Chapter 13 Optimization and Matching

Section 13.1

Definitions

weight $wt(e)$, weighted graph, distance measure in graphs, shortest path, greedy algorithm

Algorithm

Dijkstra's Shortest-Path Algorithm

Exercises

1, 2, 5

Section 13.2

Definitions

optimal (minimal) spanning tree

Algorithms

Kruskal's Algorithm	Th 13.1	$O(\|V\|^2 \log_2(\|V\|))$
Prim's Algorithm	Th 13.2	$O(\|V\|^2)$

Exercises

1, 2, 5, 6

Section 11.3

Definitions

network (transport network), source, sink, capacity, flow, saturated, cut, value of the flow $val(f)$, capacity of cut $c(P, \bar{P})$

Theorem

13.3	$C = (P, \bar{P})$ any cut. Then $c(P, \bar{P}) \geq val(f)$.
C13.1	value of flow from the source = value of flow into the sink.
13.4	$\min c(P, \bar{P}) = val(f)$

Exercises

1, 2, 4, 6, 7

Section 11.4

Definitions

assignment problem, matching, complete matching, maximal matching, perfect matching, deficiency of, deficiency of graph G, $\delta(G)$, permutation matrix, doubly stochastic

Theorem

G bipartite, partitioned as $X \cup Y$.

13.5 Complete matching of X into Y $\Leftrightarrow \forall A \subseteq X : |A| \leq |\{y \in Y | \exists x \in A : x \text{ adjacent to } y\}|$

C13.3 $\exists k \in \mathbb{Z}^+ \forall x \in X, y \in Y : deg(x) \geq k \geq deg(y) \Rightarrow$ Complete matching of X into Y.

13.6 The maximum number of vertices in X that can be matched with those in Y is $|X| - \delta(G)$.

Exercises

3, 5, 7, 13, 14

Supplementary 2, 7, 8 (Birkhoff-von Neumann Theorem)