r_w \cdot e \cdot H_l} \left(r_w \cdot c_d \cdot A_f \cdot \rho_a \cdot \bar{v} + \frac{\frac{dp_{me0,f}}{d\omega_e} \cdot V_d}{4\pi} \right) \tag{4.4}$$

where $\frac{dp_{me0,f}}{d\omega_e}$ is the speed derivative of (2.10). This parameter fixes the solution to finish in the same time that a constant speed solution, $\bar{v}$, does. However this parameter is not exactly providing the desired solution because the desired solution is not a profile close to constant speed, there are important acceleration and a deceleration phases. Nevertheless it is a good starting point to find a value and then proceed with the trial and error method in order to find the suitable β that leads to the desired time of the driving mission. This strategy, even though it requires running the algorithm several times, allows the possibility to obtain a more accurate solution with less computational time compared to the one introduced in the beginning of this section and initially tried.

4.3 Finite Time Gear Shift Model

The models presented in previous sections have used the assumption that a gear shift maneuver could be done instantaneously. One of the aims of this project is to study the influence of the gear shift maneuver as well as gear selection into the speed profiles.

For this model, the engaged gear is used as a state variable as well as the vehicle speed, and thus the optimal gear selection is found by the algorithm. The gear shift model has been inspired by the gear shift modelling that is found in [8]. In order to compute the losses of changing the gear engaged, the assumption made is that during a certain time the engine is not able to provide tractive torque. The maneuver time, e.g. time elapsed since the clutch is pressed until the engine is engaged again, has been set at one second. During this second the vehicle speed profile is computed by

$$\frac{dv}{dt} = \frac{-1}{m_v + m_r}\left(\frac{1}{2}\rho_a \cdot Af \cdot c_d \cdot v^2 + m_v \cdot g \cdot c_r\right) \tag{4.5}$$

since P_{wheel} in (2.1) is set to zero. The ODE is solved numerically using the initial speed in order to know the vehicle speed after the one second maneuver. The cost of the maneuver is assumed to be the fuel consumed during one second with the engine in idling. With the speed after a gear shift, the distance travelled during the gear shifting is known as well. Knowing the final speed and the distance travelled after a gear shift, the point in the state-space grid can be set, (v_l, x_l), and thereby the cost to go from that point to every point in the grid x_{k+1} can be computed. A gear shift computation is illustrated in Figure 4.1.

A new method is developed in this project that computes the cost to go in a different way than in [8], where the cost to go is interpolated between the nearest grid points to (v_l, x_l). In this study after a gear shift maneuver, the algorithm always ends up in a grid point and thereby interpolation is not required. However, this solution has problems when the distance travelled during the gear shift is bigger or nearly equal to the step length used in the algorithm. This happens when the initial speed value multiplied by the gear shift time is close to a multiple value of the distance step h. When this happens, two problems may occur. The

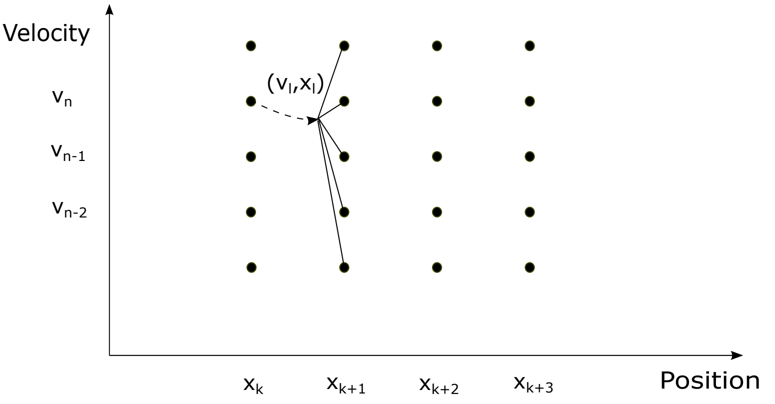

Figure 4.1. Computation of the cost to go from v_n, x_k to the states in x_{k+1} with a gear shift maneuver.

first is that if the maneuver finishes close to the next distance step, the speed points of the next state might not be reached due to a high acceleration or braking is required in the short distance that exists from the maneuver point to the next grid step. The second problem is that the gear shift maneuver ends after the next distance grid step and the cost to go cannot be computed.

In order to solve these problems, the original DP algorithm needs to be modified. If the gear shift maneuver ends after the next distance grid step, a pointer saves to which distance step the cost is computed. This pointer will be used by the algorithm to skip this step grid and compute the costs for the one that the pointer is stating. The pointer solution is also used to solve the problem that occurs when the gear shift maneuver ends close to the next state. This system of pointers, depending on the gear shifting length and the grid step distance, decides where is the next state to look for costs. The pointer makes a difference if the next step has to be reached by accelerating or braking because if braking is needed, the algorithm allows to do a gradual braking through the distance step even while performing the gearshift maneuver. The strategy is illustrated in Figure 4.2, and it assures that after the gear shift the algorithm can continue computing the optimal solution.

Regarding the results obtained by this model, it is important to explain how the gear selection is illustrated in the figures in Chapter 5. When a distance step is skipped by the algorithm due to a gearshift, the gear selected is not drawn in the figure. Moreover, when there is a gear shift, the gear shift model uses the gear engaged after the maneuver time to compute the cost to go. Therefore when there is a gear shift in the speed profile, the gear to take into consideration is the one drawn in the next distance step available.

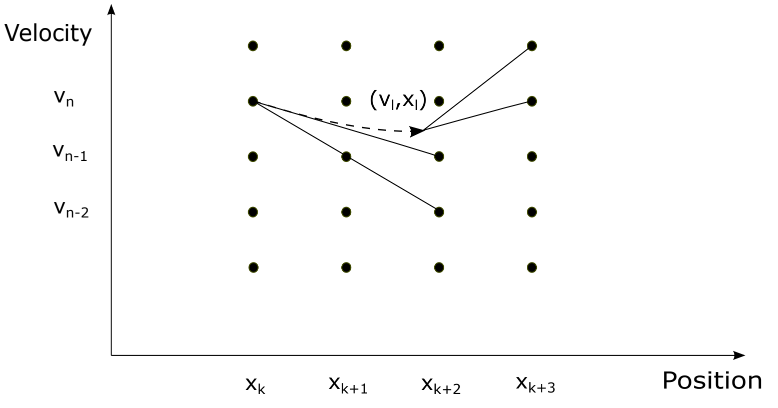

Figure 4.2. Skipping grid points strategy from (v_n, x_k) during a gear shift maneuver. In the figure, (v_l, x_l) is the point where the gear shift maneuver ends.

4.4 Finite Time Gear Shift Model with Time Constraint

This model takes into account the gear shift maneuver and adds the posibility to fix the desired trip time. However, adding a new state, e.g. the time, to an already 3 dimensional state-space (distance, speed, and gear) algorithm would be too much in computational time if the aim is to run the algorithm with dense grids in order to obtain accurate results. Hence, this model is an upgrade of the one presented in Section 4.3 with the β parameter presented in Section 4.2 in order to require less computational time.

Chapter 5

Results and Discussion

This chapter analyzes and discusses the optimization results provided by the models presented in Chapter 4. This is done by analyzing the differences between results with different constraints and parameters, and the effects of the model assumptions in the results. The discretization of the states used in this chapter, if another value is not specified, is $\delta = 0.1 m/s$ and $h = 5m$.

5.1 Constant Speed Fuel Consumption

In order to have a reference of fuel consumption with the vehicle in constant speed, the engine models are tested. The fuel consumption for a certain constant vehicle speed is computed for every gear as well as the fuel consumption for a continous variable transmission (CVT) that leads to that the engine to operate always at idling speed. The results are presented in Figure 5.1 for the Willans model and in Figure 5.2 for the map based model. It is important to know that the engine torque limits have not been veryfied in this figure for high speeds, however the important region to look at is at lower speeds.

These plots are useful to understand the differences between cruising at constant speed in each gear. It is especially useful to observe the fuel consumption behaviour if the constant speed increases and compare the results between models. Looking at the results, see Figures 5.3 and 5.4, of the models without time constraint of Chapter 4, it is interesting to observe that the cruising speed selected, arround $9m/s$ $(33km/h)$, is the one available that leads to the lowest fuel consumption. This speed is caracterized by setting the engine speed as low as possible with fifth gear without falling in the idling region. Moreover, there is a lower fuel consumption value achieved by the CVT arround $14m/s$ $(50km/h)$. It would be interesting to ensure that future gear box designs make that point available by adding a sixth gear or setting a suitable transmission relation for fifth gear. However, these optimal cruising speed values are low and this reduces the driveability and the passengers comfort.

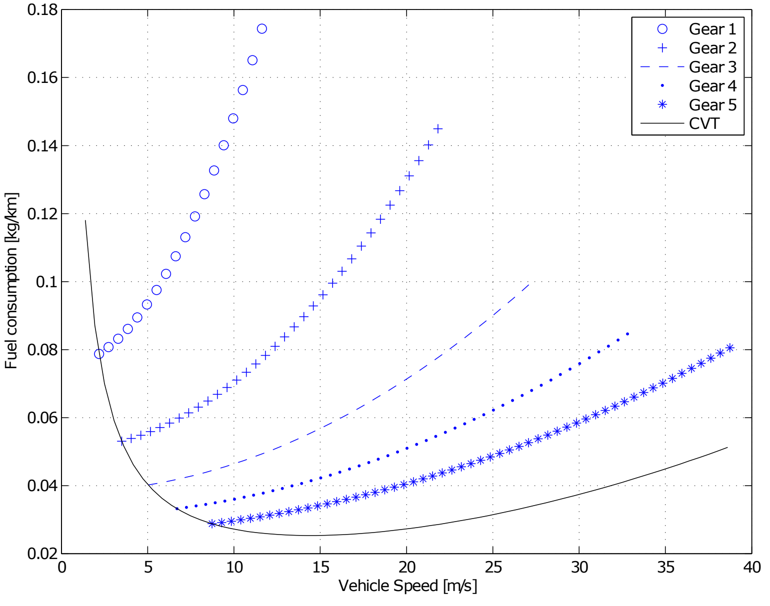

Figure 5.1. Fuel consumption for constant vehicle speed and different gears. Willans engine model is used.

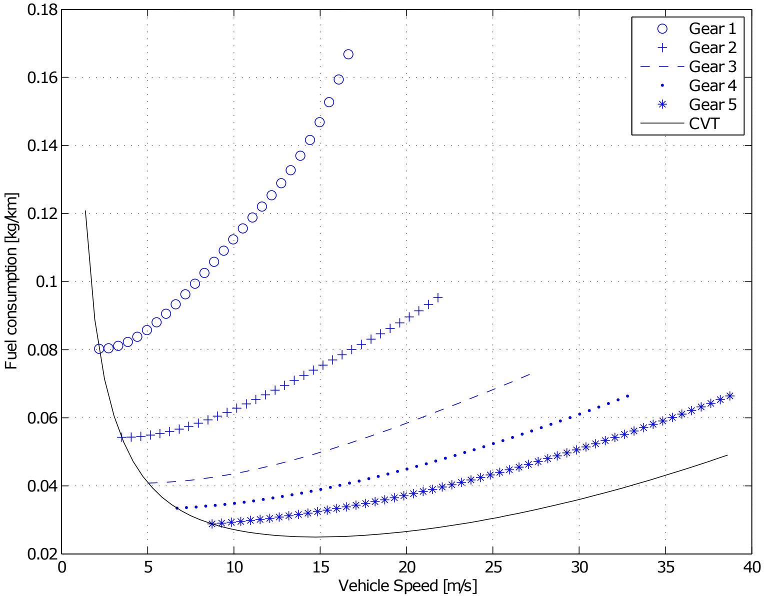

Figure 5.2. Fuel consumption for constant vehicle speed and different gears. The map based engine model is used.

5.2 Gear Shift Modelling Effects

One interesting reason for modelling the gear shift maneuver is to quantify the effects that this have to the speed profiles and the fuel consumption in relation to the model that assumes an instantaneously maneuver.

5.2.1 Trip time free

Figure 5.3 shows a comparation between the instantaneous gear shift model and the finite time gear shift model using the Willans engine model. The results are similar in general terms. However, the acceleration phase is more realistic for the finite time gear shift model than the instantaneous model. A longer distance is required to achieve the cruising speed and the speed losses during the gear shifting maneuver can be observed. The deceleration phase requires the same distance in both models, nevertheless the fuel consumption during this deceleration phase cannot be zero for the finite time gear shift model due to the downshift during this phase that requires a certain time of the engine in idle. During the deceleration phase the gear selection is different, for the finite time gear shift model there are less maneuvers and the gear zero is selected earlier.

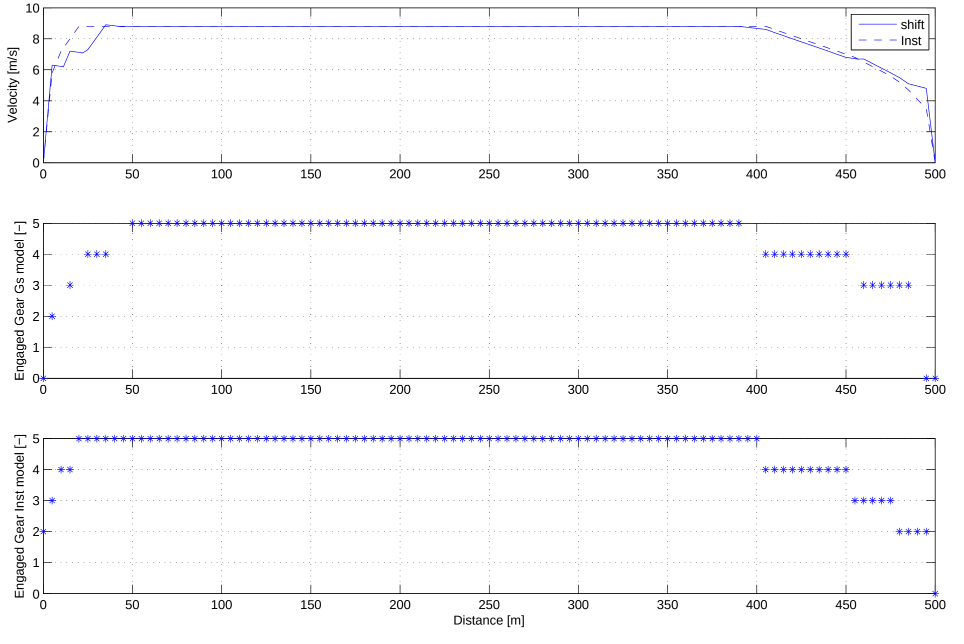

Figure 5.3. Optimal speed profiles and gear selection comparation between the gearshift model and the instantaneously model for 500m. Willans engine model is used.

On the other hand, Figure 5.4 presents the same comparation but with the map based engine model. The results are similar, the acceleration phase for the finite time gear shift model takes again a longer distance. The main difference in the deceleration phase is the gear selection. The instantaneous model selects every gear during the coasting phase, while the finite time gear shift model selects the zero gear directly from third gear.

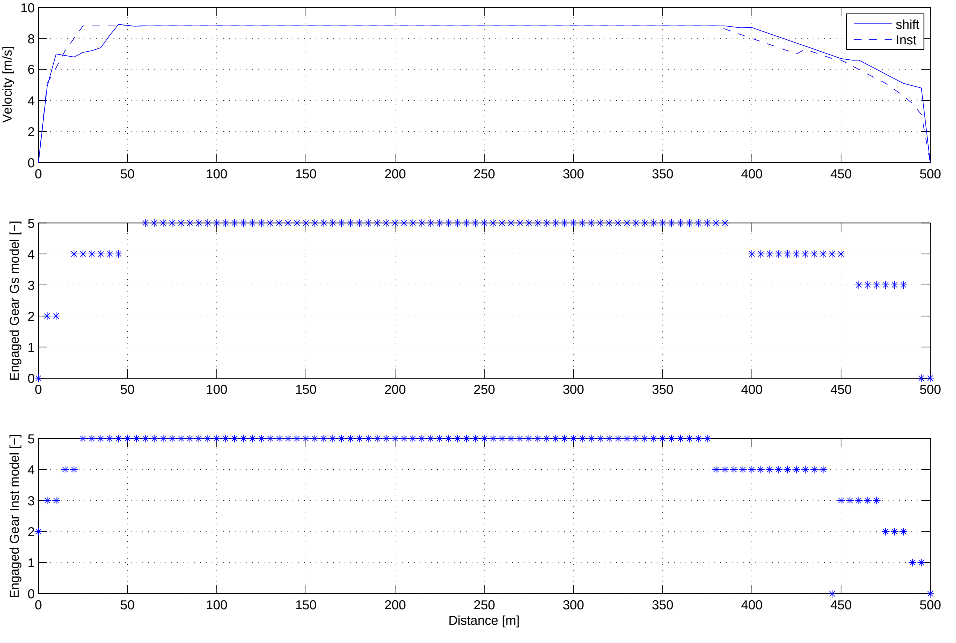

Figure 5.4. Optimal speed profiles and gear selection comparation between the gearshift model and the instantaneously model for 500m. The map based engine model is used.

Comparing the total fuel consumption for both models, it is stated that the finite time gear shift model requires more fuel in order to travel the same distance due to the fuel consumed and the speed losses during the manouver. Tables 5.1 and 5.2 show the fuel consumption for both models with each engine model.

Table 5.1. Fuel consumption for willans engine model for the two different models for gear shift described in Sections 4.1 and 4.3.

	Value	Unit
Instantaneously	4.43	$l/100km$
1 second gear shift	4.75	$l/100km$

Table 5.2. Fuel consumption for the map engine model for the Willans engine model for the two different models for gear shift described in Sections 4.1 and 4.3.

	Value	Unit
Instantaneously	4.37	$l/100km$
1 second gear shift	4.85	$l/100km$

5.2.2 Trip time constraint

In this section the time constraint is $50s$ which is a lower value than the fuel optimal solution without a time constraint presented in the previous section. Selecting a higher time constraint than the fuel optimal solution will lead the solution to follow the fuel optimal profile and then sit still at the start or the end stop.

The results for the time constrained models presented in Sections 4.2 and 4.4 are more or less similar to the ones presented in Section 5.2.1. However, it is shown that as there is a time constraint, the principal difference is that there are less shift manouvers in the deceleration phase for the gear shift model. This gear selection is due to the availability of a coasting solution within the grid. There is also an important difference between the results for the map based engine model and the Willans line that can be explained by the use of a different engine friction model. This friction model defines the torque limit that enables the fuel cut off feature and thereby the coasting solution is affected by this friction model. These coasting effects happen due to the choice of the speed discretization which are explained with more detail in Section 5.5.1. The results are shown in Figures 5.5 and 5.6.

The fuel consumption for these models are also higher for the gear shift model due to the same reasons stated in Section 5.2.1. The values for the fuel consumptions are presented in Tables 5.3 and 5.4.

Table 5.3. Fuel consumption for the Willans engine model for the two different models for gear shift described in Sections 4.2 and 4.4. Results with a time constraint of 50s.

	Value	Unit
Instantaneously	4.93	$l/100km$
1 second gear shift	5.19	$l/100km$

Table 5.4. Fuel consumption for the map engine model for the two different models for gear shift described in Sections 4.2 and 4.4. Results with a time constraint of 50s.

	Value	Unit
Instantaneously	5.04	$l/100km$
1 second gear shift	5.42	$l/100km$

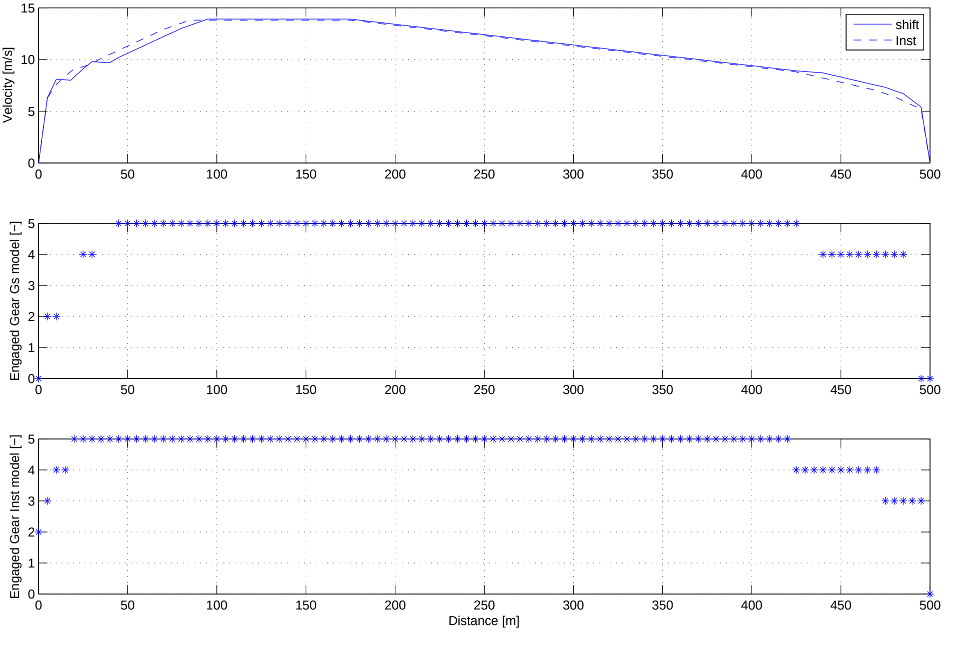

Figure 5.5. Optimal speed profiles and gear selection comparation between the gearshift model and the instantaneously model. Results for 500m, a time constraint of 50s, and using Willans engine model.

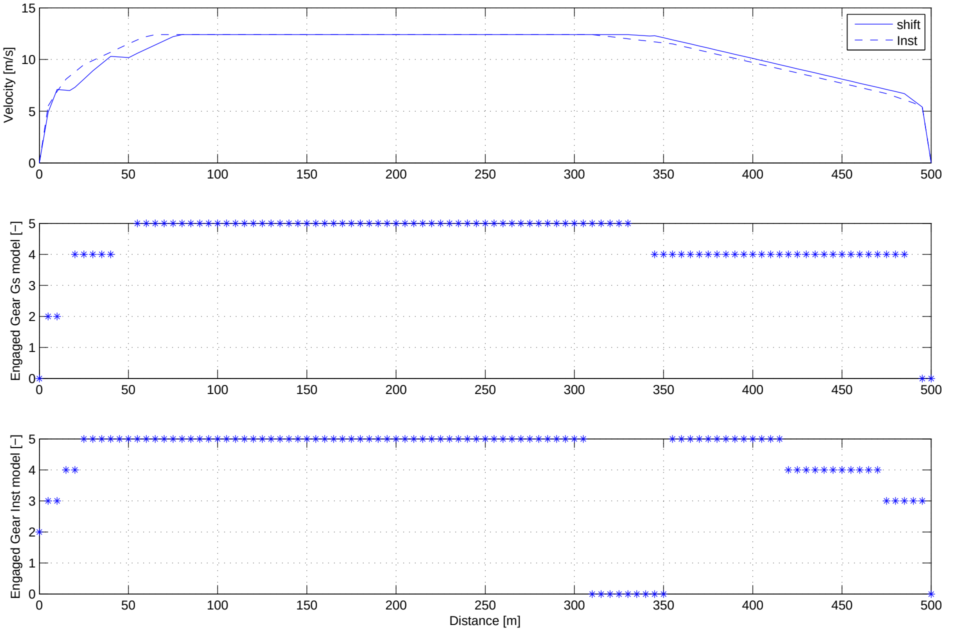

Figure 5.6. Optimal speed profiles and gear selection comparation between the gearshift model and the instantaneously model. Results for 500m, a time constraint of 50s, and using a map based engine mode.l

5.3 Engine Model Comparation

The different gear shift models presented in Chapter 4 are used to perform optimizations for both engine models explained in Section 2.2. In this Section the results of using both engine models in the different driving situations are compared.

5.3.1 Trip time free

Regarding the instantaneous gear shift model presented in Section 4.1, the comparison between the results for both engine models is presented in Figure 5.7. The main difference is that the solution for the map based engine model performs the acceleration in a longer distance. It also starts the deceleration earlier than the solution for the Willans engine model. As a result, the gear shift profile for both models differs. However, looking at the total fuel consumption values presented in Table 5.5 there is not a significant difference.

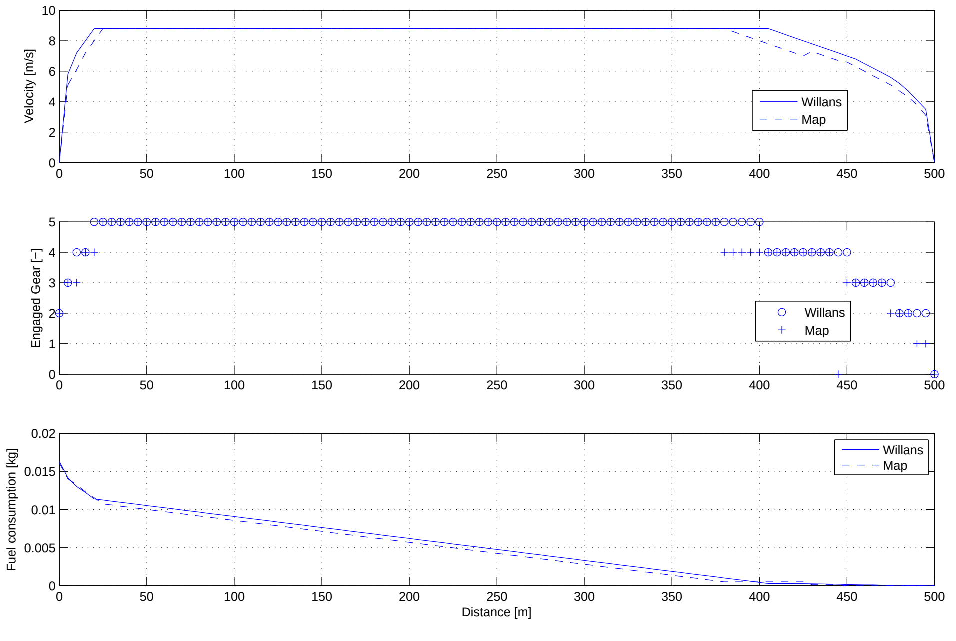

Figure 5.7. Optimal speed profiles and gear selection between the Willans engine model and the map based engine model are compared. Results for 500m without time constraint and using the instantaneously gear shift model.

Figure 5.8 shows the comparation of the results for both engine models using the gear shift model presented in Section 4.3. The differences are again in the acceleration phase. The acceleration phase for the map model takes a longer distance than for the Willans model. In addition, the gear shifting strategy is

Table 5.5. Fuel consumption for the instantaneously gear shift model.

	Value	Unit
Willans engine model	4.43	$l/100km$
Map based engine model	4.37	$l/100km$

different for both models, the Willans model engages second, third and fourth gears before reaching the cruising speed. On the other hand the map based engine model keeps the second gear longer and its acceleration is faster during these steps. Due to keeping the second gear for a longer distance, the third gear is skipped and the fourth is engaged during more distance steps.

The deceleration phase is nearly equal for both engine models. Table 5.6 contains the fuel consumption for both engine models, comparing the values with the values from Table 5.5 it is stated that the one that consumes less fuel is the Willans model.

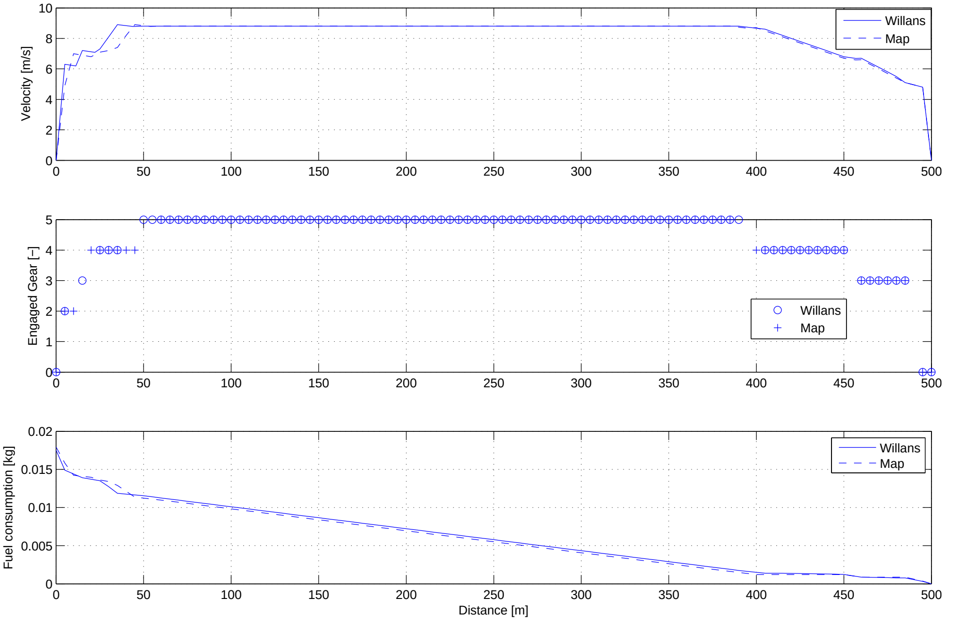

Figure 5.8. Optimal speed profiles and gear selection between the Willans engine model and the map based engine model are compared. Results for 500m without time constraint and using the gear shift model.

Table 5.6. Fuel consumption for the gear shift model.

	Value	Unit
Willans engine model	4.75	$l/100km$
Map based engine model	4.85	$l/100km$

5.3.2 Trip time constraint

Regarding the instantaneous gear shift model presented in Section 4.2, the comparison between the results for both engine models is presented in Figure 5.9. Looking at the figure, the acceleration phase for both engine models is quite similar. However, the important difference is between the cruising speed. For the Willans engine model the cruising speed is higher and the duration of it is shorter than for the map based engine model. This difference is in part due to discretization effects that are explained in Section 5.5.1, and also because each engine model uses a different expression for computing the engine friction torque.

Table 5.7 presents the fuel consumption values for both engine models, and the Willans model consumes less. Despite the significant difference in the optimal speed profiles, the fuel consumption is not that different.

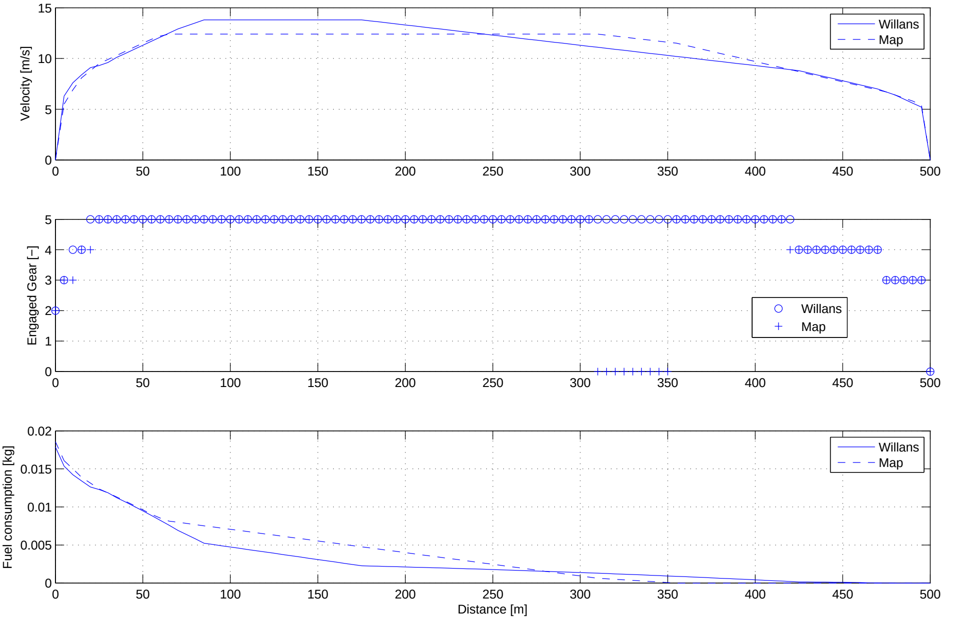

Figure 5.9. Optimal speed profiles and gear selection comparation between the Willans engine model and the map based engine model. Results for 500m with a time constraint of 50s and using the instantaneously gear shift model.

Table 5.7. Fuel consumption for the instantaneously gear shift model with time constraint.

	Value	Unit
Willans engine model	4.93	$l/100km$
Map based engine model	5.04	$l/100km$

Figure 5.10 shows the comparation of the results for both engine models using the finite time gear shift model presented in Section 4.4. There are differences in the acceleration phase, first the map based engine model performs a smoother acceleration than the one of the Willans model. However, the main difference is again in the cruising speed values and durations as explained for the instantaneous gear shift model with time constraint. This difference is in part due to discretization effects that are explained in Section 5.5.1, and also because each engine model uses a different expression for computing the engine friction torque. As a result, the deceleration phase for both models is very different, especially regarding the gear shift selection in the begining of it.

Table 5.8 contains the fuel consumption for both engine models. The Willans engine model consumes again less fuel than the map based engine model, and this difference has increased.

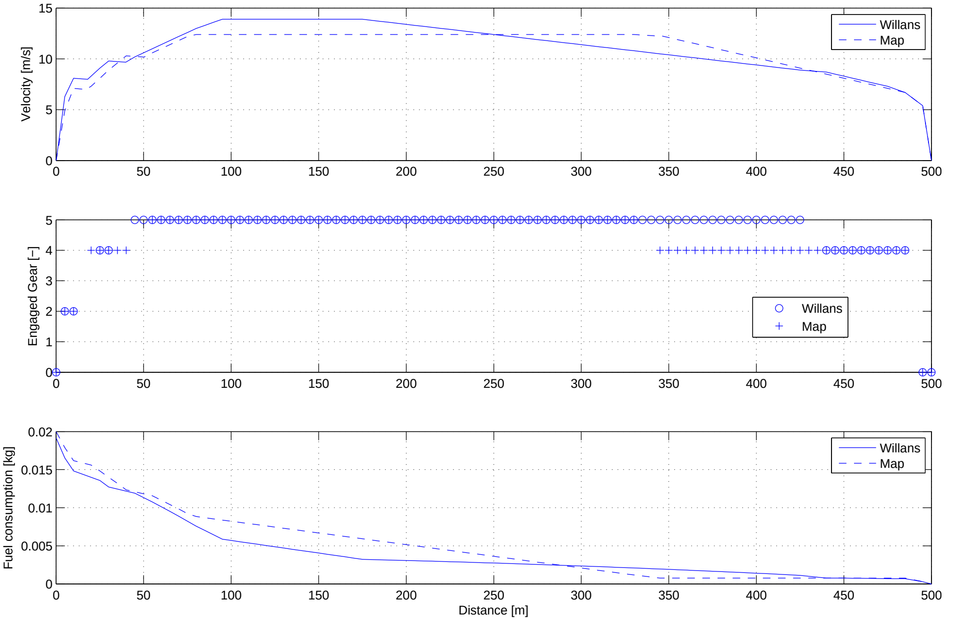

Figure 5.10. Optimal speed profiles and gear selection comparation between the Willans engine model and the map based engine model. Results for 500m with a time constraint of 50s and using the gear shift model.

Table 5.8. Fuel consumption for the gear shift model with time constraint.

	Value	Unit
Willans engine model	5.19	$l/100km$
Map based engine model	5.42	$l/100km$

5.4 Trip Length Effects

Another interesting question is if the optimal speed profile changes with the trip length.

5.4.1 Trip time free

Figure 5.11 compares the acceleration and deceleration results for a $500m$ trip with the results for a $1000m$ trip. The optimal strategies for both driving situations are exactly the same. Thus, the only difference is a longer constant cruising speed phase in order to reach the desired trip length.

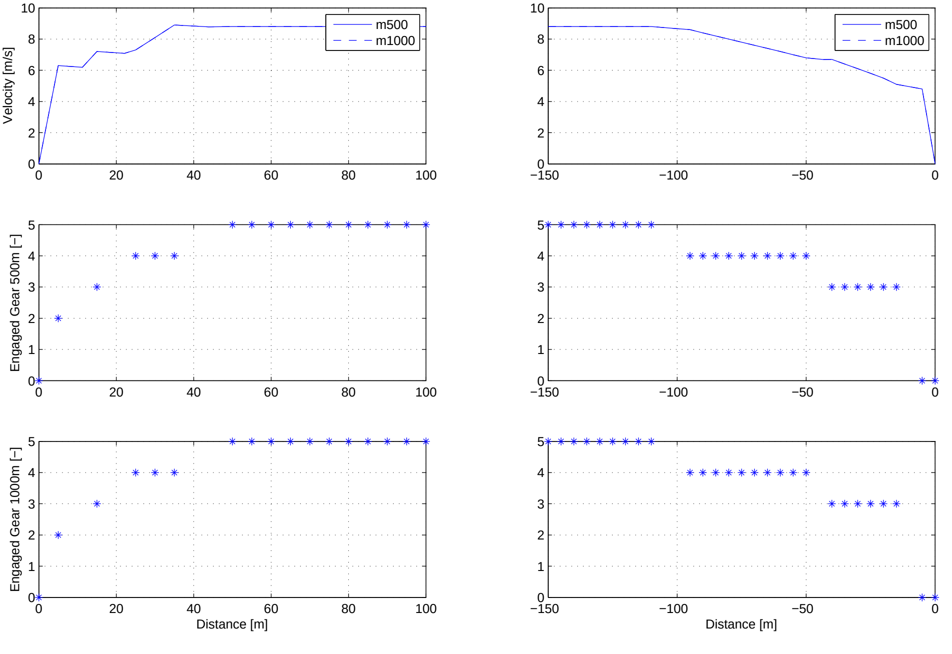

Figure 5.11. Optimal speed profiles and gear selection in the acceleration and deceleration phases, comparison between 500m trip and 1000m trip. Results using the Willans engine model.

5.4.2 Trip time constraint

When there is a time constraint, the results differ significantly more when the trip length is changed compared to Section 5.4.1. This can be explained using the supposed constant mean speed that the vehicle might follow to travel the same distance in the desired time. On one hand, if the trip is short the acceleration and deceleration phases have an important weight in the mean speed calculation and therefore the constant cruising speed has to be higher in order to fullfill the time constraint. On the other hand, if the trip is long the constant cruising speed phase has more importance in the mean speed calculation and thus, the vehicle can cruise at lower speed.

Comparing the acceleration and deceleration phases, which are presented in Figure 5.12, it can be stated that the acceleration initially is equal for both trips, and the difference comes later due to a different cruising speed. The fifth gear is engaged one distance step earlier for the results of a 1000m trip length. In the deceleration phase, that happens more or less the same but the gear selection is different for both trips. However, the grid selection is affecting the results in the deceleration phase, and this effect is explained later in Section 5.5.1.

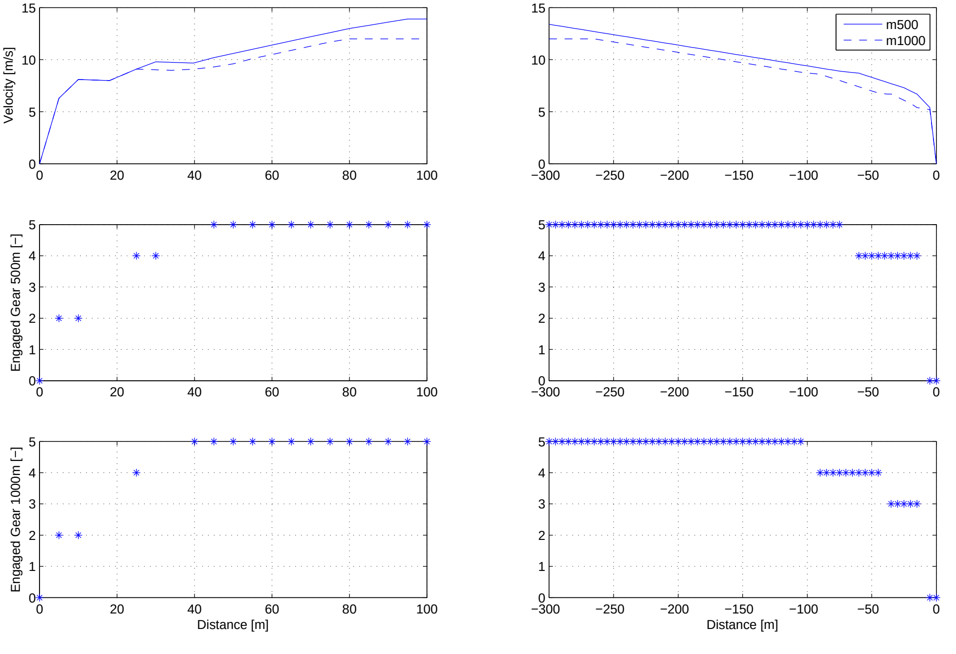

Figure 5.12. Optimal speed profiles and gear selection in the acceleration and deceleration phases, comparison between 500m trip in 50s and 1000m trip in 100s i.e. the same average speed. Willans engine model is used.

5.5 Discretization Effects

Solving a continous problem with a discrete method have its advantages, such as being able to solve a complex problem that would not be possible to solve with analytical methods. Nevertheless it also leads to errors due to taken assumptions and also because the problem has to be solved within the discretized points. If the discretization is not good enough the optimal solution might not be reachable. However, increasing the discretization leads to that the algorithm becomes slow due to the “curse of dimensionality” [1]. In order to have a reference of the computational time, the gear shift model without time constraint and using the Willans engine model with $\delta = 0.1m/s$ and $h = 5m$ takes 1h 40min. The same grid choice with the gear shift model with time constraint and using the map based engine model takes 9h 43min of computational time.

5.5.1 Coasting availability

Coasting is one interesting mode in the deceleration phase. Defining a sufficiently fine grid is important to enable coasting during the deceleration phase. Principally because if the coasting profile is not available, the algorithm has to choose another profile which leads to braking or to fuel consumption.

Making the coasting solution available is initially solved by defining an equidistant grid with a step of $0.1m/s$. However, if the speed profile reaches higher speeds, the coasting points with a distance step of $5m$ do not fall in the equidistant grid defined previously. Thus, coasting is not available during the deceleration phase. This effect is observed e.g. in Section 5.2.2 with Figure 5.5, and in Section 5.3.2 with Figure 5.10. Due to the time constraint, the speed profiles of these two models reach higher speeds and during the deceleration there is fuel consumption.

This problem could be solved by defining a grid with a smaller step, but this leads to a high computational time. Moreover, the coasting availability by making the grid more dense is not guaranteed, this is only making the error smaller. Hence, this problem is solved by defining a grid that allows coasting since this is an important operating mode of the vehicle. This coasting grid is designed by taking the speed values of the car coasting profile from an initial speed value every distance step, in this case every $5m$. It is computed by running a script that computes this coasting speed profile using the engine friction model (2.10), the engine inertia, the gearbox efficiency (2.4), and the vehicle dynamics (2.1). An illustration of a coasting profile can be seen in Figure 5.13. However, this method has a free parameter to select, the gear engaged during coasting that implicitly defines the engine speed and therefore the engine friction losses. The engaged gear is selected by using two strategies;

Strategy one consists in selecting the highest gear available that leads to the lowest engine speed value and thereby to the lowest engine friction.

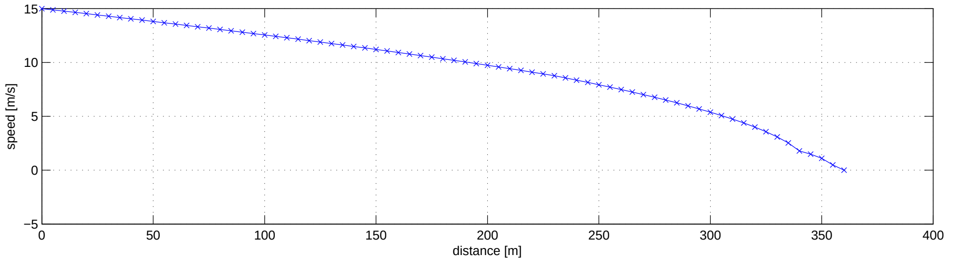

Figure 5.13. Coasting profile with $15 m/s$ as initial speed. The change in the curve is because the engine is disengaged at low speeds.

Strategy two consists in selecting the gear ratio using the gear shift strategy provided by previous optimizations carried out in Sections 5.2.2 and 5.3.2.

Results using the instantaneously gear shift model from Section 4.2 with the coasting grid following the strategy one are compared with the original results with an equidistant grid in Figure 5.14. It is possible to observe now that there is a true coasting phase that does not consume fuel. Moreover the constant cruising speed is decreased and the duration of it is longer in the case where the coasting grid is used. The gear shift strategy for both grids is similar. In order to compare

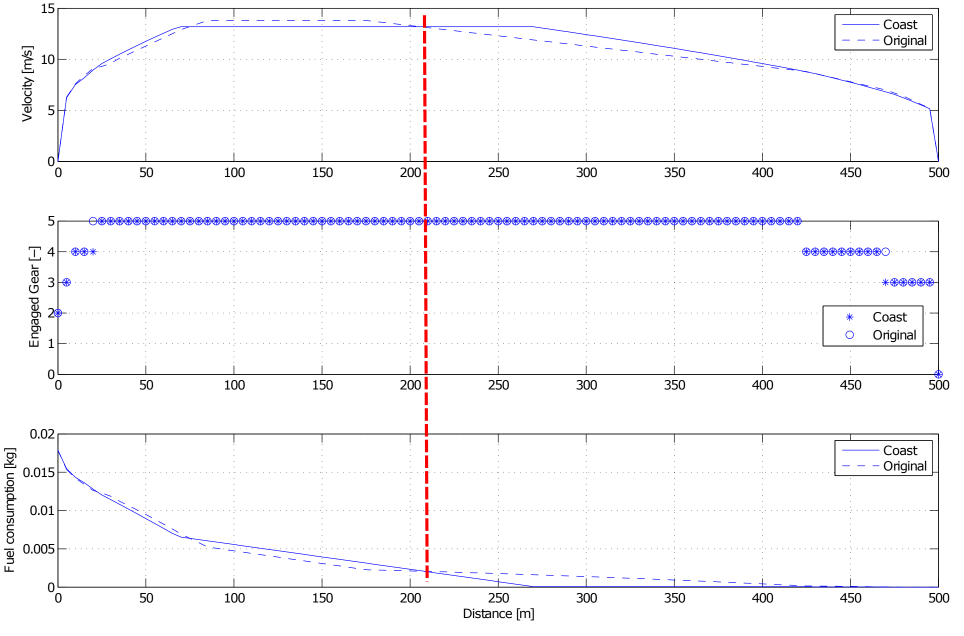

Figure 5.14. Optimal profiles comparison between the original solution and the solution with the coasting strategy one for the instantaneously gear shift model. Willans engine model.

the fuel consumption in the deceleration phase, the Table 5.9 presents the fuel consumption at the marked vertical line in Figure 5.14. Even though the results are similar, the original equidistant grid achieves a slightly lower fuel consumption. However, it has to be remembered that the two deceleration profiles are not taking the same time until stand still. As a result it can be stated that the use of the coasting grid is not relevant for the model of Section 4.2.

Table 5.9. Fuel consumption of the instantaneoulsy gear shift model at the marked point (205m). Optimization done with a time constraint of 50s.

	Value	Unit
Original	0.96	$l/100km$
Strategy one	0.99	$l/100km$

If the comparation is done with the gear shift model the results observed are similar. Figure 5.15 presents the results of the original solution of the model from Section 4.4 with the results of the model with both coasting strategies and the results obtained by merging the grids from both coasting strategies. It can be observed that the results for both strategies are very similar and that also both of them allow a true coasting deceleration phase where no fuel is consumed. Moreover the results slightly differ for the combination of grids, especially in the gear selection.

Comparing the fuel consumption of the deceleration phase there are two interesting points where the trajectories intersect, these two points are marked in Figure 5.15. Table 5.10 presents the fuel consumption for the first marked point, where the curves of the original solution and the strategy two intersect. In that point the fuel consumption for using the coasting grid is lower than the fuel consumption of using the equidistant grid, indicating that coasting is the optimal operating mode of the vehicle. Table 5.11 presents the fuel consumption for the second marked point, where the original speed profile intersects the other two speed profiles. Again using a coasting grid is beneficial for reducing the fuel consumption, especially the combinaton of both strategies that achieve the lowest value. To sum up, making the coasting deceleration available for the gear shift model contributes to decrease the fuel consumption, and thereby coasting is optimal when the gear shift maneuver is taken into consideration.

Table 5.10. Fuel consumption of the gear shift model at the first marked point (185m). Optimization done with a time constraint of 50s.

	Value	Unit
Original	1.65	$l/100km$
Strategy two	1.45	$l/100km$

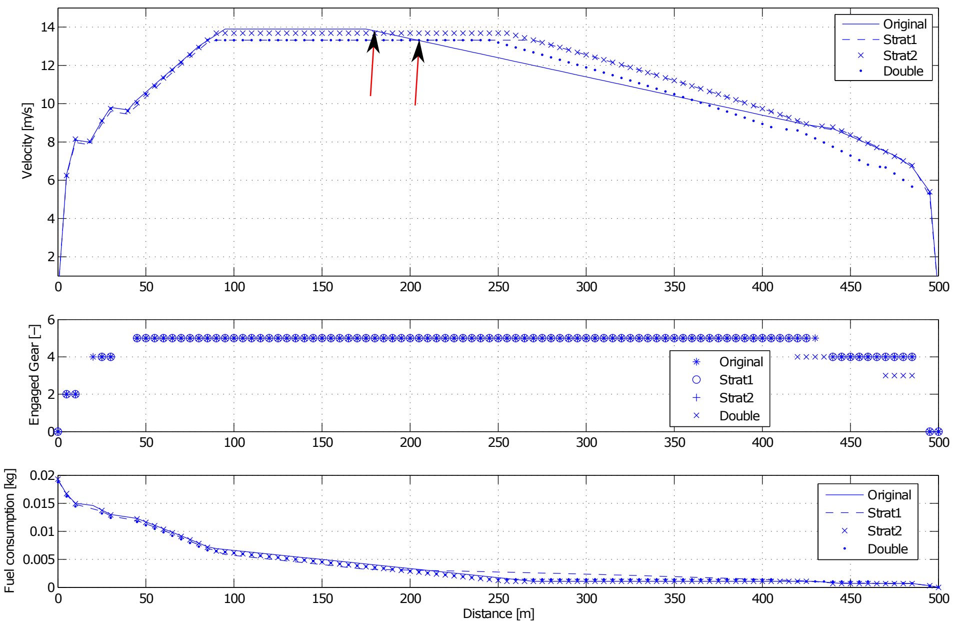

Figure 5.15. Optimal profiles comparison between the original solution, the solutions with both coasting strategies, and the solution of merging the speed grids for both strategies. The gear shift model with Willans aproximation is used.

Table 5.11. Fuel consumption of the gear shift model at the second marked point (205m). Optimization done with a time constraint of 50s.

	Value	Unit
Original	1.47	$l/100km$
Strategy one	1.40	$l/100km$
Combined strategies	1.25	$l/100km$

5.5.2 Acceleration results with a dense grid

From previous optimizations it is known that, without a time constraint, the most part of the speed profile is cruising at constant speed. Therefore it is known how much distance the acceleration phase takes and which its final speed value is. Thus, a special test for the acceleration phase with a thin grid can be carried out in order to observe changes and validate the results without increasing the computational time too much. This test is done with a distance step (h) of $1m$ and a speed interval (δ) of $0.025m/s$ for the Willans engine model. The computational time of this optimization is 9h and 58min.

The results are presented in Figure 5.16. The principal difference with the results presented in previous sections is that the first gear is selected and the skipped gear is the second one, that leads to gear shift at a lower speed than the original test. Third gear is also selected with the dense grid during a longer time instead of for only one step that is selected for the original results. Figure 5.17 presents the engine operating points during the acceleration for the dense grid. Looking at this figure it can be stated that the acceleration is done using torque values close to the maximum available.

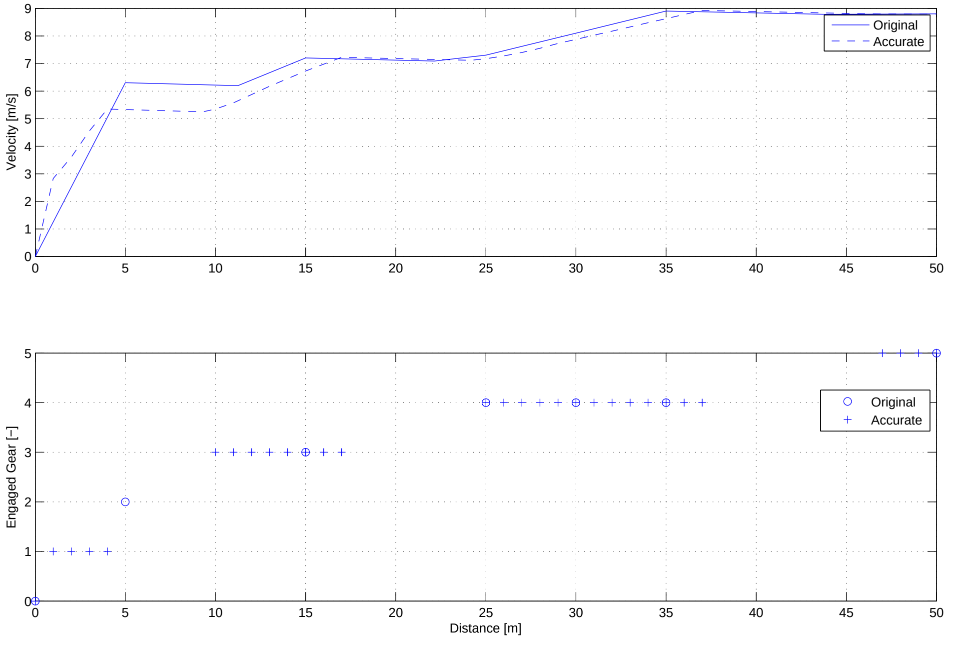

Figure 5.16. Comparison of the optimal acceleration profiles between the original solution and the solution obtained with the dense grid. The points where there is no gear is due to the gear shift maneuver, the gear used to compute the cost to go is the one which the maneuver has finished.

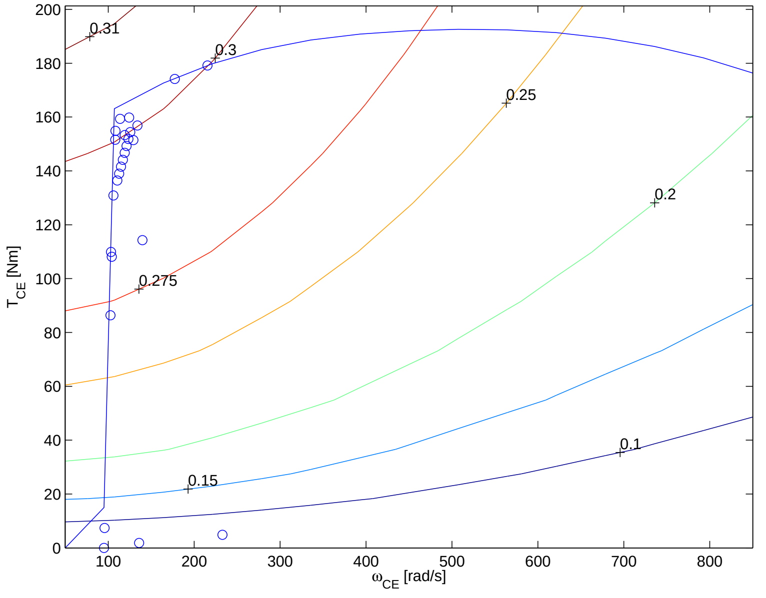

Figure 5.17. Engine operating points for the acceleration phase with the dense grid.

5.5.3 Deceleration results with a dense grid

Using the same idea as presented in Section 5.5.2, a fine grid test is done in order to see how it affects the deceleration phase. The test is done with a distance step (h) of $1m$ and a speed interval (δ) of $0.025m/s$ as well, with the Willans engine model. The computational time of this optimization is 73h and 52min. The results are presented together with the original results from the previous sections in Figure 5.18.

Figure 5.19 proves that the deceleration profile is coasting until the very end where there is a hard brake while the engine is disengaged. There are some low brake torque values before this high value due to mismatch between the grid available points and the true coasting values. The main difference between the previous results and this one is that the second gear is not skipped during the deceleration, and thereby the speed profile reaches lower speeds before the engine is disengaged. Another difference is that the gear shifts with the dense grid are always carried out before the gear shifts in the original results.

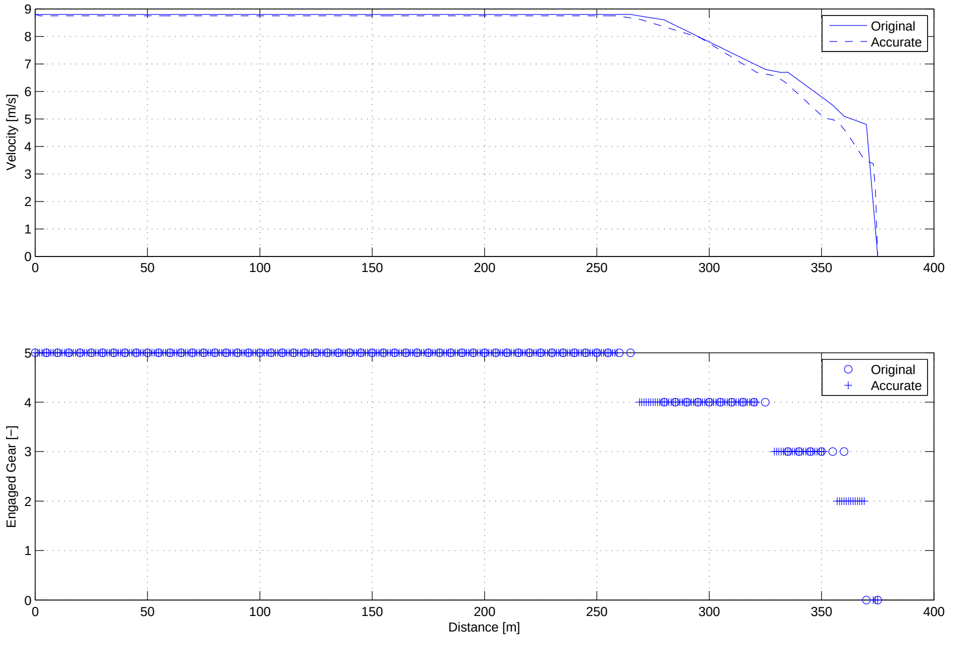

Figure 5.18. Comparison of the optimal deceleration profiles between the original solution and the solution obtained with the dense grid.

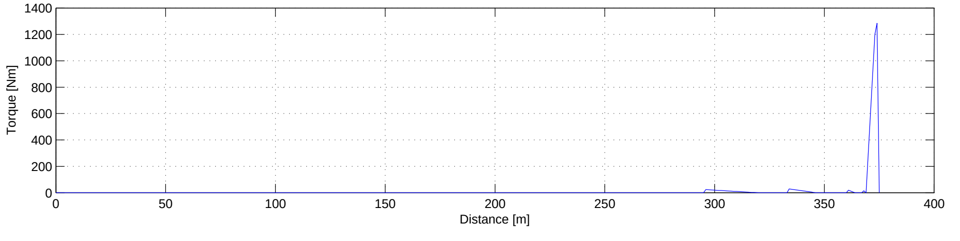

Figure 5.19. Brake torque profile for the dense grid.

5.6 Road Topology Effects

In Section 1.1 some research papers are introduced that test the influence of road topology in optimal speed profiles. In [16] and [10] the speed profile has variations when the driving trip is to go up and down a hill. On the other hand [3] stated that maintaining constant speed is the optimal way to face a hill if the use of the brakes is not required to maintain constant speed and the engine is powerful enough.

This section uses the model from Section 4.3, with some modifications in order to compute the power required regarding the road slope, and the Willans engine model to find the optimal speed profile in a road with a hill. Three different hill profiles with different heights are used to compare the results between them, Figure 5.20 shows the results. The three different hill heights are $3.5m$, $7m$, and $10.5m$. Figure 5.21 present the engine operating points for every hill.

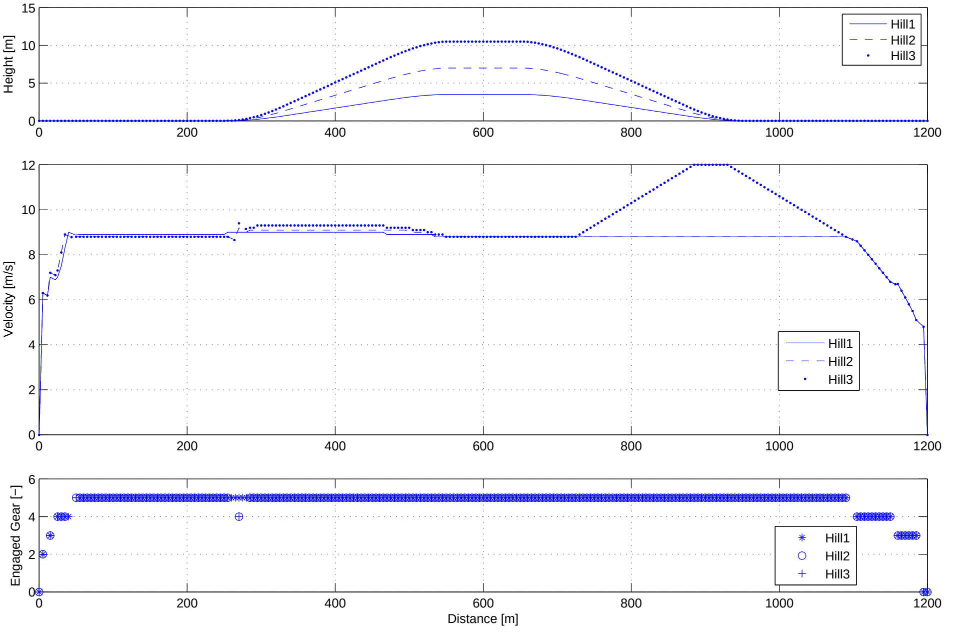

Figure 5.20. Comparison between three different hill heights with the gearshift model without time constraints. The Willans based engine is used.

The speed profiles obtained present a solution close to the constant cruising speed from Section 4.3. However there is a slight increase of the cruising speed while the vehicle is going uphill. This is because if the vehicle keeps the lowest speed with fifth gear, which is the optimal solution of Section 4.3, the torque required for going uphill exceeds the maximum torque available. Due to this the cruising speed during uphill has to be slightly higher during this phase of the trip. This increase in the cruising speed matches with what is stated in [3], keeping constant speed in uphill regions is optimal if the engine is powerful enough.

Looking to the downhill region there is a clear difference between the first two hill profiles and the third one which is the highest. This can be explained also using the previous research done in [3]. For the two first hills, keeping the cruising speed is still optimal because braking is not needed to keep constant speed during the downhill. For the last test, where the hill is higher, going downhill and keeping constant speed would require the use of the brakes which is not an optimal strategy. Due to this the speed increases and then it is used for a longer coasting phase before stopping the vehicle.

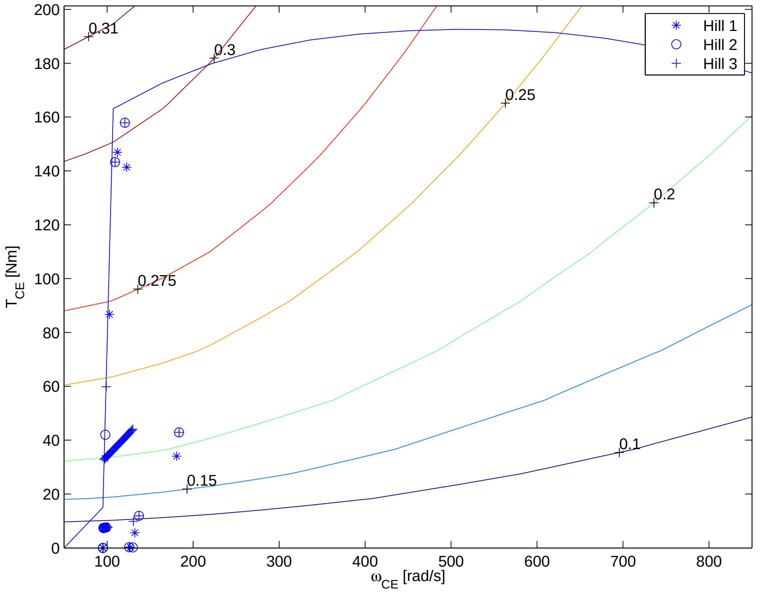

Figure 5.21. Engine operating points for every hill height.

The fuel consumption and the trip duration of the three tests are shown in Table 5.12. It is known that as the initial road level and the final road level is the same, the consumption values should be similar between the tests. For the higher hill, the losses due to air drag in higher speed and engine friction will be higher and thereby the consumption of the 10.5m hill test is slightly higher. Figure 5.21 presents the engine operating points for each hill tested, these points are located in lower engine speeds and very close to the maximum torque curve. Many points for Hill 2 and Hill 3 coincide because the acceleration to the cruise speed is done still in level road and the speed profile for both is equal.

Table 5.12. Fuel consumption and trip duration for each hill test

	Fuel consumption	Trip time
3.5m Hill	4.27 $l/100km$	143.33 s
7m Hill	4.28 $l/100km$	143.14 s
10.5m Hill	4.37 $l/100km$	135.88 s

5.7 Sensitivity of the Vehicle Parameters

This section presents how the optimal speed profiles change when changing one or more vehicle parameters. This is done in order to determine the importance or sensibility of this parameter for the speed profiles found in the previous sections.

5.7.1 Vehicle Mass

An interesting parameter to study is the vehicle mass and its influence on the optimal speed profiles. In order to quantify this influence, the model from Section 4.3 is used to find the optimal speed profiles for a several vehicle masses. The results for $1350kg$, $1500kg$, $1650kg$, and $1800kg$ are presented in Figure 5.22

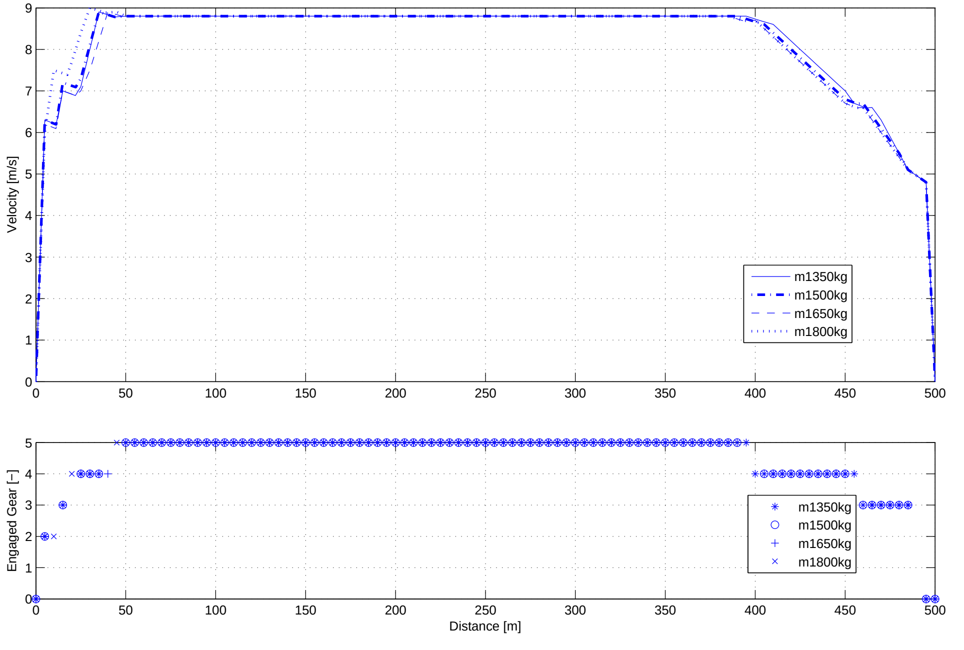

Figure 5.22. Fuel consumption values for different vehicle masses. Willans engine model is used.

Observing the results from Figure 5.22, the results for $1350kg$, $1500kg$, and $1650kg$ are quite similar throughout the whole distance. However, the optimal speed profile for a vehicle mass of $1800kg$ differs significantly in the acceleration phase compared with the other masses. The main difference is that the acceleration is harder and faster and the second gear is maintained when the other profiles change to third gear.

Looking at the fuel consumption, it is obvious that it is directly related to the vehicle mass, thereby if the vehicle mass increases the fuel consumption will increase as well. Figure 5.23 presents the fuel consumption for several vehicle masses, the relation follows quite good an affine function until $1350kg$, where the slope slightly changes. Optimizations for $1200kg$, $1350kg$, $1500kg$, $1650kg$,$1800kg$, and $1950kg$ were carried out to obtain the figure.

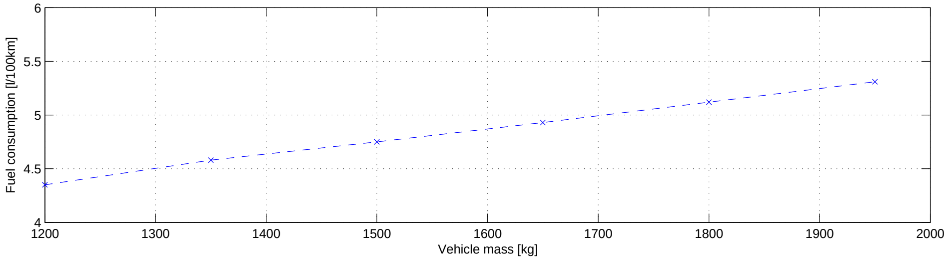

Figure 5.23. Optimal speed profiles for several engine sizes. Willans engine model.

5.7.2 Engine Size

In order to see the influence of the engine size, the model from Section 4.3 is used to perform several optimizations with several engine volumes. Figure 5.24 shows the results for engines with a displacement volume of $2.3l$, $2l$, $1.5l$, and $1l$ respectively.

Observing the results it can be seen that the optimal speed profiles change significantly if the engine displacement is changed. Looking at the acceleration phase, as the engine size is decreasing and thereby the available power is decreasing, the acceleration requires more distance with the smaller engines and the gears are engaged for a longer distance. However, this does not happen for the $2l$ engine, the acceleration is faster than for the $2.3l$ engine but it is mainly done in second gear and the third one is skipped. This can be explained because as having less power available, the optimal acceleration has to be performed in second gear instead of the acceleration done by the $2.3l$ engine that does not skip third gear.

Other differences can be observed in the deceleration phase, e.g. the coasting phase is started earlier with the small engine size. This happens due to the less friction losses of the smaller engines that allows the vehicle to coast for a longer time. The constant cruising speed is the same for the $2.3l$ and $2l$ engines. However, due to the torque limitation at lower engine speed is modelled as an affine function with a very high slope, the cruising speed has to increase a bit for the two smaller engines in order to have enough engine torque available.

To sum up, Table 5.13 contains the fuel consumption and trip duration for

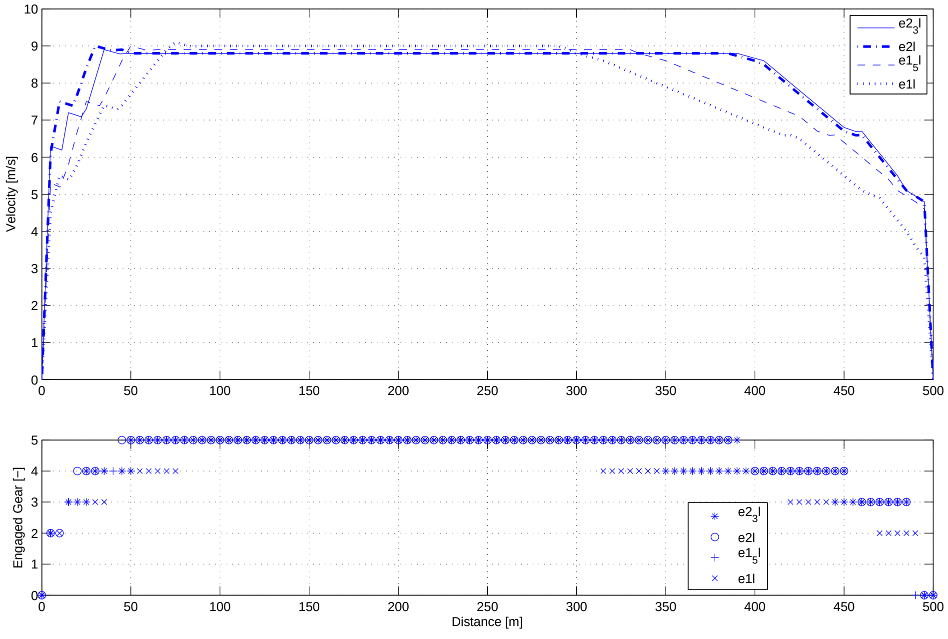

Figure 5.24. Optimal speed profiles and gear selection for several engine sizes. Willans engine model.

each engine size tested. As the driving mission does not require a very powerful engine, and thus using a smaller one is still enough, the fuel consumption can be decreased by downsizing the engine. Moreover the friction losses of the engine during the whole drive mission are decreased, as well as the overall efficiency of the engine during the acceleration phase is improved due to the engine torque is closer to more efficient areas of the engine map, this can be seen in Figure 5.25. As a result, the use of a smaller engine is beneficial for fuel consumption reduction.

Table 5.13. Fuel consumption and trip duration for each engine size.

	Fuel consumption	Trip time
2.3l engine	4.75 $l/100km$	64.53 s
2l engine	4.40 $l/100km$	64.45 s
1.5l engine	3.84 $l/100km$	66.91 s
1l engine	3.19 $l/100km$	73.47 s

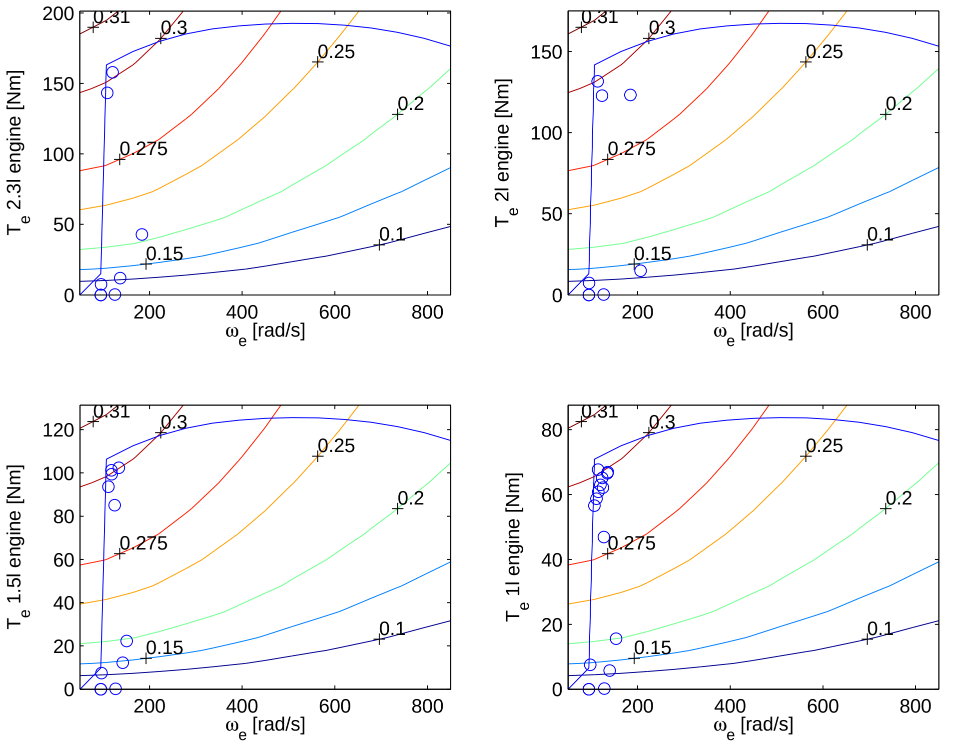

Figure 5.25. Engine operating points for several engine sizes. Willans engine model.

Chapter 6

Conclusions

The goal with this work was to develop knowledge for driving behavior in order to decrease the fuel consumption in different driving situations. In general terms and without a time constraint, the optimal acceleration is done by achieving the desired cruising speed as fast as possible. The gear shift maneuvers should be done once the next upper gear is available to engage. The cruising phase should be done by keeping constant speed with the highest gear engaged and the lowest possible engine speed. The deceleration phase ought to be done by coasting strategy and using the fuel cut-off feature. Despite the engine models used are simplified, the results are consistent and coincide with previous research in the subject.

Modelling of the gear shift is useful in order to obtain more accurate results, specially regarding the gear shift strategy in the acceleration and deceleration phases. The fuel consumption values differ if the gear shift dynamics is taken into consideration or not. On the other hand, the results show that varying the trip length has no influence in the optimal solution if there is no time constraint.

Adding a time constraint to the optimization problem can increase the cruising speed and thus accelerate and decelerate takes longer distance. Due to that longer distance, the acceleration and deceleration phases are more important in the fuel consumption calculation than when there is not a time constraint.

Coinciding with what is stated in previous research [3], driving up and down a hill optimally should be done by keeping a constant cruising speed if the engine is powerful enough to overcome the uphill distance without a gear shift, and the brakes are not required during the downhill.

6.1 Future work

It would be interesting to test the models presented in this Master Thesis with more accurate engine models, e.g. a measured engine map model. This would be useful in order to validate the presented results, and for further research regarding sensitivity analysis of the vehicle parameters.

Bibliography

[1] R.E. Bellman and S.E. Dreyfus. *Applied Dynamic Programming*. Princeton University Press, 1962.

[2] D.P. Bertsekas. *Dynamic Programming and Optimal Control*, volume I. Athena Scientific, 1995.

[3] David J. Chang and Edward K. Morlok. Vehicle speed profiles to minimize work and fuel consumption. *Journal of Transportation Engineering*, 131(3):173–182, 2005.

[4] Sandra Gausemeier, Karl-Peter Jäker, and Ansgar Trächtler. Multi-objective optimization of a vehicle velocity profile by means of dynamic programming. In *Prep. IFAC symp. Advances in Automotive Control*. AAC2010, 2010.

[5] L. Guzzella and A. Amstutz. CAE-tools for quasistatic modeling and optimization of hybrid powertrains. *IEEE Transactions on Vehicular Technology*, 48(6):1762–1769, 1999.

[6] Lino Guzzella and Christopher H. Onder. *Introduction to Modeling and Control of Internal Combustion Engine Systems*. Springer Verlag, 2004.

[7] Lino Guzzella and Antonio Sciarretta. *Vehicle Propulsion Systems - Introduction to Modeling and Optimization*. Springer Verlag, 2 edition, 2007.

[8] Erik Hellström, Jan Åslund, and Lars Nielsen. Design of an efficient algorithm for fuel-optimal look-ahead control. *Control Engineering Practise*, 18(11):1318–1327, 2010.

[9] Erik Hellström, Maria Ivarsson, Jan Åslund, and Lars Nielsen. Look-ahead control for heavy trucks to minimize trip time and fuel consumption. *Control Engineering Practise*, 17(2):245–254, 2009.

[10] J. N. Hooker. Optimal driving for single-vehicle fuel economy. *Transportation Research*, Part A(22A(3)):183–201, 1988.

[11] Tiago R. Jorge, Joao M. Lemos, Miguel S. Barao, and Alberto Bemporad. Hybrid dynamic optimization for cruise speed control. In *Prep. 18th IFAC World Congress*, 2011.

[12] Hong Tu Luu, Lydie Nouvelière, and Said Mammar. Dynamic programming for fuel consumption optimization on light vehicle. In *Prep. IFAC symp. Advances in Automotive Control*. AAC2010, 2010.

[13] Felicitas Mensing, Rochdi Trigui, and Eric Bideaux. Vehicle trajectory optimization for application in eco-driving. In *Prep. 18th IFAC World Congress*, 2011.

[14] B. Saerens, J. Vandersteen, T. Persoons, J. Swevers, M. Diehl, and E. Van den Bulck. Minimization of the fuel consumption of a gasoline engine using dynamic optimization. *Applied Energy*, 86:1582–1588, 2009.

[15] Bart Saerens, Moritz Diehl, and Eric Van den Bulck. Optimal control using pontryagin's maximum principle and dynamic programming. In *Automotive Model Predictive Control*, chapter 8. Springer Verlag, 2010.

[16] A. B. Schwarzkopf and R. B. Leipnik. Control of highway vehicles for minimum fuel consumption over varying terrain. *Transportation Research*, 11(4):279–286, 1977.