Vehicle Propulsion Systems
Lecture 2
Fuel Consumption Estimation & ICE

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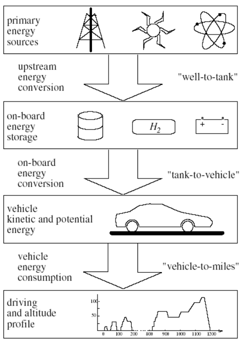
Outline

Energy System Overview



Primary sources

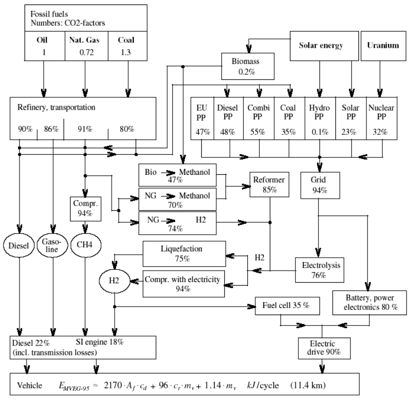
Different options for on-board energy storage

Powertrain energy conversion during driving

Cut at the wheel!

Driving mission has a minimum energy requirement.

W2M – Energy Paths



Outline

Energy Consumption of a Driving Mission

- Remember the partitioning
  –Cut at the wheels.
- How large **force** is required at the wheels for driving the vehicle on a mission?

Repetition – Work, power and Newton's law

Translational system – Force, work and power:

$$W = \int F\,dx, \qquad P = \frac{d}{dt}W = F\,v$$

Rotating system – Torque ($T = F\,r$), work and power:

$$W = \int T\,d\theta, \qquad P = T\,\omega$$

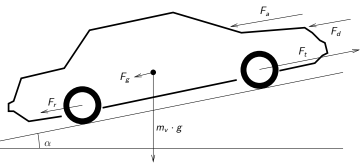
Newton's second law:

Translational	Rotational
$m\,\frac{dv}{dt} = F_{driv} - F_{load}$	$J\,\frac{d\omega}{dt} = T_{driv} - T_{load}$

The Vehicle Motion Equation

Newton's second law for a vehicle

$$m_v \frac{d}{dt} v(t) = F_t(t) - (F_a(t) + F_r(t) + F_g(t) + F_d(t))$$



- F_t – tractive force
- F_a – aerodynamic drag force
- F_r – rolling resistance force
- F_g – gravitational force
- F_d – disturbance force

Aerodynamic Drag Force – Loss

Aerodynamic drag force depends on:
Frontal area A_f, drag coefficient c_d, air density ρ_a and vehicle velocity $v(t)$

$$F_a(t) = \frac{1}{2} \cdot \rho_a \cdot A_f \cdot c_d \cdot v(t)^2$$

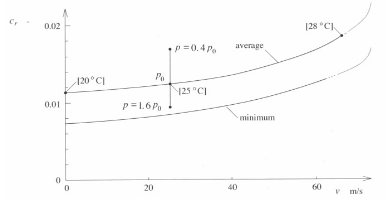
Approximate contributions to F_a

- 65% car body.
- 20% wheel housings.
- 10% exterior mirrors, eave gutters, window housings, antennas, etc.
- 5% engine ventilation.

Rolling Resistance Losses

Rolling resistance depends on:
load and tire/road conditions

$$F_r(v, p_t, \text{surface}, \ldots) = c_r(v, p_t, \ldots) \cdot m_v \cdot g \cdot \cos(\alpha), \qquad v > 0$$

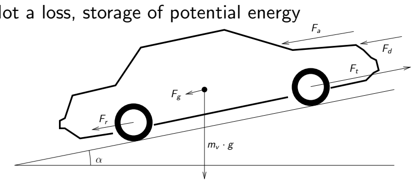


The velocity has small influence at low speeds.
Increases for high speeds where resonance phenomena start.
Assumption in book: c_r – constant

$$F_r = c_r \cdot m_v \cdot g$$

Gravitational Force

- Gravitational load force
  –Not a loss, storage of potential energy

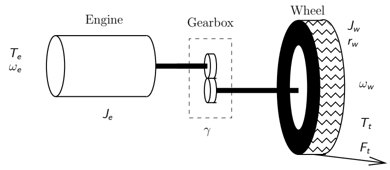


- Up- and down-hill driving produces forces.

$$F_g = m_v\, g\, \sin(\alpha)$$

- Flat road assumed $\alpha = 0$ if nothing else is stated (In the book).

Inertial forces – Reducing the Tractive Force



$$T_e - J_e \frac{d}{dt}\omega_e = T_{gb} \qquad T_{gb} \cdot \gamma - J_w \frac{d}{dt}\omega_w = T_t$$

Variable substitution: $T_w = \gamma\, T_e, \quad \omega_w\, \gamma = \omega_e, \quad v = \omega_w\, r_w$

Tractive force:

$$F_t = \frac{1}{r_w}\left[(T_e - J_e \frac{d}{dt}\frac{v(t)}{r_w}\gamma) \cdot \gamma - J_w \frac{d}{dt}\frac{v(t)}{r_w}\right] = \frac{\gamma}{r_w} T_e - \left(\frac{\gamma^2}{r_w^2} J_e + \frac{1}{r_w^2} J_w\right) \frac{d}{dt} v(t)$$

The Vehicle Motion Equation:

$$\left[m_v + \frac{\gamma^2}{r_w^2} J_e + \frac{1}{r_w^2} J_w\right] \frac{d}{dt} v(t) = \frac{\gamma}{r_w} T_e - (F_a(t) + F_r(t) + F_g(t) + F_d(t))$$

Vehicle Operating Modes

The Vehicle Motion Equation:

$$m_v \frac{d}{dt} v(t) = F_t(t) - (F_a(t) + F_r(t) + F_g(t) + F_d(t))$$

- $F_t > 0$ traction
- $F_t < 0$ braking
- $F_t = 0$ coasting

$$\frac{d}{dt} v(t) = -\frac{1}{2\, m_v} \rho_a\, A_f\, c_d\, v^2(t) - g\, c_r = -\alpha^2\, v^2(t) - \beta^2$$

Coasting solution for $v > 0$

$$v(t) = \frac{\beta}{\alpha} \tan\left(\arctan\left(\frac{\alpha}{\beta}\, v(0)\right) - \alpha\, \beta\, t\right)$$

How to check a profile for traction?

The Vehicle Motion Equation:

$$m_v \frac{d}{dt} v(t) = F_t(t) - (F_a(t) + F_r(t) + F_g(t) + F_d(t)) \qquad (1)$$

- Traction conditions:
  $F_t > 0$ traction, $F_t < 0$ braking, $F_t = 0$ coasting
- Method 1: Compare the profile with the coasting solution over a time step

$$v_{coast}(t_{i+1}) = \frac{\beta}{\alpha} \tan\left(\arctan\left(\frac{\alpha}{\beta}\, v(t_i)\right) - \alpha\, \beta\, (t_{i+1} - t_i)\right)$$

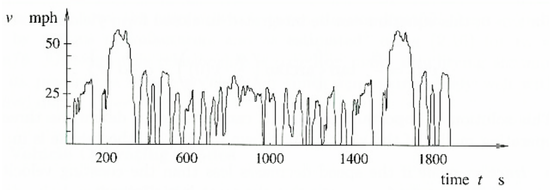
- Method 2: Solve (1) for F_t

$$F_t(t) = m_v \frac{d}{dt} v(t) + (F_a(t) + F_r(t) + F_g(t) + F_d(t))$$

Numerically differentiate the profile $v(t)$ to get $\frac{d}{dt} v(t)$.
Compare with Traction condition.

Driving profiles

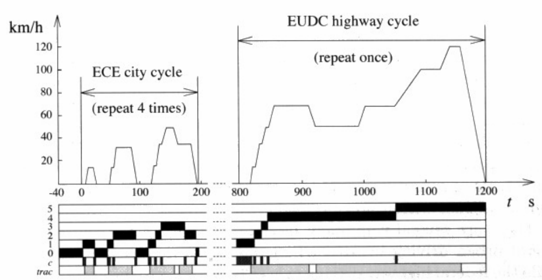
Velocity profile, American FTP-75 (1.5*FUDS).



Driving profiles in general

- First used for pollutant control now also for fuel consumption.
- Important that all use the same cycle when comparing.
- Different cycles have different energy demands.

Driving profiles – Another example



Velocity profile, European MVEG-95 (ECE*4, EUDC)

No coasting in this driving profile.

Mechanical Energy Demand of a Cycle

Only the demand from the cycle

- The mean tractive force during a cycle

$$\bar{F}_{trac} = \frac{1}{x_{tot}} \int_0^{x_{tot}} \max(F(x), 0)\, dx = \frac{1}{x_{tot}} \int_{t \in trac} F(t) v(t) dt$$

  where $x_{tot} = \int_0^{t_{max}} v(t) dt$.
- Note $t \in trac$ in definition.
- Only traction.
- Idling not a demand from the cycle.

Evaluating the integral

Discretized velocity profile used to evaluate

$$\bar{F}_{trac} = \frac{1}{x_{tot}} \int_{t \in trac} F(t) v(t) dt$$

here $v_i = v(t_i)$, $t_i = i \cdot h$, $i = 1, \ldots, n$.
Approximating the quantites

$$\bar{v}_i(t) \approx \frac{v_i + v_{i-1}}{2}, \qquad t \in [t_{i-1}, t_i)$$

$$\bar{a}_i(t) \approx \frac{v_i - v_{i-1}}{h}, \qquad t \in [t_{i-1}, t_i)$$

Traction approximation

$$\bar{F}_{trac} \approx \frac{1}{x_{tot}} \sum_{i \in trac} \bar{F}_{trac,i}\, \bar{v}_i\, h$$

Evaluating the integral

Tractive force from *The Vehicle Motion Equation*

$$F_{trac} = \frac{1}{2} \rho_a A_f c_d v^2(t) + m_v g c_r + m_v a(t)$$

$$\bar{F}_{trac} = \bar{F}_{trac,a} + \bar{F}_{trac,r} + \bar{F}_{trac,m}$$

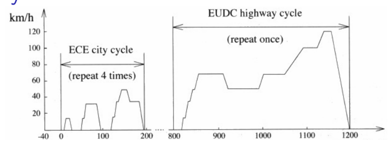
Resulting in these sums

$$\bar{F}_{trac,a} = \frac{1}{x_{tot}} \frac{1}{2} \rho_a A_f c_d \sum_{i \in trac} \bar{v}_i^3\, h$$

$$\bar{F}_{trac,r} = \frac{1}{x_{tot}} m_v g c_r \sum_{i \in trac} \bar{v}_i\, h$$

$$\bar{F}_{trac,m} = \frac{1}{x_{tot}} m_v \sum_{i \in trac} \bar{a}_i\, \bar{v}_i\, h$$

Values for cycles



Numerical values for the cycles: {MVEG-95, ECE, EUDC}

$$\bar{X}_{trac,a} = \frac{1}{x_{tot}} \sum_{i \in trac} \bar{v}_i^3\, h = \{319, 82.9, 455\}$$

$$\bar{X}_{trac,r} = \frac{1}{x_{tot}} \sum_{i \in trac} \bar{v}_i\, h = \{0.856, 0.81, 0.88\}$$

$$\bar{X}_{trac,m} = \frac{1}{x_{tot}} \sum_{i \in trac} \bar{a}_i\, \bar{v}_i\, h = \{0.101, 0.126, 0.086\}$$

$$\bar{E}_{\text{MVEG-95}} \approx A_f c_d\, 1.9 \cdot 10^4 + m_v c_r\, 8.4 \cdot 10^2 + m_v\, 10 \qquad kJ/100km$$

Tasks in Hand-in assignment

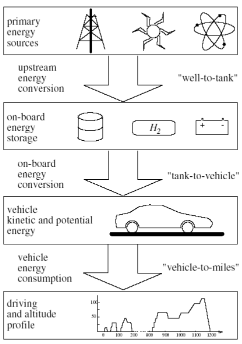
Approximate car data

$$\bar{E}_{\text{MVEG-95}} \approx A_f c_d\, 1.9 \cdot 10^4 + m_v c_r\, 8.4 \cdot 10^2 + m_v\, 10 \qquad kJ/100km$$

	SUV	full-size	compact	light-weight	PAC-Car II
$A_f \cdot c_d$	1.2 m^2	0.7 m^2	0.6 m^2	0.4 m^2	.25 · .07 m^2
c_r	0.017	0.017	0.017	0.017	0.0008
m_v	2000 kg	1500 kg	1000 kg	750 kg	39 kg
$\bar{P}_{\text{MVEG-95}}$	11.3 kW	7.1 kW	5.0 kW	3.2 kW	
$\bar{P}_{max}$	155 kW	115 kW	77 kW	57 kW	

Average and maximum power requirement for the cycle.

Energy System Overview



Primary sources

Different options for on-board energy storage

Powertrain energy conversion during driving

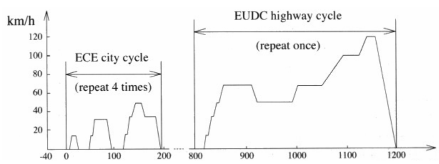
Cut at the wheel!

Driving mission has a minimum energy requirement.

Outline

Energy demand again – Recuperation

- Previously: Considered energy demand from the cycle.
- Now: The cycle can give energy to the vehicle.



Recover the vehicle's kinetic energy during driving.

Perfect recuperation

- Mean required force

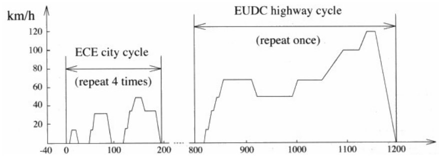
$$\bar{F} = \bar{F}_a + \bar{F}_r$$

- Sum over all points

$$\bar{F}_a = \frac{1}{x_{tot}} \frac{1}{2} \rho_a A_f c_d \sum_{i=1}^{N} \bar{v}_i^3 h$$

$$\bar{F}_r = \frac{1}{x_{tot}} m_v g c_r \sum_{i=1}^{N} \bar{v}_i h$$

Perfect recuperation – Numerical values for cycles



Numerical values for MVEG-95, ECE, EUDC

$$\bar{X}_a = \frac{1}{x_{tot}} \sum_i \bar{v}_i^3 h = \{363, 100, 515\}$$

$$\bar{X}_r = \frac{1}{x_{tot}} \sum_i \bar{v}_i h = \{1, 1, 1\}$$

$$\bar{E}_{\text{MVEG-95}} \approx A_f c_d \, 2.2 \cdot 10^4 + m_v c_r \, 9.81 \cdot 10^2 \qquad kJ/100km$$

Comparison of numerical values for cycles

- Without recuperation.

$$\bar{X}_{trac,a} = \frac{1}{x_{tot}} \sum_{i \in trac} \bar{v}_i^3 h = \{319, 82.9, 455\}$$

$$\bar{X}_{trac,r} = \frac{1}{x_{tot}} \sum_{i \in trac} \bar{v}_i h = \{0.856, 0.81, 0.88\}$$

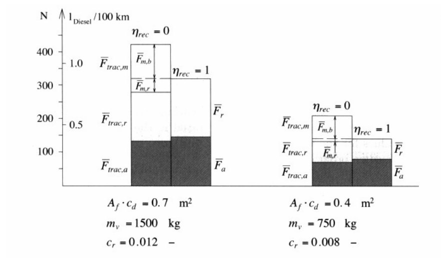
$$\bar{X}_{trac,m} = \frac{1}{x_{tot}} \sum_{i \in trac} \bar{a}_i \bar{v}_i h = \{0.101, 0.126, 0.086\}$$

- With perfect recuperation

$$\bar{X}_a = \frac{1}{x_{tot}} \sum_i \bar{v}_i^3 h = \{363, 100, 515\}$$

$$\bar{X}_r = \frac{1}{x_{tot}} \sum_i \bar{v}_i h = \{1, 1, 1\}$$

Perfect and no recuperation



Mean force represented as liter Diesel / 100 km.

Sensitivity Analysis

- Cycle energy reqirement (no recuperation)

$$\bar{E}_{\text{MVEG-95}} \approx A_f c_d \, 1.9 \cdot 10^4 + m_v c_r \, 8.4 \cdot 10^2 + m_v \, 10 \qquad kJ/100km$$

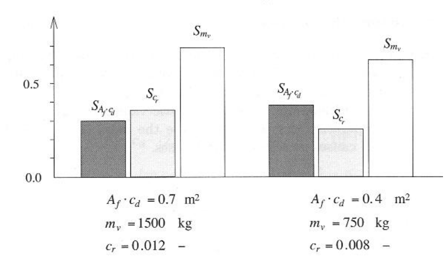
- Sensitivity analysis

$$S_p = \lim_{\delta p \to 0} \frac{\left[\bar{E}_{\text{MVEG-95}}(p + \delta p) - \bar{E}_{\text{MVEG-95}}(p)\right] / \bar{E}_{\text{MVEG-95}}(p)}{\delta p / p}$$

$$S_p = \lim_{\delta p \to 0} \frac{\left[\bar{E}_{\text{MVEG-95}}(p + \delta p) - \bar{E}_{\text{MVEG-95}}(p)\right]}{\delta p} \frac{p}{\bar{E}_{\text{MVEG-95}}(p)}$$

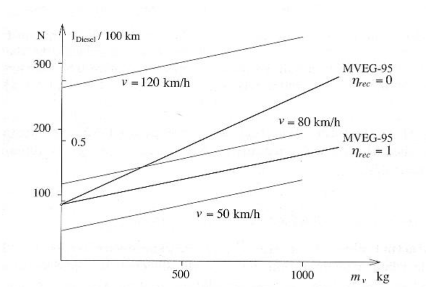
- Vehicle parameters:
  - $A_f c_d$
  - c_r
  - m_v

Sensitivity Analysis

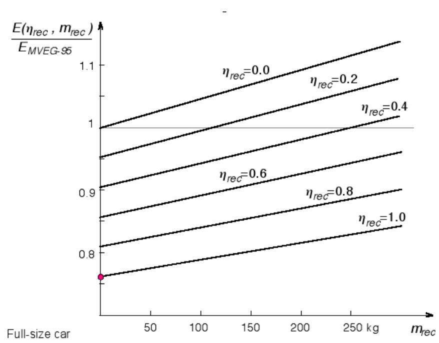


Vehicle mass is the most important parameter.

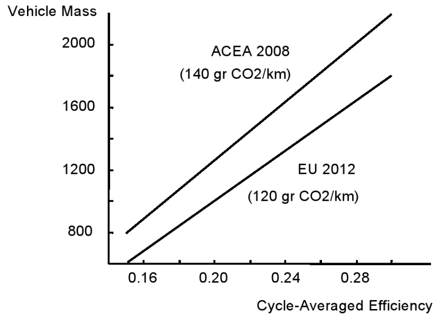
Vehicle mass and fuel consumption



Realistic Recuperation Devices



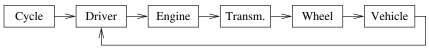
Vehicle Mass and Cycle-Avearged Efficiency



Outline

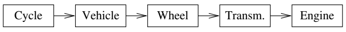
Two Approaches for Powertrain Simulation

- Dynamic simulation (forward simulation)



–"Normal" system modeling direction
–Requires driver model

- Quasistatic simulation (inverse simulation)



–"Reverse" system modeling direction
–Follows driving cycle exactly

- Model causality

Dynamic approach

- Drivers input u propagates to the vehicle and the cycle
- Drivers input ⇒ . . . ⇒ Driving force ⇒ Losses ⇒ Vehicle velocity ⇒ Feedback to driver model
- Available tools (= Standard simulation) can deal with arbitrary powertrain complexity.

Quasistatic approach

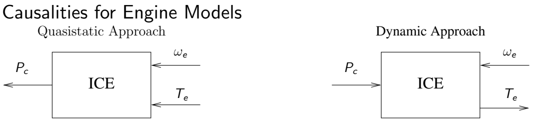
- Backward simulation
- Driving cycle ⇒ Losses ⇒ Driving force ⇒ Wheel torque ⇒ Engine (powertrain) torque ⇒ . . . ⇒ Fuel consumtion.
- Available tools are limited with respect to the powertrain components that they can handle. Considering new tools such as Modelica opens up new possibilities.
- See also: *Efficient Drive Cycle Simulation*, Anders Fröberg and Lars Nielsen (2008) . . .

Outline

Causality and Basic Equations

High level modeling – Inputs and outputs

- Causalities for Engine Models



- Engine efficiency

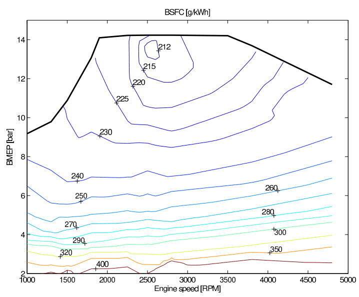
$$\eta_e = \frac{\omega_e T_e}{P_c}$$

- Enthalpy flow of fuel (Power $\dot{H}_{fuel} = P_c$)

$$P_c = \dot{m}_f \, q_{LHV}$$

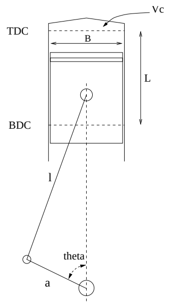
Engine Efficiency Maps

Measured engine efficiency map – Used very often



–What to do when map-data isn't available?

Engine Geometry Definitions



Cylinder, Piston, Connecting rod, Crank shaft

- Bore, B
- Stroke, $S = 2\,a$
- Number of cylinders z
- Cylinder swept volume, $V_d = \frac{\pi B^2 S}{4}$
- Engine swept volume, $V_d = z\,\frac{\pi B^2 S}{4}$
- Compression ratio $r_c = \frac{V_{max}}{V_{min}} = \frac{V_d + V_c}{V_c}$

Definition of MEP

See whiteboard.

Normalized Engine Variables

- Mean Piston Speed ($S_p = mps = c_m$):

$$c_m = \frac{\omega_e\, S}{\pi}$$

- Mean Effective Pressure (MEP=p_{me} ($N = n_r \cdot 2$)):

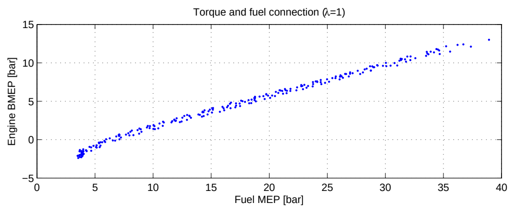
$$p_{me} = \frac{N\,\pi\, T_e}{V_d}$$

- Used to:
  - Compare performance for engines of different size
  - Design rules for engine sizing.
    At max engine power: $c_m \approx 17$ m/s, $p_{me} \approx 1\text{e}6$ Pa (no turbo)
    $\Rightarrow$ engine size
  - Connection:

$$P_e = z\,\frac{\pi}{16}\,B^2\,p_{me}\,c_m$$

Torque modeling through – Willans Line

- Measurement data: x: p_{mf} y: $p_{me} = BMEP$

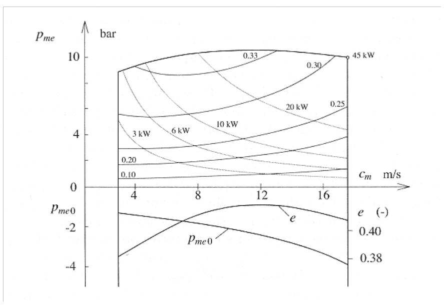


- Linear (affine) relationship – Willans line

$$p_{me} = e(\omega_e) \cdot p_{mf} - p_{me,0}(\omega_e)$$

- Engine efficiency: $\eta_e = \frac{p_{me}}{p_{mf}}$

Engine Efficiency – Map Representation



Willans line parameters: $e(\omega_e)$ $p_{me,0}(\omega_e)$