Modelling and verification in the time domain of the driveline and suspension at idle with the purpose of transient studies.

Examensarbete utfört i Fordonssystem
Vid tekniska högskolan i Linköping av

Leif Hermansson
Michael Krasser

Reg: LiTH-ISY-EX-1866

Handledare: **Bengt Jacobson**, Chalmers

Examinator: **Lars Nielsen**
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Abstract

The department Noise, Vibration and Harshness at Saab Automobile AB in Trollhättan has finite element (FE) models of the complete car. These models, which are modelled in the frequency domain, do not show the non-linear, transient behaviour of the driveline.

The objective of this master thesis is to build a model of the driveline with suspension for a car with automatic transmission in the time domain focusing on idling with the gear selector in drive and the wheels braked.
This master thesis should also give a good foundation for further analysis of other car models and load cases in the time domain. Easy changeable models are therefore desired.

The model has been built in the object orientated modelling tool Dymola and shows good simulation results. Dymola's use of model classes and equation based modelling will result in easy changeable models.
The model extended with the engine and gearbox control system would be well suited to study transient behaviour of the complete car.

Key word: driveline, idle, Dymola, object orientated, modelling, time domain

Abstract

Acknowledgements

"Galen nog att slå mot det som förtrycker, fördummar, stagnerar. Galen nog att våga leva, prata, vråla, skratta, dansa, sjunga. Galen nog att ta hem maskrosorna och göra vin av dem. Galen nog att stå för sina handlingar. Galen nog att vara kär i jorden och tillvaron här. Galen nog att bränna ett skepp då och då. Galen, så galen att vingarna växer ut." /Ulf Lundell

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Notations

NVH: Department of Noise Vibration and Harshness
FE: Finite Element
rpm: Revolutions per minute
T.D.C: Top Dead Centre
B.D.C: Bottom Dead Centre
e.c.s: Engine control system
ACC: Air Condition Control

Idling: engine running, gear selector in drive and brakes locked.

F = Force   [N]
P = Pressure   [N/m^2]
T = Torque   [Nm]
V = Speed   [m/s]
X = Position   [m]

α = angular acceleration [rad/s^2]
η = degree of efficiency
η_l = loss factor
γ = loss angel   [rad]
φ = angle [rad]
λ = torque converter parameter   [Nm/(rad/s)^2]
μ = torque converter parameter [N/m^2]
σ = standard deviation
ν = relative angular speed
ω = angular speed [rad/s]

Vectors are written with **bold**.

1. Introduction

The automobile business is tough and competitive. The demands on low fuel consumption to decrease exhaust emission forces manufactures to test their cars in wind tunnels to minimise the air resistance. Therefore the cars are beginning to look quite similar. Due to the lack of this competitiveness, car manufacturers are obliged to find other means of attributes to sell the car. This is the object of the NVH (Noise, Vibration and Harshness) department at Saab Automobile AB. NVH's work is to investigate and improve the noise and vibration behaviour of Saab's cars. At NVH almost all studies are made in the frequency domain. They have got complete finite element (FE) models of the cars. The problem with these models is that they can only be used to study linear, non-transient behaviours.

This master thesis report is part of the education program in applied physics and electronic engineering.

Introduction

2. Problem description

2.1 Background

Noise and vibration analysis of complete car models are mainly done in the frequency domain as linear models at NVH with finite element methods. These models have not got the ability to show the non-linear behaviour which some of the subsystems have. Neither can these models handle transient behaviour such as changing the electrical load or gear shifting. The object for this master thesis is to examine the possibility of building a model of the driveline to be able to examine non-linear and transient behaviour in noise and vibration. The modelling is to be done in the time domain preferably using a modelling program existing at Saab.

2.2 Goals

The goal is to build a model of the driveline and suspension for a car with automatic transmission in the time domain focusing on idling with the gear selector in drive and the wheels braked. The output of the model should be the forces in the bushings between car body and engine block, subframe, left and right wheel.

These forces are to be transformed into the frequency domain and used to excite the existing FE model of the total car. This latter work is not part of this thesis. This master thesis should also give a good foundation for further analysis of other car models and load cases in the time domain. Easy changeable models are therefore desired.

The car that is to be modelled is a Saab 9^5 with automatic transmission and 2.3 litres low-pressure turbo engine.

2.3 Boundaries

The model is primarily built to study forces in the bushings between car body and engine block, subframe, left and right wheel for twice the idle frequency. This is due to known problems that occurs due to the engine firing two times every revolution.

The model of the four stroke engine is only valid for a constant load. This is due to the fact that the measurements of the cylinder pressure that should have been used to examine load changes were not completed in time to be used in this thesis.

Problem description

3. Model

3.1 Modelling approach

A structured way of modelling is to break down the model into submodels. It also makes it easier to understand, to make changes and to reuse modelling knowledge. First the model is divided into two parts, 1D and 3D.

In the one dimensional part the driveline is modelled. It contains the four stroke engine, the torque converter, the gearbox and the differential as subsystems. How they are connected and their causality is shown in figure 3.1. The shafts are considered to be stiff enough to be neglected in the model. In the four stroke engine power losses due to electrical loads, friction and power steering are modelled. The torque which is created here can be seen as the excitation of the whole system. It is a function of φ and varies approximately with twice the desired idling frequency. Torque and angular velocity are enough to describe the subsystems and therefore used as variables.

The three dimensional part contains engine block, subframe, left wheel, right wheel, car body and rear suspension as subsystems. The forces in the bushings between car body and engine block, subframe, left and right wheel are to be used as an excitation of the FE model. How they are connected is also shown in figure 3.1. The causality of the three dimensional part is very hard to show. One problem is that at each connection point there has to be eighteen variables. Position, velocity, angel, angular velocity, force and torque, in a Cartesian coordinate system, to describe the system properly.

The drive shafts are working with equal torque directly on the wheels. This means that the differential has to be connected to left and right wheel. In reality the one dimensional part is part of the engine block. Here they are modelled as separate subsystems, where torque from the four stroke engine, torque converter and gearbox works with reaction torque on the engine block. Rear suspension is added to the model as a subsystem to make it complete.

The gyroscopic moment of the one dimensional part, due to the change of it's axis of rotation, is considered to be small and is therefore neglected.

Figure 3.1 *The model is separated into a one dimensional part (driveline) and a three dimensional part. The causality in the one dimensional part and the causality between the one and three dimensional part is clearly defined and described by ω and T as variables.*

3.2   Modelling the four stroke engine

The engine has been the subject of lots of discussions during the modelling work. The reason is that all noise and vibration that occurs during idle is due to the combustion process. This is different from driving a car, because then there occur noise and vibrations from the road and air resistance. In other words if the combustion model generates incorrect signals in the area to study, the rest of the model does not matter. The four stroke engine is modelled as three parts. These are combustion, control system and power consumer, see figure 3.2. Before explaining how the four-stroke engine is modelled, a short description of how a four-stroke engine works is needed.

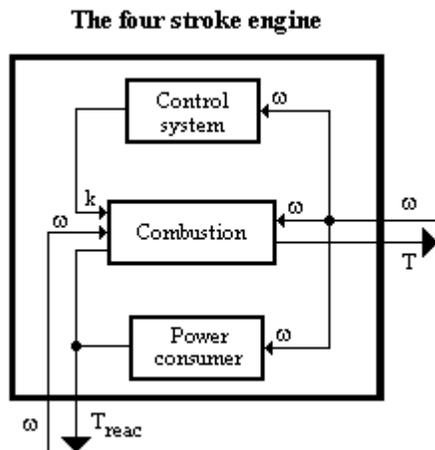**Figure 3.2** *The four stroke engine is modelled as combustion, engine control system and power consumer. A simple engine control system is used to do the simulations in future works (chapter 7.2), it is not used in any other simulations.*

3.2.1 Introduction to the four-stroke engine

The engine to be modelled is a four-stroke engine with four cylinders. To complete a combustion cycle each cylinder passes through four stages.

1. Intake stroke
The piston starts in the top dead centre (T.D.C). During the downward stroke the injection valve opens and the fuel/air mixture is injected into the cylinder. Just before the piston reaches the bottom dead centre (B.D.C) the injection valve closes.

2. Compression stroke
The piston starts in B.D.C. Both the injection valve as well as the exhaust valve is closed. During the upward stroke the piston compresses the fuel/air mixture. Just before the piston reaches T.D.C the combustion is initiated by ignition from the spark plug.

3. Power stroke
The piston starts in T.D.C. Since both valves still are closed the cylinder pressure increases rapidly due to the combustion. This high pressure forces the piston down during the downward stroke to finally reach B.D.C where the power stroke ends. The total work produced by one cylinder is the work made by this stroke minus the work   needed during the other three strokes.

4. Exhaust stroke
The piston starts at B.D.C. During the upward stroke the exhaust valve opens and the piston forces the exhaust gas to flow out of the cylinder. This stroke ends with the piston at T.D.C. Now a complete combustion cycle is done and a new cycle starts with an intake stroke.

During the power stroke the pressure in the cylinder forces the piston to move downwards. Due to this pressure the connecting rod creates a torque around the crankshaft, see chapter 3.2.3. To make this torque smoother the four cylinders are delayed 180 degrees i.e. half a revolution with one another. This delay is made in such a way that when cylinder one is in it's power stroke, cylinder two is in it's exhaust stroke, cylinder three is in it's compression stroke and cylinder four is in it's intake stroke. When the crankshaft has made half a revolution all the cylinders has entered the next stroke and so on. Since it takes each cylinder 720 degrees i.e. two revolutions to complete a cycle and there are four cylinders the engine fires two times each revolution. Accordingly the crankshaft torque fluctuates with twice the number of revolutions at idle.

3.2.2 Modelling the cylinder pressure during combustion

The combustion model is based on measured pressure data during idle. To make the explanation of the model easier the pressure during all four strokes needs to be described. In figure 3.3 measured pressure curves as function of the crankshaft angle are shown for one cylinder and all four strokes. Here it can be seen that during the intake stroke the pressure is low. Then the injection valve closes and the pressure starts building up as the cylinder enters the compression stroke and the piston starts compressing the air/fuel mixture. Then as the ignition is given the pressure increases rapidly during the first part of the power stroke. Then the
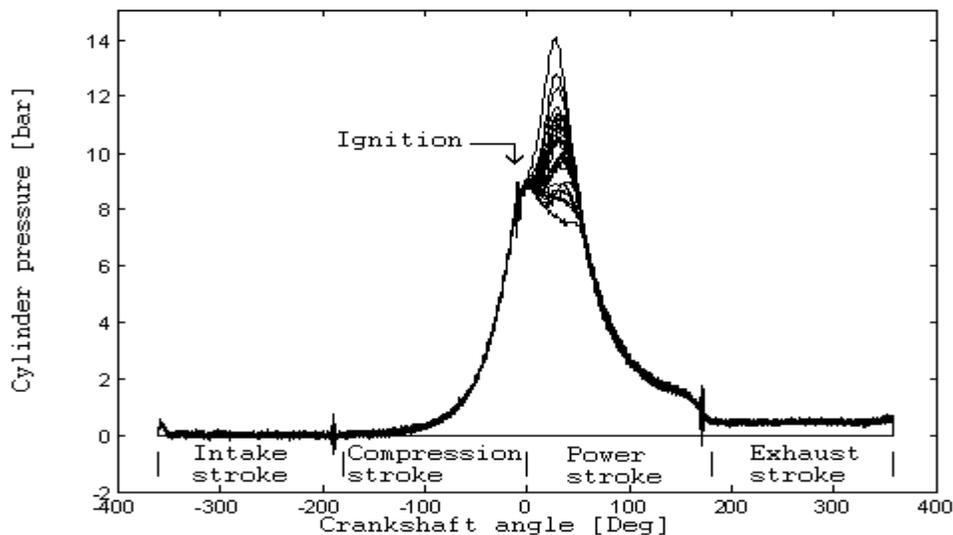

Figure 3.3 *30 measured pressure curves as function of the crankshaft angle when idling at 866 rpm. This shows the typical appearance of the pressure for one cylinder and all four strokes. Especially note the big difference in pressure from one cycle to the other during the first part of the power stroke (0° to 60°).*

pressure decreases, due to consumption of fuel and the increasing volume, during the latter part of the power stroke. Finally the cylinder enters the exhaust stroke in which the pressure is quite low and constant. It is also worth noting that during the intake stroke, compression stroke and the exhaust stroke, the pressure hardly differs anything at all from one revolution to the other. In the first part of the combustion however the pressure differs a lot from one revolution to the other. This is due to the chaotic part of the combustion, there might be a bad spark, the fuel/air mixture might not be optimal and so on. The peaks that occurs at -190°, -10°, 170° and 350° is due to the spark disturbing the manometer which is placed on the spark plug. The peak at -10° is from the cylinder's own spark plug and the three others are from the other cylinders [15]. The idea of the modelling of the combustion is to use this knowledge about the cylinder pressure and get typical pressure curve for one cylinder and crankshaft angles between -360° and 360°. Then converting the pressure to torque and finally add the contribution from all four cylinders. Therefore a calculation of a mean value (P_m) and a standard deviation (σ_p) of the pressure for each crankshaft angle (φ_c) is done.

$$P_m(\varphi_c) = \sum_{cyl} \left(\sum_k P(\varphi_c) \right) / (4 \cdot n) \tag{3.1}$$

$$\sigma_p(\varphi_c) = \sqrt{ \left(\sum_{cyl} \left(\sum_k (P(\varphi_c) - P_m(\varphi_c))^2 \right) / (4 \cdot n - 1) \right)} \tag{3.2}$$

$$\varphi_c = [-360^{o} \text{ to } 360^{o}]$$
$$cyl = [1 \text{ to } 4]$$
$$k = [1 \text{ to } n]$$
$$n = \text{number of measured pressure curves}$$

In figure 3.4 can be seen that the calculated mean value has got the typical appearance described . In the plot of the standard deviation it can be seen that σ_p is low for all crankshaft angles except 0° to 60°. The small standard deviation that occurs for crankshaft angles less than 0° and larger than 60° is due to error in measurement and the fact that the pressure might be slightly different from one revolution to the other. Since it is small it will be neglected. Therefore the standard deviation is considered to be zero for all crankshaft angles except 0° to 60°. The model of the pressure in one cylinder (P_{mod}) is

$$P_{mod} = P_m + P_c \tag{3.3}$$
$$P_c = 0 \quad \text{for } \varphi_c < 0^{o} \text{ and } \varphi_c > 60^{o} \tag{3.4}$$

How should P_c be modelled for $0^{o} < \varphi_c < 60^{o}$? In figure 3.5 the appearance of P minus Pm is shown. The shape of the curves reminds of half a period of a sine scaled with a random constant for each revolution. Therefore the curve shape (P_s) of P_c is modelled to be half a period of a sine. In figure 3.5 it can also be seen that the probability is higher for small amplitudes of P_c. This lead to the assumption that the amplitude of P_c had a distribution that was normal(0,sigma). To verify this the pressure amplitudes at different crankshaft angles where sorted in ascending order. Then it was counted how many amplitudes that could be found in equally sized

amplitude class intervals. Figure 3.6 shows this at two different crankshaft angles. Here can be seen that the estimated amplitude distribution appears to be normal and that the curves have different standard deviation. The standard deviation is bigger for 25° than it is for 45°. The reason to this can be seen in figure 3.5. The amplitudes at 25° has got a bigger spread than the amplitudes at 45°. In the model this is accomplished by the sine having a bigger amplitude at 25° than it has got at 45°, when the interval $0^{o} < \varphi_c < 60^{o}$ has been converted to half a period of a sine. Therefore the standard deviation of the modelled amplitude can be calculated as the maximum value of $\sigma_p(\varphi)$. This means that the total model of the pressure in one cylinder during the combustion is modelled as

$$P_{mod} = P_m + P_c \qquad\qquad (3.5)$$

$$P_c = \begin{cases} 0 & \varphi_c < 0^{o} \\ k*P_s & 0^{o} < \varphi_c < 60^{o} \\ 0 & \varphi_c > 60^{o} \end{cases} \qquad\qquad (3.6)$$

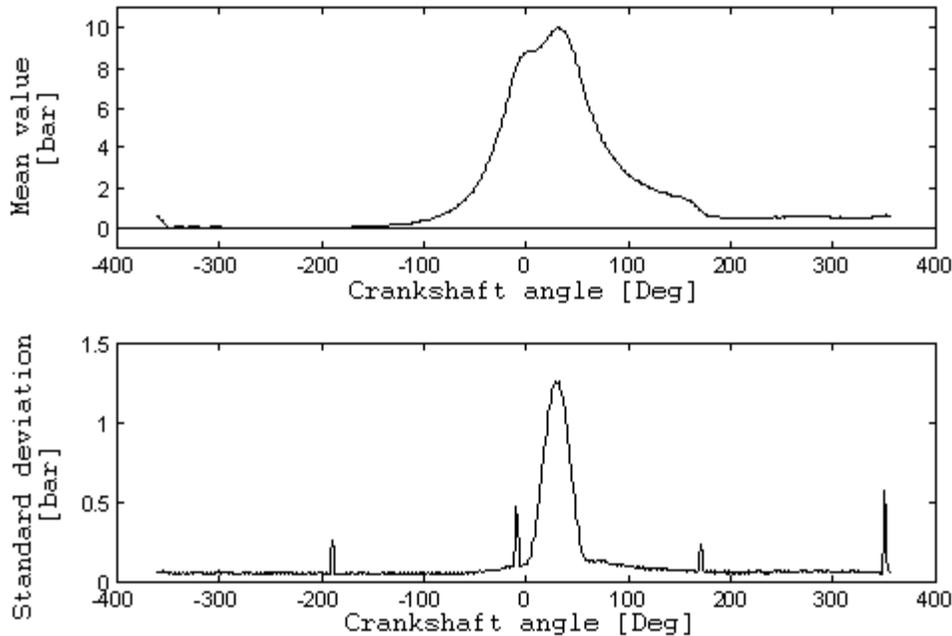$$k \text{ is normal } (0,\max(\sigma_p(\varphi_c)))$$

Figure 3.4 *Calculated mean value (upper) and standard deviation (lower) for the cylinder pressure. The peaks that occurs in the standard deviation at -190°, -10°, 170° and 350 ^{o} is due to the spark disturbing the manometer .*

Figure 3.5 *The chaotic part of the combustion. This is the measured pressure with the mean value subtracted.*

Figure 3.6 *Shows the estimated distributions at 25⁰ (upper) and 45⁰ (lower) from measured data. They can be considered to be normally distributed.*

Because the pressure during the compression stroke is the same during every revolution, this way of modelling the engine can be seen as if the amount of injected fuel and air is the same during every revolution. If the variation in the amount of fuel and air is small however the pressure during intake, compression and exhaust

stroke can be assumed to be constant and only the pressure during the power stroke varies. P_c implies that the fuel/air mixture and the spark might differ a bit from one revolution to the other.

3.2.3 Converting the cylinder pressure to crank shaft torque

The modelled cylinder pressure in the previous chapter has to be converted into a torque acting on the crankshaft. In figure 3.7 the crankshafts dynamics can be seen. The pressure in the cylinder creates the force

$$F_{cyl} = P_{cyl} \cdot \pi \cdot r^2 \tag{3.7}$$

The part of the force that is transferred through the connecting rod is

$$F_c = F_{cyl}/\cos(\alpha_c) \tag{3.8}$$

The part of the force that creates the torque around the crankshaft is

$$F_m = F_c \cdot \sin(\alpha_c + \varphi_c) \tag{3.9}$$

Since the pressure is a function of the crankshaft angle φ_c it is desired to solve α_c as a function of φ_c. From the geometry in figure 3.7 can be seen

$$L_S/2 \cdot \sin(\varphi_c) = L_c \cdot \sin(\alpha_c) \tag{3.10}$$

Solving equation (3.10) for α_c gives

$$\alpha_c = \arcsin(L_S/(2 \cdot L_c) \cdot \sin(\varphi_c)) \tag{3.11}$$

The torque around the crankshaft is

$$T_c = F_m \cdot L_S/2 \tag{3.12}$$

This means that the whole expression for the crankshaft torque for one cylinder becomes

$$T_c = P_{cyl} \cdot \pi \cdot r^2/\cos(\alpha_c) \cdot \sin(\alpha_c + \varphi_c) \cdot L_S/2 \tag{3.13}$$

with α according to equation (3.11).
Recalling that the cylinders are delayed 180° the total crankshaft torque becomes

$$T_{tot}(\varphi_c) = T(\varphi_c) + T(\varphi_c + 180^o) + T(\varphi_c + 360^o) + T(\varphi_c + 540^o)$$
$$\tag{3.14}$$

F_{cyl} : Force due to cylinder pressure
L_c : Connecting rod length
L_s : Stroke length
r : Radius of cylinder
φ_c : Crankshaft angle
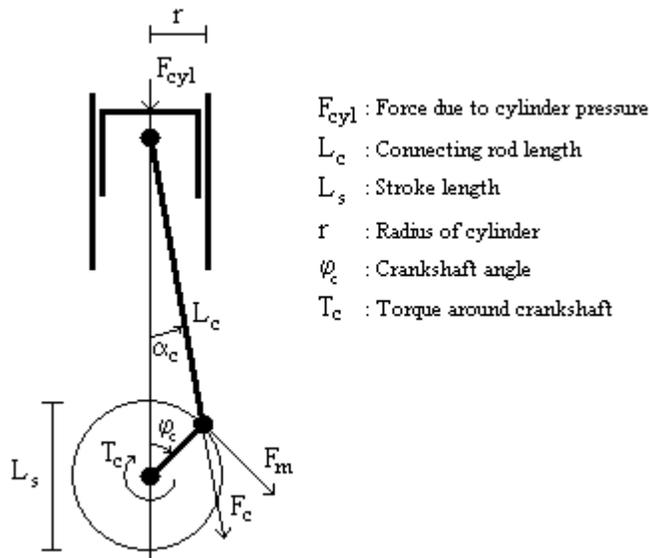T_c : Torque around crankshaft

Figure 3.7 *Description of piston, cylinder and shafts for calculating the torque around the crankshaft from cylinder pressure.*

3.2.4 Engine control system

The engine control system (e.c.s) is not in the model of the engine. The reason is that there is high security on the e.c.s to the 9^5 and therefore Saab would not release it to this thesis. This left nothing but two alternatives. The first is to try to build a control system and the second is to just ignore the e.c.s The choice fell on the second alternative, except for a simplified e.c.s, which is used in chapter 7.2 This may seem strange since the e.c.s should make sure that the desired rpm is maintained at idle. However this is the strongest reason not to build a control system. Building a control system would result in giving the engine a characteristic decided by the built control system and not the real e.c.s Another reason for this choice is that when looking at plots such as figure 3.5 but sorted in time order one can not see any clear trends that imply that the e.c.s should change the amount of fuel and air during idle. This is of course only true when loads such as ACC, power steering and light is constant. If the electrical loads where to change the output from the e.c.s would change the amount of fuel and air to maintain the rpm. Therefore the model of the combustion can only be used for constant electric loads because the e.c.s is not modelled.

3.2.5 Power consumer

The work created by the engine during idle can be divided into four major parts: energy losses due to electrical loads such as air condition and light, friction in the engine, the power steering and the torque converter. In the power consumer the three first parts are modelled.

To be able to calculate power losses due to friction and power steering measurements have been made. When the gear selector is put into neutral and the engine holds it's idle rpm the engine power as function of the generator current is plotted. The same is then done with the gear selector in position drive. A principal sketch with two straight lines adjusted to the measurements is shown in figure 3.8. The friction power losses are actually a function of the engine speed but are here considered to be constant since the engine is idling at an almost constant speed. Due to this assumption the power losses due to friction and power steering is equal to the engine power when the generator current is zero and the gear selector is in position neutral.

3.3   Torque converter

When building a car with an automatic gearbox you will need a torque converter, see [1] and [2]. There are two purposes of the converter. The first is to make a viscous coupling between the engine and gearbox. In other words make it possible to keep the car at idling without the engine shutting off. The second is to increase the output torque at low speed ratios to provide more torque for the vehicle. The torque converter modelled is of trilok type and consists of four major parts, pump, turbine, reactor and the oil inside. The pump, reactor and turbine are impellers that determines the oil flow inside the converter. The engine crankshaft is connected to the pump and the gearbox shaft is connected to the turbine. When the pump speed (ω_p) is higher than the turbine speed (ω_t) the pump forces the oil to circulate. The oil circulation then forces the turbine to increase its speed. The reactor is used to
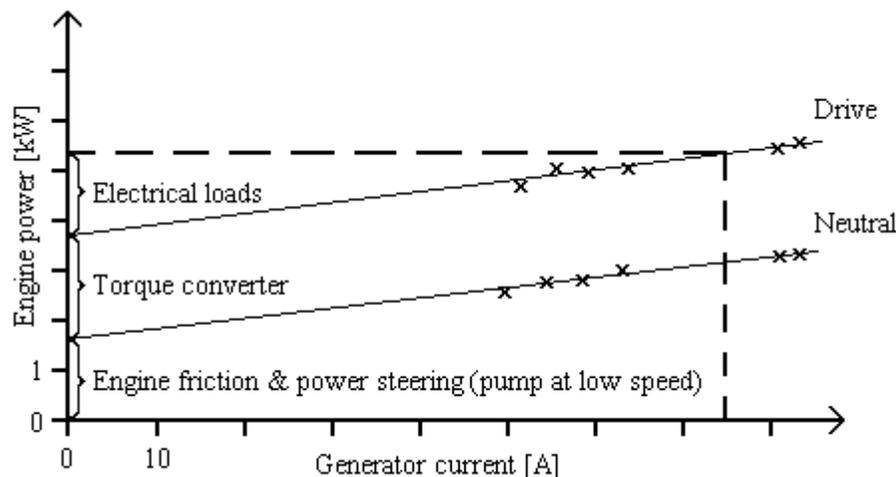

Figure 3.8 *A principal sketch of two straight lines adjusted to measurements made with gear selector in position neutral and drive. Power losses due to friction and power steering can here be calculated as well as how much engine power that is needed at a certain generator current.*

increase the turbine torque (T_t) and the degree of efficiency (η) when ν is small, see equation (3.15) to (3.18) and figure 3.9. The reactor does this by changing the oil flow direction. When ω_t increases the influence by the reactor decreases and finally it acts as an free wheel. The model we have used for the torque converter is also based on measured data [3]. To describe the torque converter we need the two characteristics λ and μ which are functions of ν. They are shown in figure 3.9. These two characteristics are described by the following equations, where T_p is the pump torque:

$$\nu = \omega_t/\omega_p \qquad\qquad (3.15)$$
$$\lambda(\nu) = T_p/\omega_p^2 \qquad\qquad (3.16)$$
$$\mu(\nu) = T_t/T_p \qquad\qquad (3.17)$$

In the μ-curve in figure 3.9 can be seen that the reactor acts as a torque multiplier when ν is less than 0.95. When ν becomes greater than 0.95 the reactor works as a free wheel. The degree of efficiency of the torque converter is included in these characteristics. This is shown in equation (3.18). The degree of efficiency is output power divided with input power.

$$\eta = P_{out}/P_{in} = \omega_t \cdot T_t/(\omega_p \cdot T_p) = \{(3.15) \text{ and } (3.16)\} = \nu \cdot \mu(\nu) \qquad\qquad (3.18)$$
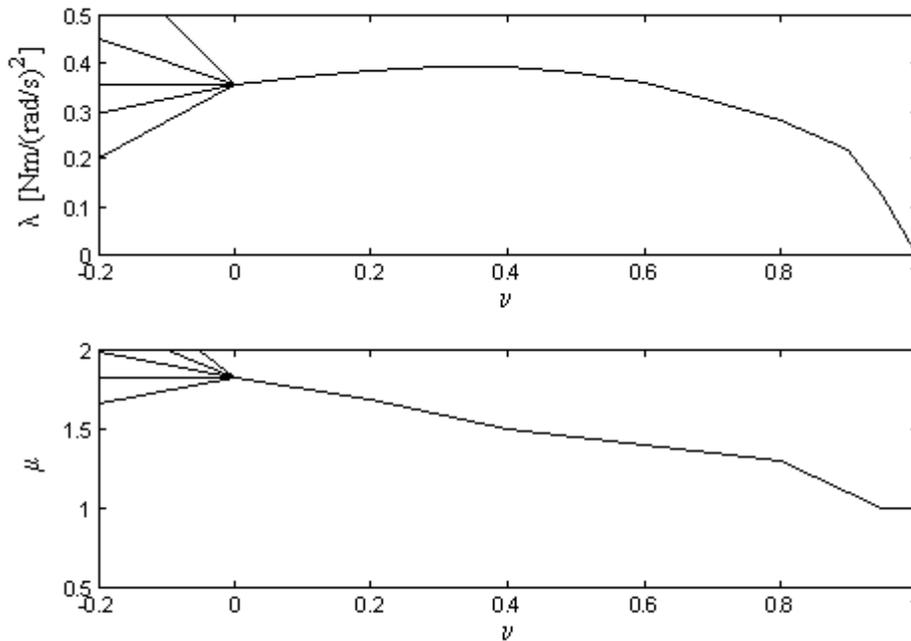

Figure 3.9 *Upper curve shows $\lambda(\nu) = T_p/\omega_p^2$, where $\nu = \omega_t/\omega_p$ and the other curve shows $\mu(\nu) = T_t/T_p$. Measurements are only made for $0 < \nu < 1$ therefore the figure also shows possible extensions of the curves for $\nu < 0$.*

The λ and μ curves are determined by measuring the torque's T_p and T_t when varying the speed ω_p and ω_t. Since it is impossible to do this measurement for all

combinations of ω_p and ω_t the test is made while keeping one of the variables ω_p and ω_t constant. This means that a set of λ and μ curves will only be valid for a fixed ω_p or ω_t and varying the other one. Measuring at a different fixed ω_p and ω_t shows that the λ and μ curves hardly depends on which of these two is constant and which varies [16], which also can be made credible by a theoretical one dimensional flow analysis [4]. Therefore we neglect this effect and consider the λ and μ curves to be the same for all combinations of ω_p and ω_t.

The real reason why these curves are measured are to show the torque converter's performance characteristics in a compact way and to be able to make comparisons between different kinds of torque converters. Since the normal way to use a car is driving, these curves are only measured for positive ω_t and consequently positive ν [17]. During idling however ω_t oscillates around zero. Since the wheels are not moving ω_t is equally negative and positive. This means that ν also will oscillate around zero because ωp is equal to the idling speed. Considering the movement which ν does around zero to be small, a linear part is added to extend the curves into the zone where ν is negative. Figure 3.9 shows a couple of possible curves. An evaluation of how these curves effect the model will be done in chapter 5.2.

3.4 Gearbox

During idling there is no gear change. The gearbox is therefore very easy to model which it would not have been during a gear change.

The gears in a gearbox have to be made of a very hard material because they are exposed to a lot of power. There will always be a little gap between the teeth of two gears engaging in each other. This gap is the source of a phenomena called rattle which occurs when a change of torque over the gearbox is made. This phenomena is very interesting when you are studying vibrations coming from the gearbox. During idling however the torque never changes direction and therefore this phenomena does not have to be modelled. In figure 3.10 a typical curve over the torque is shown.

Losses in the gearbox due to friction, oil circulation etc are considered to be small and therefore neglected in the model. Considering these facts the model of the gearbox is reduced to the equations (3.19) and (3.20)!

$$T_{out} = i \cdot T_{in} \tag{3.19}$$
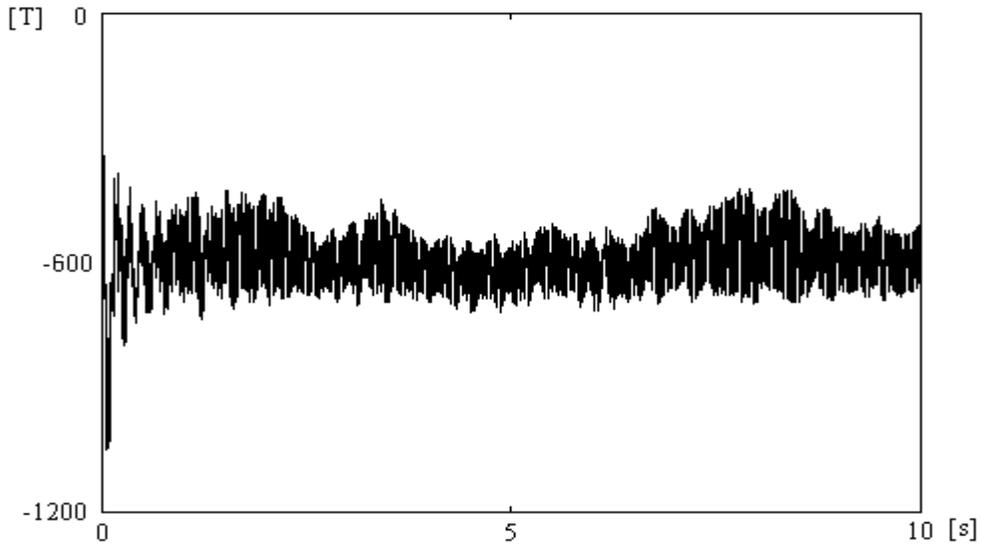$$w_{in} = i \cdot w_{out} \tag{3.20}$$

Figure 3.10 *The torque over the gearbox during a simulation of 10 s.*

3.5 Differential, including the final gear

The reason that a differential is needed is to allow the right and left wheel to have different angular velocity. Figure 3.11 shows a principal sketch of how a differential is built. To be able to model the differential the equations that relates angular velocity and torque, in the cardan shaft, left and right drive shaft is needed. With references according to figure 3.11 the derivation of these equations becomes as follows

$$\omega_H = n_1/n_2 \cdot \omega_G \quad (3.21) \qquad T_H = n_2/n_1 \cdot T_G \quad (3.24)$$
$$\omega_L = \omega_H + \omega_D \cdot n_3/n_4 \quad (3.22) \qquad T_L = T_H/2 + T_D/2 \cdot n_4/n_3 \quad (3.25)$$
$$\omega_R = \omega_H - \omega_D \cdot n_3/n_4 \quad (3.23) \qquad T_R = T_H/2 - d_H/2 \cdot n_4/n_3 \quad (3.26)$$

Using equation (3.21), (3.22) and (3.23) to solve ω_G and equation (3.24), (3.25) and (3.26) to solve T_G gives

$$\omega_G = n_2/n_1 \cdot (\omega_R + \omega_L)/2 \quad (3.27)$$
$$T_G = n_1/n_2 \cdot (T_R + T_L) \quad (3.28)$$

Using the fact that the input effect at the cardan shaft must be equal to the sum of the output effect at the left and right drive shaft, not considering the friction losses, gives

$$\omega_G \cdot T_G = \omega_R \cdot T_R + \omega_L \cdot T_L \quad (3.29)$$

Using equation (3.27) and (3.28) to get the same left side as in equation (3.29) gives

$$\omega_G \cdot T_G = \omega_R \cdot (T_R + T_L)/2 + \omega_L \cdot (T_R + T_L)/2 \qquad (3.30)$$

Identification from (29) and (30) gives

$$T_L = T_R \qquad (3.31)$$

Accordingly the equations needed to model the differential are equation (3.27), (3.28) and (3.31).
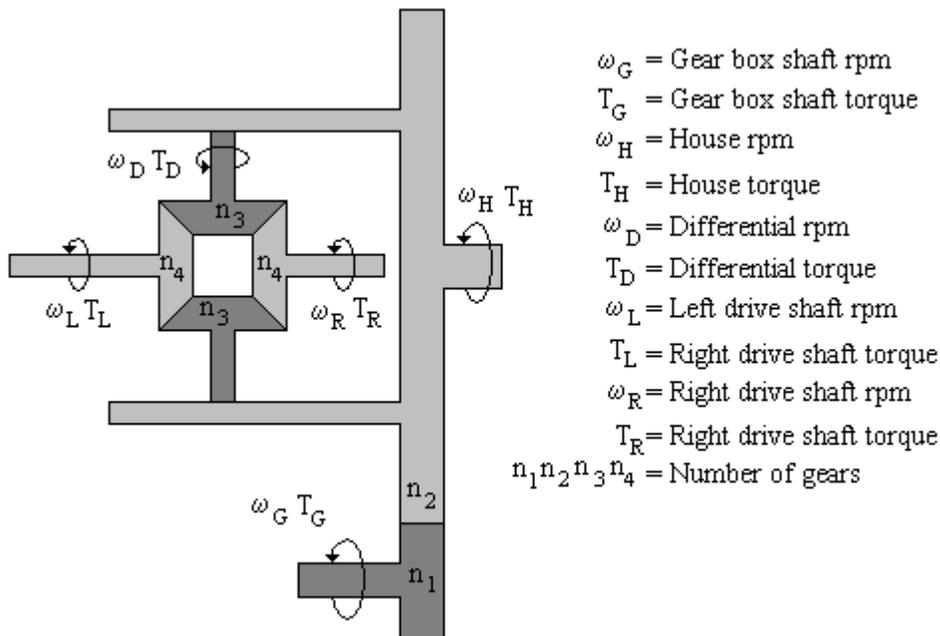

Figure 3.11 *A principal sketch of how a differential and the final gear is built.*

3.6 Three dimensional modelling

When building the three dimensional part of the model bushings, rods and body are used as corner stones. They are not hard to model, but still important to explain because of the simplifications which are done.

3.6.1 Bushing

All bushings are modelled as a damper and a spring placed parallel to each other and the model has the following attributes:

• Pretensioned by the forces working on the car due to the gravitational force makes the force oscillate around a non zero level. In the model we only use the offset from this level.

• No torque is transferred from one side of the bushing to the other. Bushings are made to handle forces not torque. The torque they do handle is negligible.

• Small movements at idling makes it possible only using the linear part of the spring's characteristics.

• Their dynamic characteristics are not modelled because the model is made to study the static behaviour of the car at idle.

With references given in figure 3.12 [5].

$$\mathbf{F} = k \cdot (\mathbf{X}_A - \mathbf{X}_B) + d \cdot (\mathbf{V}_A - \mathbf{V}_B) \tag{3.32}$$
$$\mathbf{T}_A = \mathbf{T}_B = 0 \tag{3.33}$$

All data for the bushings are given as functions of frequency and have to be recalculated to the time domain. In the frequency domain they are usually given as spring constant (k') and loss factor (η_l) or loss angle (γ), see figure 3.12.

$$\mathbf{F} = k^* \cdot \mathbf{X} = k' \cdot (1 + \eta_l\, i) \cdot \mathbf{X} \tag{3.34}$$

The interesting frequency (f) is twice the desired idling revolution and it is therefore used when k and d are calculated [6].

$$k = k^* \cdot \cos(\gamma) = k' \tag{3.35}$$

$$d = k^* \cdot \sin(\gamma)/f = k' \cdot \eta_l\, /f = k' \cdot \tan(\gamma)/f \tag{3.36}$$

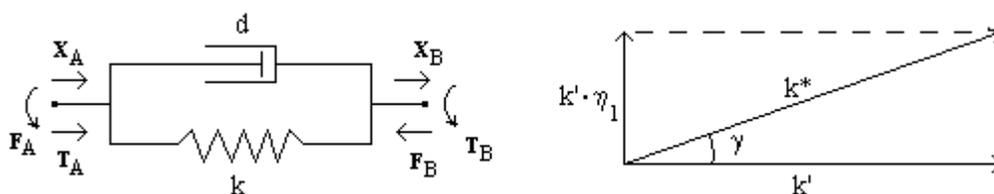The loss angel is approximately five for rubber bushings, which is used when the loss angel is not given [18].

Figure 3.12 *References for equation (3.32) to (3.36).*

3.6.2 Rod

The rod is modelled as a component without mass [5]. It's only used to transfer velocities, force and torque from one co-ordinate to another. During idling only small movements are made and therefore the rod is linearised regarding $\sin(\varphi) = \varphi$ and $\cos(\varphi) = 1$.

$$\mathbf{r}_{AB} = \mathbf{X}_B\text{-}\mathbf{X}_A \tag{3.37}$$
$$\mathbf{X}_B = \mathbf{X}_A + \boldsymbol{\varphi}_A \times \mathbf{r}_{AB} + \mathbf{r}_{AB} \tag{3.38}$$
$$\boldsymbol{\varphi}_B = \boldsymbol{\varphi}_A \tag{3.39}$$
$$\mathbf{V}_B = \mathbf{V}_A + \boldsymbol{\omega}_A \times \mathbf{r}_{AB} \tag{3.40}$$
$$\boldsymbol{\omega}_B = \boldsymbol{\omega}_A \tag{3.41}$$
$$\mathbf{F}_B = \mathbf{F}_A \tag{3.42}$$
$$\mathbf{T}_B = \mathbf{T}_A + \mathbf{F}_B \times \mathbf{r}_{AB} \tag{3.43}$$

3.6.3 Body

The mass is modelled with following equations [5]:

$$\sum \mathbf{F} = m \cdot d\mathbf{V}/dt \tag{3.44}$$
$$\sum \mathbf{T} = \mathbf{J} \cdot \boldsymbol{\alpha} + \boldsymbol{\omega} \times (\mathbf{J} \cdot \boldsymbol{\omega})$$
(3.45)

Since the forces in the bushings are offsets from a pretensioned level, the gravitational term is not included. Due to this the pendelum phenomen will not be regarded.

4. Implementation

Simulink [7] is a simulation tool for making models in the time domain. Because Simulink is an existing and well known program at NVH, the one dimensional part of the model without engine control system, electrical loads and differential was first implemented in Simulink. But then, when trying to implement the rest of the model, problems occur! When making a model in Simulink it must always be defined what is input and what is output. Instead of spending a lot of time writing and solving the equations for Simulink and repeating the work, when some changes in the model have to be done, it is better to change modelling tool.

Dymola [8] is an equation based object orientated modelling tool. Dymola is used because of the following advantages:
- Reuse of modelling knowledge by using libraries containing model types or model classes.
- Equations are written as ordinary differential equations and algebraic equations in each model type. This will be an easy and natural way when modelling components.
- Connections between submodels are described by defining cuts which model physical coupling. Understanding of cuts is essential when using Dymola and will therefore be explained in appendix A.1.
- The causality between different model types in a model is handled by Dymola Translator. In Simulink you would have to transform the equations differently depending on which variables are known and unknown.
- The software can give model output on e.g Simulink format (Simstruct format). Through this feature the model, or a part of it, could be exported to Simulink.

One disadvantage with Dymola is that it is not always easy to find out how the equations are solved. Specially not when the model becomes very complex. Another disadvantage is that when a subsystem is under or over determined, Dymola will tell you that the whole system is under or over determined, but not where in the system. In Simulink this does not occur because the equations have to be clearly defined.

The result of making the implementation in Dymola is an easy changeable model, which also is a part of the goal for this thesis.

How the submodels are connected to each other when the implementation in Dymola is done is shown in figure 4.1 to 4.6. The names which are used in Dymola are put in brackets. How the submodels are written can be seen in appendix B where a HTML file created by Dymola is shown.
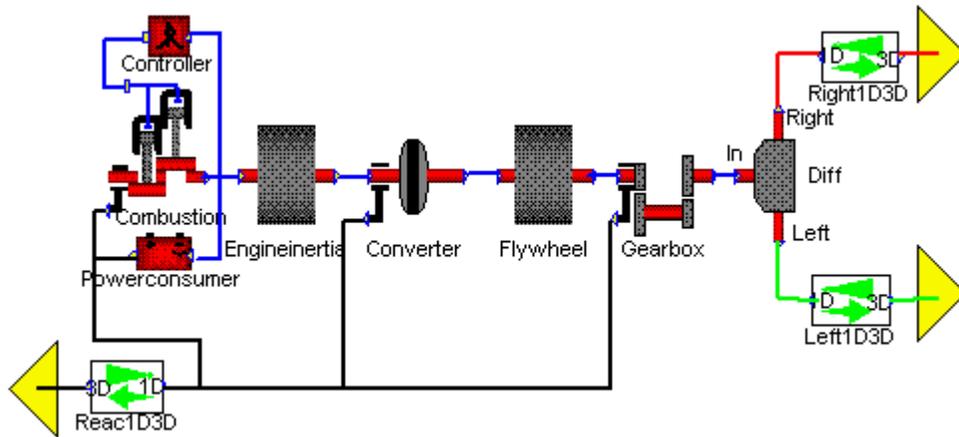
Figure 4.1 *Here is the one dimensional part (Engine1D) shown. The difference to figure 3.1 is that the four stroke engine is separated into the three submodells (Combustion, controller and Powerconsumer) and that the torque converter is modelled by two flywheels (Engineinertia and Flywheel) and a look-up table (Converter).*
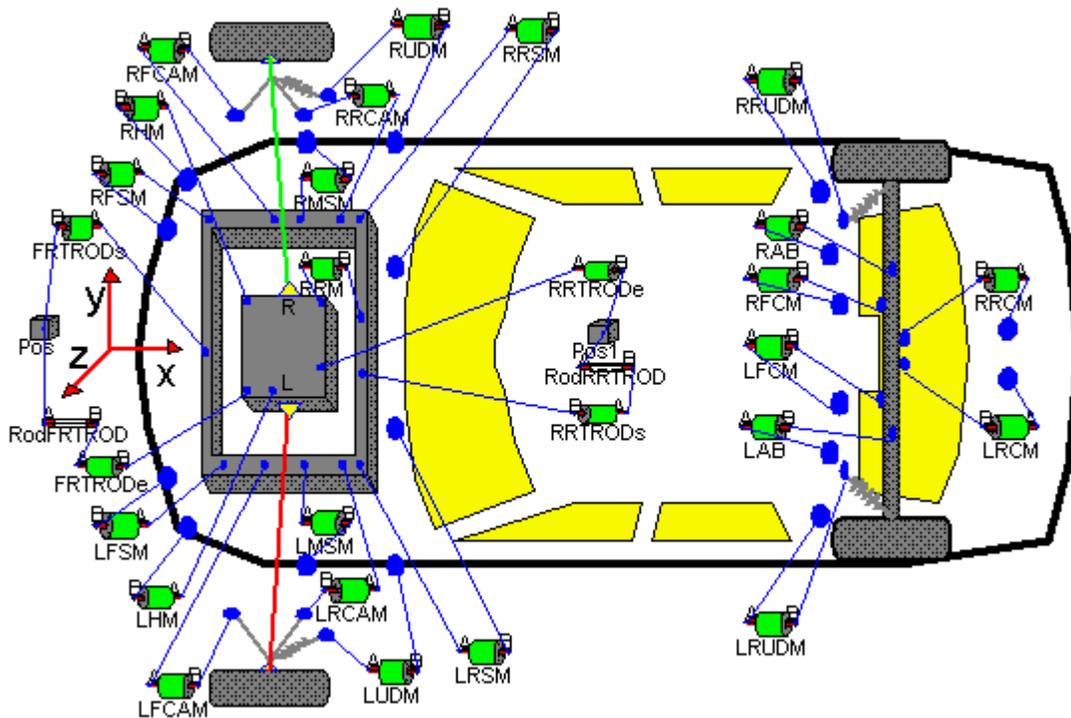

Figure 4.2 *Here is the three dimensional part shown (Car). The submodells left wheel (LWSusp), subframe (Subframe), engine block (Engine3D), right wheel (RWSusp), car body (Carbody) and rear suspension (RearSusp) are the same as in figure 3.1.*

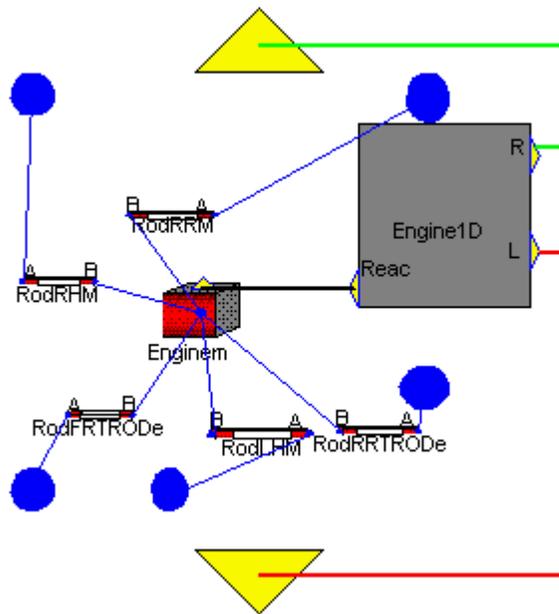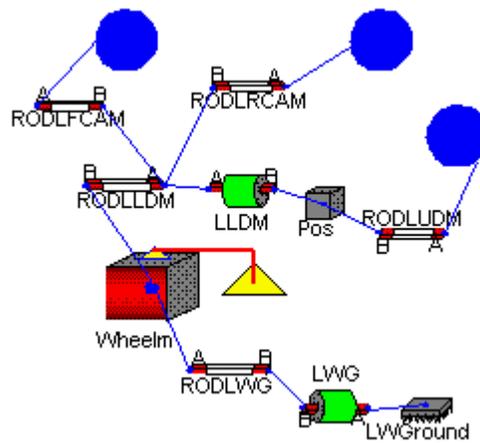**Figure 4.3** *The engine block (Engine3D) which contains the one dimensional part (Engine1D).*

Figure 4.4 *The Left wheel and it's suspension (LWSusp). The right wheel and it's suspension (RWSusp) is not shown because it is built in the same way.*
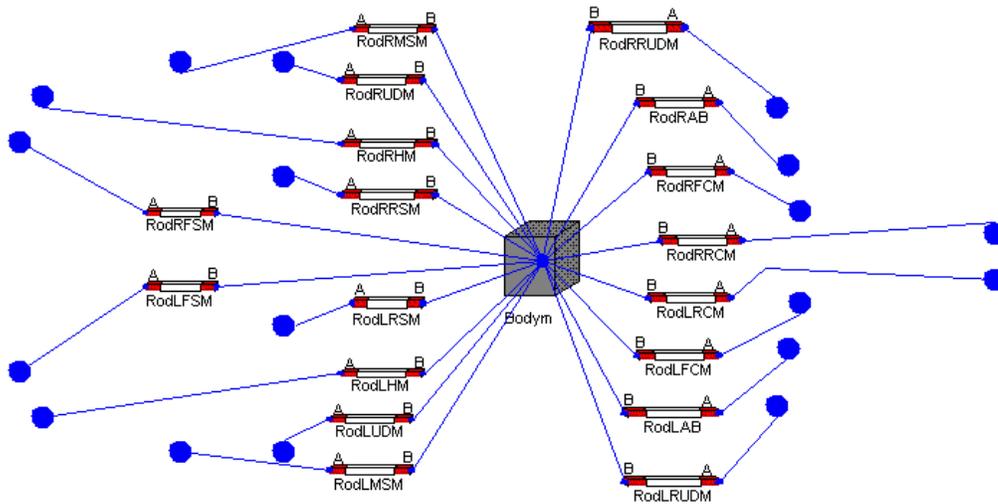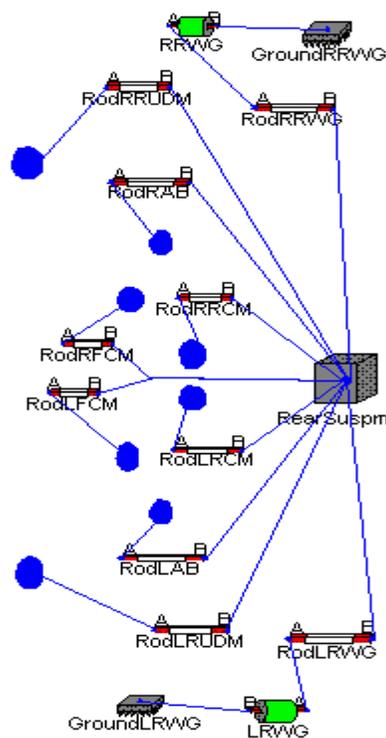
Figure 4.5 *The car body (Carbody).*

Figure 4.6 *The rear suspension (RearSusp) is only modelled as a mass. The wheels and the cross member have not been modelled as separate masses, because the influence on the model was negligible and it also takes a lot of extra computer power during a simulation.*

5. Verification

5.1   The four stroke engine

The model of the input to the system i.e. the crankshaft torque that is done in chapter 3.2 needs to be validated to ensure that it generates the correct crank shaft torque.

In figure 5.1 four plots can be seen. The upper left plot shows the measured cylinder pressure data converted to crankshaft torque and summarised for all four cylinders. Here can be seen that the torque hardly varies from one revolution to the other except for φ_c between 0° and 60°. In the upper right corner simulations from the mathematical model is plotted. Here can be seen that when φ_c is between 0° and 60° the mathematical model is a good approximation of the measured data. During the other crank shaft angles the mathematical model is the mean value of the measured data. In the lower left corner simulations from the implemented model in Dymola is shown. The reason why these curves have sharp edges is that it is
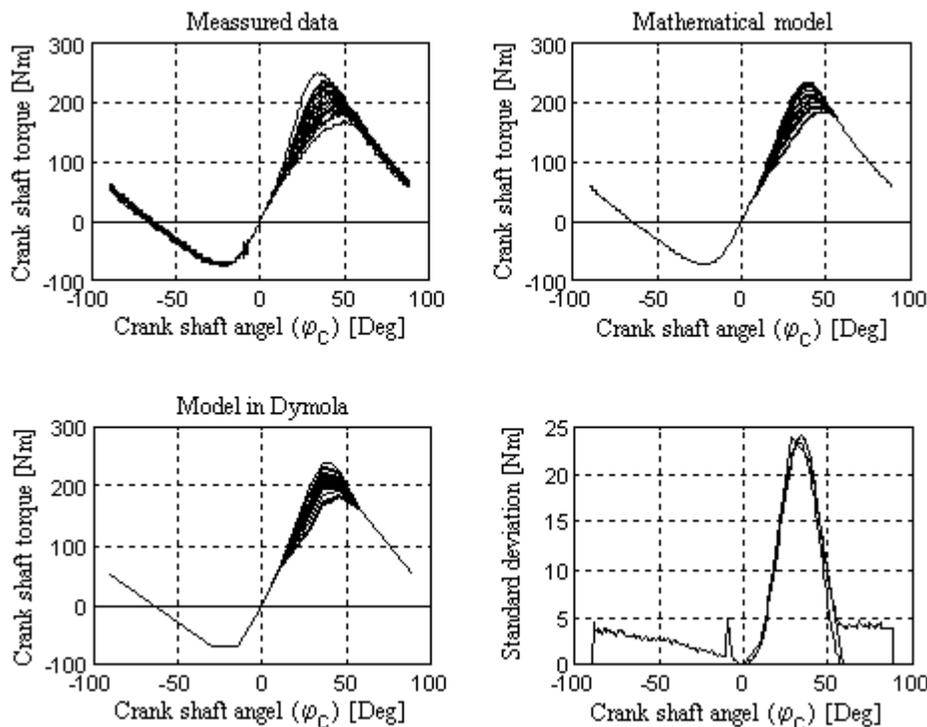

Figure 5.1 *The three plots of the crank shaft torque are plotted with 24 curves and the standard deviation is calculated from 100 cycles for each of them and drawn in the same plot. For $0^0 < \varphi_c < 55^0$ the standard deviation of the modelled is equal to the measured. Outside of this range it is modelled to be zero.*

implemented as a look up table. Between the chosen points the torque is considered to be a straight line. The sharp edges is due to using as few points as possible to keep down the simulation time. Making the simulations on a faster computer or allowing longer simulation times would allow these curves to be modelled smoother and more like the mathematical results.

In the lower right corner a plot of the standard deviation for all three torque's are made. Here can be seen that for φ_c between 0° and 60° the standard deviation is close to identical for all three torque's. For the other crank shaft angles the standard deviation for the mathematical and the Dymola model is considered to be zero while the measured torque has got a standard deviation that is not zero. The reason why the standard deviation is big for crank shaft angles other than between 0° and 60° is that the mean value of the four cylinders are not the same. The model of the engine is made under the assumption that the mean pressure in all four cylinders are equal. This is however not completely true. Measurements show that the mean pressure of the cylinders are different. This assumption is made to keep down simulation time and because the standard deviation is much lower for crank shaft angles other than between 0° and 60°.

If simulations are made on a more powerful computer however the cylinders should be separately simulated. This is not difficult because it only means that Dymola has to work with four instead of one look up table in the engine model and they have to be four times longer. The way the model handles this is that the mean value and standard deviation is calculated for all four cylinders. Therefore the standard deviation will be correct for φ_c between 0° and 60°. The result of this is that the engine can be modelled as one cylinder firing four times during two revolutions.

The model appears to be correct since the models standard deviation is almost identical to the measured standard deviation for φ_c between 0° and 60°.

5.1.1 Power consumer

The power consumer is modelled to make it possible to take care of three major power consumers. These are internal friction in the four stroke engine, the power steering at low speed and electrical loads. Plots such as in figure 3.8 gives the amount of these values to be put into the model. When this is done for the measured data used in this model the total amount of power in the consumers is to low. This is shown when simulating because the engine speed becomes higher than the desired one. To adjust this the engine friction power loss was raised until the desired engine speed was maintained. This is not a correct way of solving this problem. The fact that the engine speed becomes to high when using the measured engine friction power loss inclines that there are other losses in the car not included in the model. The correction of the engine losses is about 7-8 % of the total engine power. There might be several reasons why the model does not correspond to the measured values of the real engine. One is that there are error in measurements and error in the adjustment of the curve (straight line) in the plot in figure 3.8. Another reason is that the power losses in the gearbox can not be neglected.

5.2   Torque converter

Since there are no measurements done on λ and μ for negative υ there has to be a validation of how the model is affected by the fact that υ becomes negative. To examine the effect of negative υ two different extensions of the λ and μ curves are tested. The two cases A and B can be seen in figure 5.2. λ and μ in case A are chosen from simulations to quickly reduce transients due to the simulation being started. In case B, λ and μ are chosen to be natural extensions of the curves. Other possible combinations has been tested but as will be discussed in this chapter they only affect simulations which contains transients.

Since the model is not made for studying transients, due to the lack of the control system, it should be enough to study υ when all transients are gone. To show how important the modelling of negative υ is when studying transients the transient behaviour of the torque converter will also be discussed.

In figure 5.3 the transient behaviour of the torque converter is shown for the two different extensions. Here can be seen that case A is less oscillative than case B. This can be seen both in υ and in the engine angular acceleration. In the engine angular acceleration the transient behaviour of the torque converter can be clearly seen as the envelope off the curve. That the rpm differs however is not due to the difference in the torque converter. This is due to the chaotic part of the combustion. The rpm shown in the figure is at the pump side of the torque converter. Since υ is equal to the turbine rpm divided with pump rpm and the pump rpm is big in relation to the turbine rpm, the turbine rpm has the same appearance as υ. The plots show that if transient studies are to be made the torque converters behaviour for negative υ has to be determined.

Depending on at which crank shaft angle the simulation is started the rpm fluctuates. The start of the simulation of the model could be compared with reality, it would then simulate that the gear selector be put into drive. Since the studies will be done after the transients are gone this angle has been chosen to minimise the decrease of the rpm. This is done to keep the simulation time down.
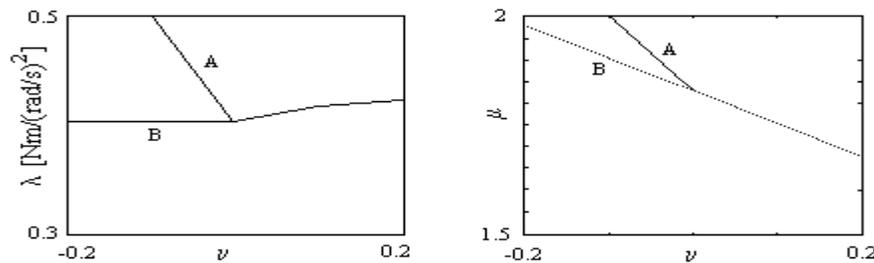

Figure 5.2 *The figure shows the two different set of extensions of λ and μ for negative v for which the validation has been done*
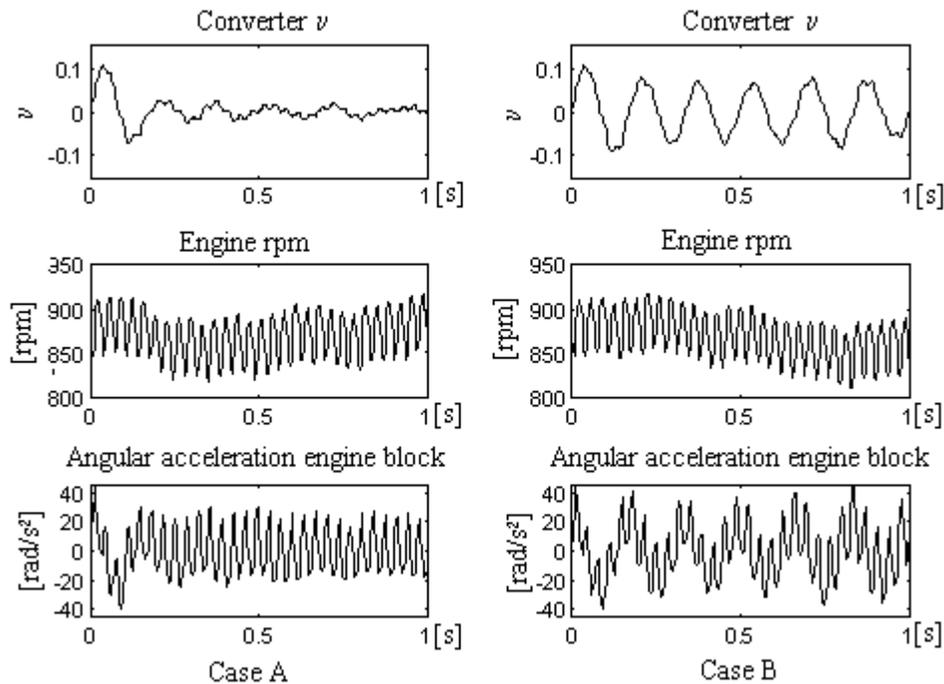
Figure 5.3 *The figure shows how the transient part depends on the converter parameters λ and μ. Case A (left column) is less oscillative than Case B (right column). This shows that when studying transients λ and μ for negative υ has to be examined.*

In figure 5.4 the behaviour of the torque converter can be seen when the transient almost is gone. Comparing the angular acceleration it can be seen that the amplitude and behaviour is almost the same. The engine angular acceleration is now depending more on the rpm than υ, see the discussion about the engine angular acceleration depending on the engine rpm in chapter 5.3. The reason to this is that υ is ten times smaller now than during the start of the engine at t=0 seconds. This means that when the transient is gone λ and μ can be considered as constants. The main reason why λ and μ are not modelled as constants is that this would result in longer simulations to reach steady state. Because of this case A is chosen as the continuing of the λ and μ curves for negative υ. Another reason for not modelling λ and μ as constants is that this model can be used for normal driving when υ does not become negative.

5.3 Angular acceleration of the engine block

To be able to do a correct validation there should have been measurements done of the forces in the bushings between car body and engine block, subframe, left and right wheel, because they are considered to be the output of the model. Now these
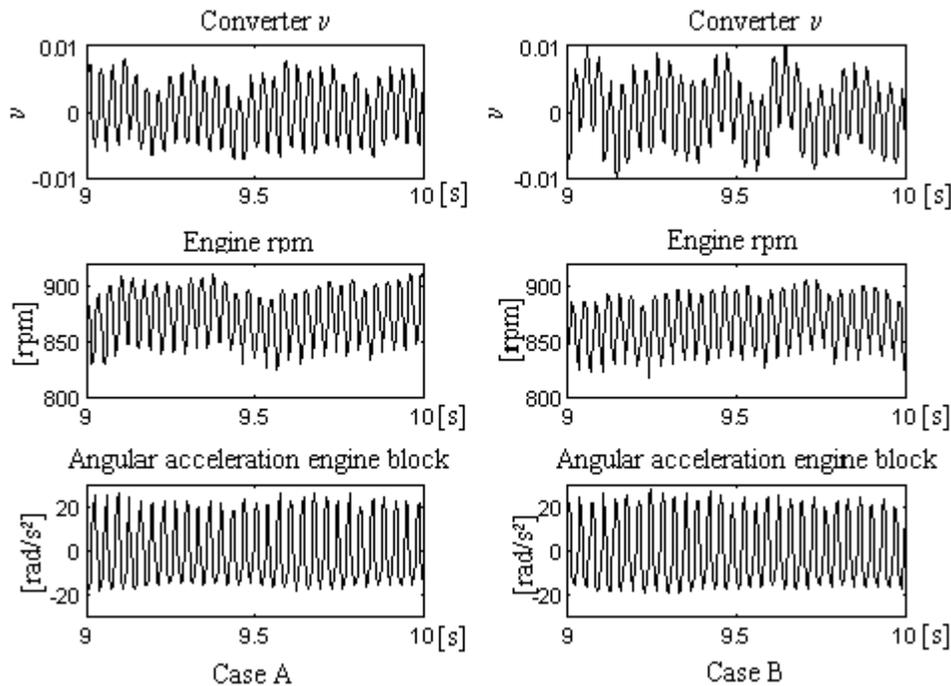
Figure 5.4 *The figure shows that when the transient part is over the results becomes the same for both Case A (left column) and Case B (right column). This means that the parameters λ and μ can be considered to be constant. This is due to the small variation in the turbine speed which makes υ small.*

measurements where not planned to be made in the test that is used. The only measurements that could be used is the angular acceleration around the y-axis of the engine block and the engine rpm at idle. The angular acceleration is a result of the reaction torque from the one dimensional engine and the bushings from the three dimensional part. In other words if the angular acceleration is correct the forces in the bushings should be correct to.

There are two things that can be validated with the angular acceleration i.e. the appearance of the curves and the amplitude of the curves. In figure 5.5 the measured and simulated rpm and angular acceleration during four seconds can be seen. In the plot of the measured rpm can be seen that it has got a self-oscillation with approximately 1 Hz. This self-oscillation can not be clearly seen in the simulated models rpm. It is believed that it is due to the engine control system, even though it is not seen in plots, this is discussed in chapter 3.2.4. The simulated rpm becomes lower than the desired rpm (866 rpm) between 7 and 9 seconds. The reason to this is that the mean value of the normally distributed noise used to model the chaotic part of the power stroke tends to drift. The way to correct this would be to model the engine control system.

In the plot of the angular acceleration can be seen that the simulated acceleration is higher between 7 and 9 seconds than the measured one. This is due to the fact that the simulated rpm is lower between 7 and 9 seconds. The amplitude becomes higher when the rpm decreases and smaller when the rpm increases. The reason why the measured acceleration does not vary with 1 Hz is that the acceleration transmitter works as a high pass filter with a cut off frequency at 3 Hz [19]. Looking at the measured acceleration it is possible to see a vague rest of the 1 Hz. This means that to be able to compare the measured and simulated acceleration the measured acceleration should be high pass filtered with 3 Hz. This is not done because the output file from Dymola has not got a linear time scale and this makes filtering in Matlab [9] difficult.

The appearance of the measured and the simulated angular acceleration during 0.25 seconds can be seen in figure 5.6. In the part that is chosen the simulated and measured part have the same rpm, due to the discussion above that the amplitude is depending of the rpm. In the plots it can be seen that the model manages to simulate the shape of the acceleration. This means that the upper part is pointy and the lower part is round. As a result of this the amplitude is higher on the upper than the lower part. This is due to the engine block's vibration around an equilibrium. The higher frequency in the measured acceleration is not modelled and therefore it will not appear in the simulated acceleration.
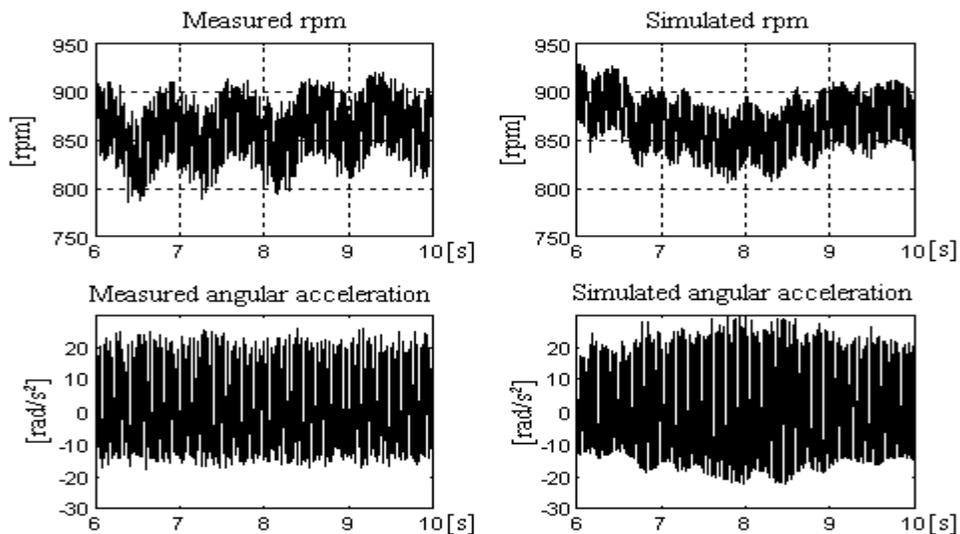

Figure 5.5 *In the measured rpm a 1 Hz self-oscillation can be seen. This is probably due to the engine control system. The simulated rpm is lower between 7 and 9 seconds which is due to the drifting of the mean value of the normally distributed noise in the model of the chaotic part of the power stroke. The reason why the simulated angular acceleration has a higher amplitude than the measured is that the simulated rpm is higher due to the lack of the engine control system.*
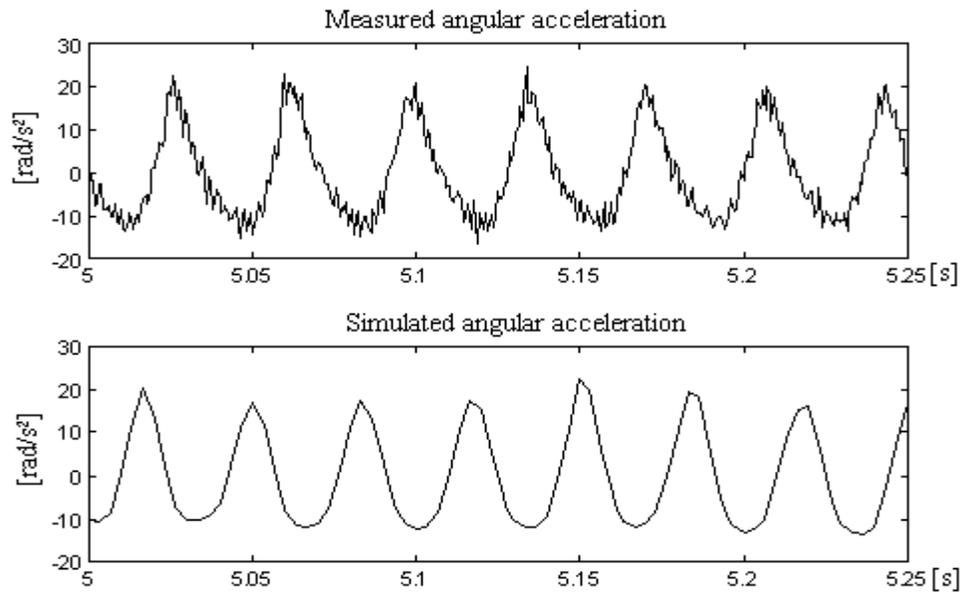
Figure 5.6 *In the plot can be seen that the appearance of the simulated angular acceleration is the same as the measured. That is that it has got a round bottom and a pointy upper part. As a result of this the amplitude is higher on the upper than the lower part.*

Verification

6. Conclusion and results

The model shows good results when the simulations are done. The model is able to simulate the angular acceleration of the engine block both in shape and size. This is not the output of the system but as discussed in chapter 5.3 the angular acceleration should be enough to tell if the model is acceptable or not. These results should be enough to show that it is possible to do studies of a complete car in the time domain.

To get all the advantages by simulating in the time domain the model should be used to analyse transients. This would give the opportunity to study the non-linear behaviour of the driveline. To be able and to do this the engine control system has to be modelled. This would enable studies of altering loads such as ACC and Power steering. The model can also be used to study gear changes. Then both the engine control system and the gearbox control system has to be modelled.

When these changes are to be done the advantages of using an object orientated modelling tool such as Dymola will be shown:

• Easy changeable models. The use of model classes allows changes at a lower hierarchical level without needing to change higher levels.
• Reuse of modelling knowledge by using libraries containing model classes.
• Equations can be written as ordinary differential equations and algebraic equations in each model class.

Conclusions and results

7. Suggestions for future work

7.1 Model improvements

There are some improvements that might be needed on the existing model. One is to examine what happens with the torque converter when ν is negative. There are two ways to do this. The first is to measure λ and μ for negative ν. This would be a direct way and the results could easily be implemented in the existing model of the torque converter as an extension of the look up table. The other is to find the equations that describe the torque converter. There has been done a thesis which considers the one dimensional dynamics of the oil flow inside the torque converter [10]. This should be a good foundation when modelling the torque converter in a physical way instead of using measured data. The latter might be a better choice when the model is built to examine transients because this model can be made especially for studying the transient behaviour of the torque converter.

Another improvement might be to make a model of the engine control system. This would probably make the input to the existing model more accurate. It is not sure however because the e.c.s will probably have a low effect in the frequency that this model is made to study i.e. approximately twice the revolution at idle.
If the e.c.s is not to be modelled the measurements of the cylinder pressure for different loads could be used to study different constant loads.

A third improvement might be to consider the gearbox losses.

7.2 Transient studies

Modelling the e.c.s would make transient studies possible which is one of the largest advantages when modelling in the time domain instead of the frequency domain. If the existing model were rebuilt to study transients the model of the engine could be made much easier. Then since the model should be built to examine lower frequencies it should be enough to model the torque from the engine as the mean torque during one revolution. There would be three major load cases that could be studied namely altering load during idling, altering between idling with gear selector in/not in drive and gear shifting while driving the car.
The changes that has to be made on the model to be able to simulate altering loads is that the e.c.s has to be modelled. Further the influence from the torque converter when υ is negative has to be checked and modelled. As can be seen in the validation chapter this is important and can not be neglected. When this is done simulations such as the one shown in figure 7.1 could be done. Here it can be seen that after 2 seconds there is a change in electric loads (e.g. ACC). The model would then show how this effects e.g. forces in the bushings, rpm and the engine block
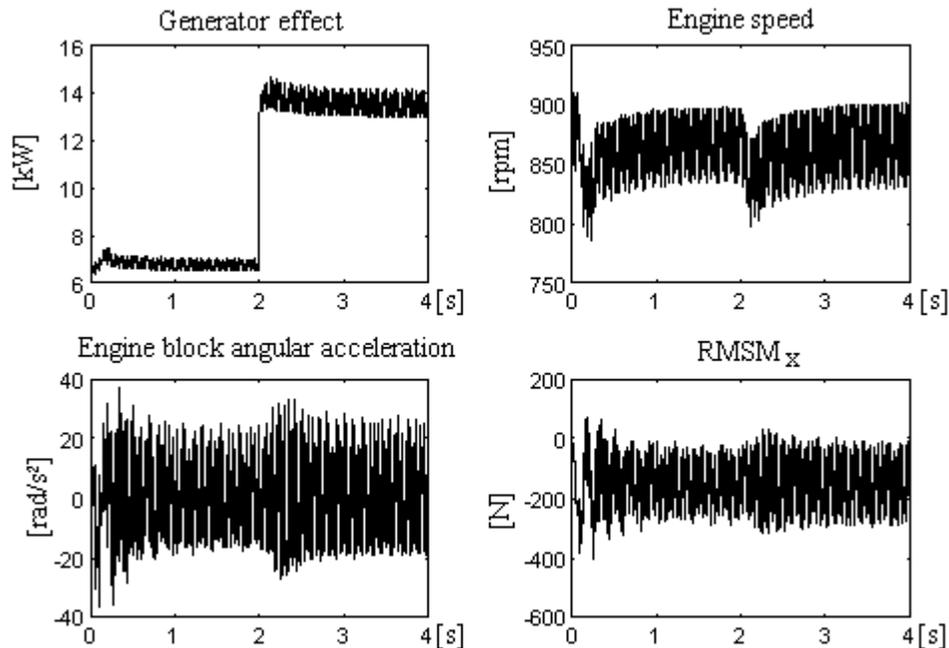
Figure 7.1 *Shows what could be studied if the engine control system is modelled. After 2 seconds there is a change of the generator effect (upper left). Then the changes can be seen in the engine speed (upper right), engine block angular acceleration (lower left) and also in the force in the RMSM$_X$ bushing (lower right).*

angular acceleration. The simulation is made with the existing model and an simple control system.

When these changes are made the next step would be to model the gear box in a different way to enable simulating how the idling with gear selector in/not in drive and gear shifting while driving the car would effect the car. There are several works done to model the gear box. Especially there is a German project [11] that uses Dymola as an implementation tool. This work would be a good foundation when modelling the transient, non-linear behaviour of the gear box.

When studying transients the bushings will probably have to be modelled as non-linear due to larger movements. This is no problem however since this just means that the parameters has to be implemented as look up tables.

Another modelling approach would be to combine Dymola with Adams [12]. The advantage with this approach is that there are existing models of the three dimensional part in Adams at Saab. The driveline would then be modelled in Dymola. A project is done at Chalmers University of Technology to combine these two programs [13]. Very recently, it seemed that interfaces between

Adams/Simulink and Adams/Systembuild are going to be developed which also can be of interest.

When these changes are to be done the largest advantage of using an object oriented modelling tool such as Dymola will be shown. The reason is that the model is built of easily changeable blocks that can be replaced one by one without having to change the rest of the model.

Suggestions for future works

8. References

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References

A. Dymola

Dymola is an object orientated modelling tool and has model, model class and library window class as different classes. In a library window class all models and model classes are stored during the work, they can also store other libraries. Model is at the highest hierarchical level when modelling a system. There can only be one model in a system. In a model or a model class the equations can be directly written. They can also contain other model classes.

The libraries with model and model classes which were created during the implementation are shown in figure A.1 to A.4. There are a lot of prebuilt components stored in libraries which are available in Dymola, but they are not used in this thesis. To get a good understanding of what happens in the model and of Dymola as modelling tool the libraries and components were created from the beginning.

Figure A.1 The library window class Mainlib contains the library window classes Comp1DLib, Comp3DLib, Compcar and Extra, the model 9^5 and an information box.
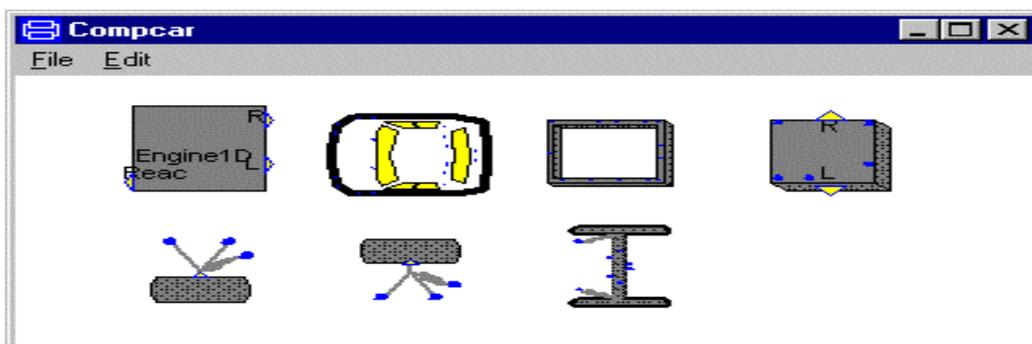

Figure A.2 *The library window class Compcar contains the model classes Engine1D, Carbody, Subframe, Engine3D, LWSusp, RWSusp and RearSusp.*
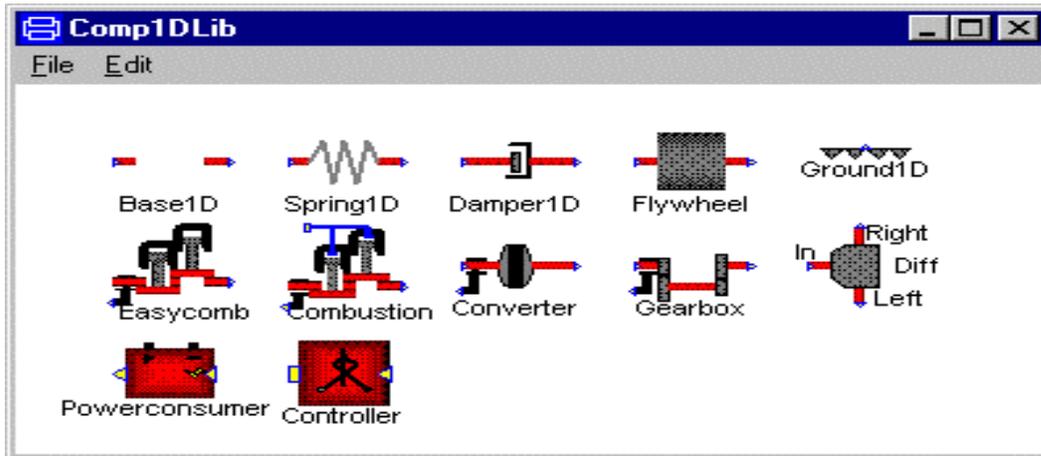
Figure A.3 *The library window class Comp1Dlib contains the model classes which are used to model the one dimensional part.*
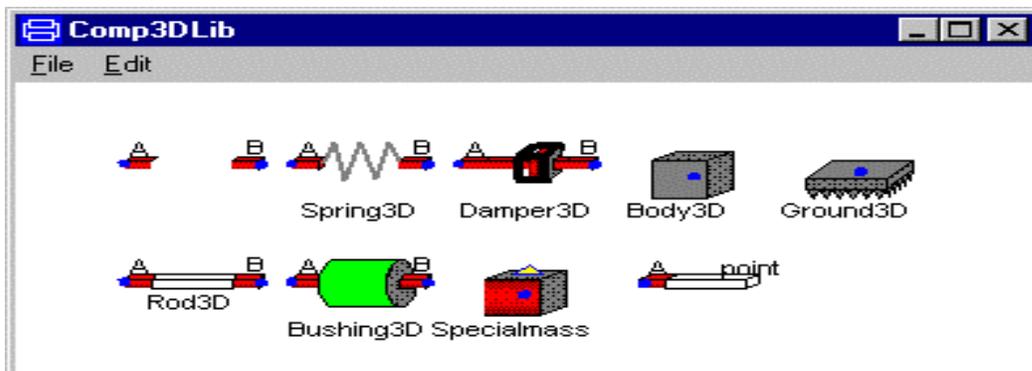

Figure A.4 *The library window class Comp3Dlib contains the model classes which are used to model the three dimensional part.*

A.1 Cuts

Cuts are essential when modelling in Dymola. They are the connection points between submodels which are created of model classes. When two or more submodels are connected to each other in a cut the variables, which are defined in the cuts, are related to each other. This means that they are set equal or summarised to zero depending on how they are defined in the cuts. An analogy to the electrical area is that in a connection point the voltage level is the same for all components. The current which flows through the components are summarised to zero in the connection point (Kirchoffs law I). In bond graphs [14] the use of effort and flow variables is used to describe electrical, mechanical and thermical systems. Understanding of the theory presented in bond graphs makes it easier to understand Dymolas use of cuts.

A.2 Modelling example

To give an easy example of how a model class is created in Dymola a torsion spring will be modelled. In figure A.5 the following modelling steps are illustrated.

1. Choose global reference directions. In this example ω and T are chosen as variables.
2. Choose internal reference directions.
3. Set the variables in the cuts depending on how the references are chosen in 1 and 2 in the diagram layer, see figure A.6. If the variable is written on the left side of the slash sign then that kind of variables are set equal when connecting them in cuts. Are the variables written on the right side, then they are summarised to zero.
4. Draw a figure of a spring in the icon layer, see figure A.6.
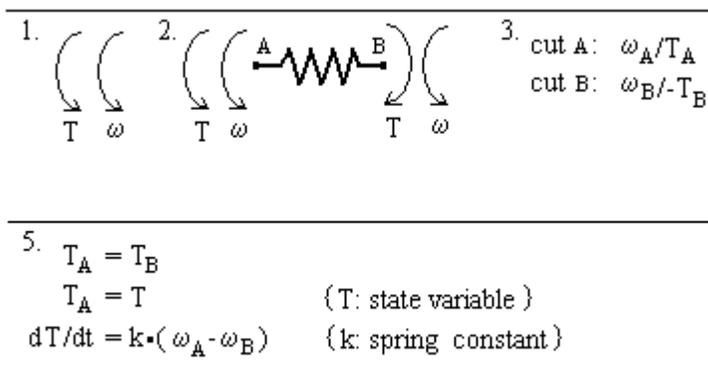5. Write the equations with the references chosen in 2 in the equation layer, see figure A.7.

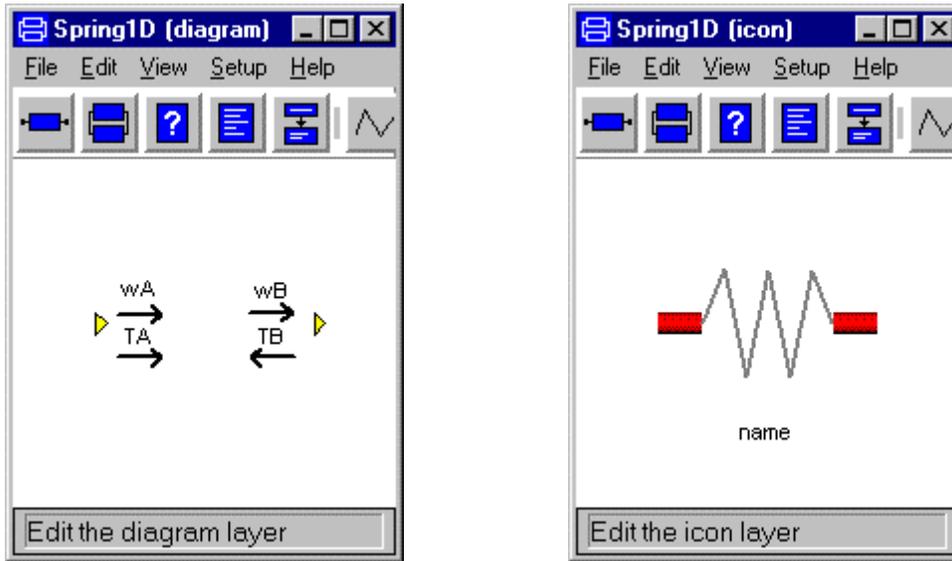**Figure A.5** *Modelling a torsion spring.*

Figure A.6 *In the diagram layer the cuts are defined. A picture of how the spring will be presented in the model is drawn in the icon layer.*
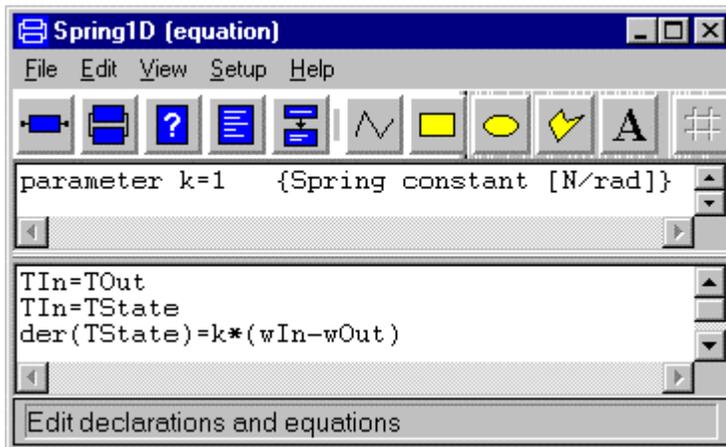

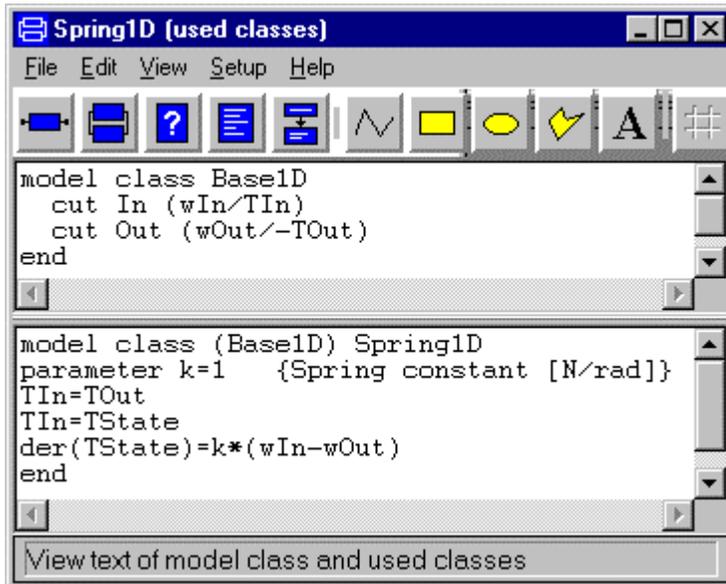**Figure A.7** *In the equation layer are the equations and declarations written.*

Figure A.8 *In the used classes layer the equations, declarations and used classes are shown.*
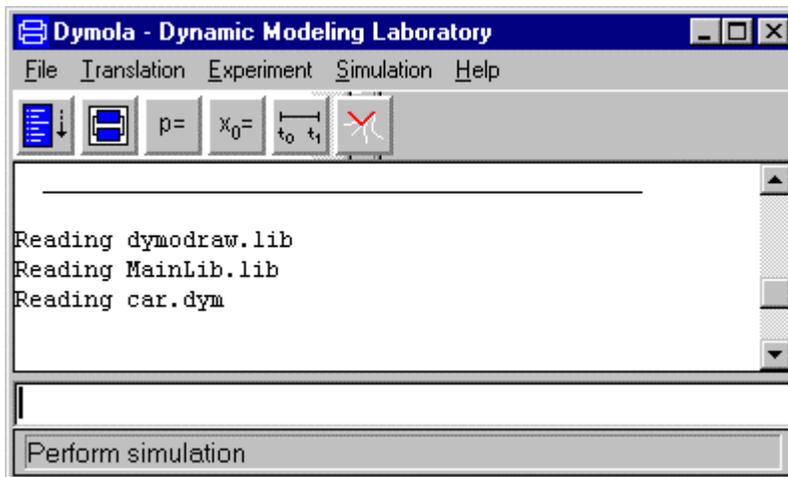

Figure A.9 *In the Dynamic Modelling Laboratory the model is compiled, simulations are started and parameters or initial conditions altered.*