A sequential test selection algorithm for fault isolation

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Abstract: A sequential test selection algorithm is proposed which updates the set of active test quantities depending on the present minimal candidates. By sequentially updating the set of active test quantities, computational time and memory usage can be reduced. If test quantities are generated on-line, a sequential test selection algorithm gives information about which test quantities that should be created. The test selection problem is defined as an optimization problem where a set of active test quantities is chosen such that the cost is minimized while the set fulfills a required minimum detectability and isolability performance. A quantitative diagnosability measure, distinguishability, is used to quantify diagnosability performance of test quantities. The proposed test selection algorithm is applied to a DC-circuit where the diagnosis algorithm generates residuals on-line. Experiments show that the sequential test selection algorithm can significantly reduce the number of active test quantities during a scenario and still be able to identify the true faults.

Keywords: Fault detection and isolation, Sequential test selection, Quantitative diagnosability analysis, Model-based diagnosis.

1. INTRODUCTION

A diagnosis algorithm uses test quantities to detect and isolate faults present in the system. An overview of a diagnosis algorithm is shown in Fig. 1. If different test quantities are sensitive to different sets of faults a fault isolation algorithm can be used to isolate and identify which faults that are present given the alarmed test quantities, see de Kleer and Williams (1987). Often, different sets of faults, *candidates*, are consistent with the set of alarmed tests. The goal is to have a set of tests which can identify the present faults.

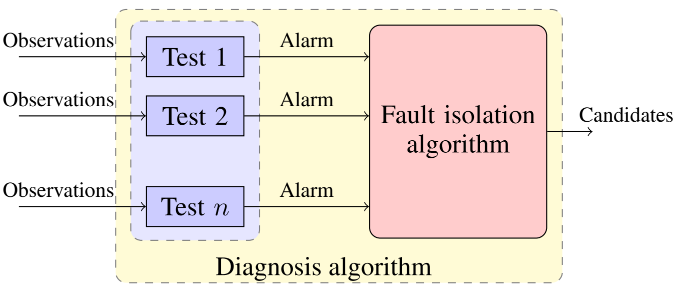

Fig. 1. A diagnosis algorithm consists of a set of test quantities and a fault isolation algorithm to compute (minimal) candidates.

Since different test quantities are good at detecting and isolating different sets of faults, not all available test quantities are necessary to be active during a scenario in the diagnosis algorithm. By choosing different sets of test quantities depending on the present candidates, the total number of active test quantities can be reduced while still having sufficient diagnosability performance.

A sequential test selection algorithm becomes interesting, for example, if there exists a large number of available test quantities or if new test quantities should be automatically selected and generated on-line. To use all available test quantities during a scenario could be too computationally expensive. Therefore, a smart test selection algorithm can be used to reduce the number of active test quantities while maintaining a satisfactory diagnosability performance. As an example, a system considered here is a re-configurable DC-circuit where a number of relays are used to distribute power from three batteries to two load banks. A number of sensors are used to measure voltages and currents in the circuit. The DC-circuit is a part of the diagnosis test bed ADAPT which is described in Poll et al. (2007), see Fig. 2. The number of test quantities that can be generated based on a model of the DC-circuit is large because of high redundancy in the system.

A quantitative diagnosability measure called distinguishability which is introduced in Eriksson et al. (2011b) and Eriksson et al. (2011a) is used when implementing the diagnosis algorithm to automatically generate residual generators with maximum fault to noise ratio. Distinguishability will be used to quantify detectability and isolability performance of the residual generators in order to choose the residual generators which fulfills a required quantitative diagnosability performance.

There are other works where sequential diagnosis are used to improve fault isolation, for example de Kleer and Williams (1987). In Krysander et al. (2010), a sequential test selection algorithm, FlexDX, updates the active set of test quantities during run-time to isolate the faults. In Svärd (2012), a greedy test selection algorithm is used off-line to find a set of test quantities which fulfills a required single fault isolability performance. In these previous works, only deterministic fault detectability and isolability performance are considered. In this work, a measure for quantitative diagnosability performance is used to quantify the performance of a test quantity when choosing the active set of test quantities.

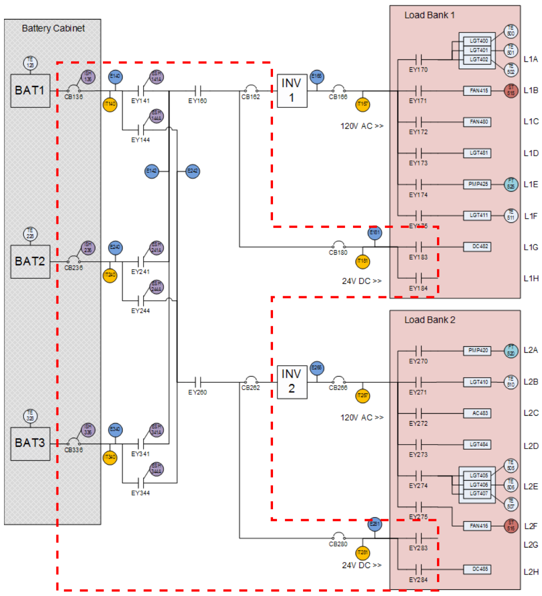

Fig. 2. Schematics of the diagnosis test bed ADAPT. The dark area to the left represents the three batteries and the two dark areas to the right are two load banks. Sensors in the circuit are marked as circles. The DC-circuit considered in this work is inside the dotted line.

First, the problem formulation is presented in Section 2. Then some previous results regarding distinguishability are presented in Section 3 and a generalization of distinguishability for multiple faults is presented in Section 4. A sequential test selection algorithm is presented in Section 5. Some experimental results when applying the test selection algorithm in a diagnosis algorithm for a DC-circuit are shown in Section 6. Then some tuning aspects of the test selection algorithm are discussed and compared to other works in Section 7 and finally some conclusions are presented in Section 8

2. PROBLEM FORMULATION

The purpose here is to develop a sequential test selection algorithm to automatically compute which available test quantities that should be active in the diagnosis algorithm during run-time depending on the present candidates. A candidate is a hypothesis about the system state given the alarmed test quantities, de Kleer and Williams (1987).

A candidate represents a set of faults $d = \{f_1, f_2, \dots, f_l\}$, that is consistent with the alarmed test quantities. If d is a candidate where no subset $d' \subset d$ is a candidate then d is called a minimal candidate.

Let $\mathcal{T} = \{T_1, T_2, \dots, T_n\}$ be the set of all available test quantities T. Each test quantity T is sensitive to a set of faults f_i. The cost to use a test quantity T_i is defined as $c(T_i)$. The cost could, for example, be the computational cost of generating and using T_i. Let D be a set of minimal candidates d_i, i.e., $D = \{d_1, d_2, \dots, d_k\}$. We want to find a cheapest set of tests $\mathcal{T}^* \subseteq \mathcal{T}$ where each minimal candidate $d \in D$ can be rejected by at least one test quantity $T \in \mathcal{T}^*$. The test selection problem can be formulated as a binary integer programming (BIP) problem,

$$\begin{aligned} \min \sum_{T \in \mathcal{T}} & c(T) x_T \\ s.t. \sum_{T \in \mathcal{E}_{d_i,d_j}} & x_T \geq 1, \forall \mathcal{E}_{d_i,d_j} : d_j \in D \\ & x_T = \{0,1\}, \forall T \end{aligned} \tag{1}$$

where $\mathcal{E}_{d_i,d_j} \subseteq \mathcal{T}$ contains a set of tests which can reject d_j for a new minimal candidate $d_i = \{f_a\} \cup d_j$ where $f_a \notin d_j$ and x_T is a binary variable determining whether a test quantity T should be active or not. The solution $\mathcal{T}^*$ contains all tests $T \in \mathcal{T}$ where $x_T = 1$.

To assure that each test $T \in \mathcal{E}_{d_i,d_j}$ have satisfactory performance, distinguishability is used to quantify the diagnosability performance for each test T. Each set $\mathcal{E}_{d_i,d_j}$ can be chosen such that, for example, only test quantities which have a sufficiently high distinguishability when rejecting d_j for d_i are included.

3. BACKGROUND THEORY

Here, some useful results from Eriksson et al. (2011b) and Eriksson et al. (2011a) are recalled. A definition of distinguishability and some useful properties are presented here which will be used for quantifying detectability and isolability performance. There are also some results of how to generate residual generators with maximum fault to noise ratio, FNR. For more details and proofs, the reader is referred to previous mentioned papers.

3.1 *Distinguishability*

The models considered here are discrete-time linear models written in the form

$$Lz = Hx + Ff + Ne \tag{2}$$

where z are known signals, x are unknown signals, f are fault signals and e is Gaussian distributed with known co-variance matrix Λ. Discrete-time descriptor models when observed for a given time interval n can be written as (2), see Eriksson et al. (2011a). Thus, model dynamics and fault time profiles $\theta = (\theta(t-n+1), \theta(t-n+2), \dots, \theta(t))^T$, describing how a fault changes over the time interval can be considered. It is assumed that the model (2) fulfills

$$(H \;\; N) \text{ is full row-rank.} \tag{3}$$

The model (2) is written in an input-output form, see Polderman and Willems (1998), by multiplying with $\mathcal{N}_H$ from the left, where the rows of $\mathcal{N}_H$ is an orthonormal basis for the left null-space of H, which gives

$$\mathcal{N}_H Lz = \mathcal{N}_H Ff + \mathcal{N}_H Ne. \tag{4}$$

If the linear model (2) fulfills assumption (3) the co-variance matrix of the vector $\mathcal{N}_H Ne$ will be non-singular.

To be able to quantify diagnosability performance, a stochastic representation of different fault modes is required. Let $\tau = \mathcal{N}_H Lz$, then let $p(\tau, \mu)$ denote a multivariate probability density function where $\mu(\theta) = \mathcal{N}_H F_i \theta$ is the mean of τ where F_i is the ith row of F corresponding to fault mode f_i.

Let Θ_i denote the set of all fault time profiles θ of fault $f_i = \theta$ which could be explained by fault mode f_i. Each fault mode can thus be described by a set of pdf's $p(\tau, \mu)$ given the following definition.

Definition 1. Let $\mathcal{Z}_{f_i}$ denote the set of all pdf's, $p(\tau, \mu)$, for all fault time profiles $\theta \in \Theta_i$, describing τ which could be explained by the fault mode f_i, i.e.,

$$\mathcal{Z}_{f_i} = \{p(\tau, \mathcal{N}_H F_i\theta)|\forall\theta \in \Theta_i\}. \quad (5)$$

■

Definition 1 is a stochastic counterpart to observation sets in the deterministic case, see Frisk et al. (2009). A specific pdf given that $f_i = \theta$ is denoted $p^i_\theta = p(\tau, \mathcal{N}_H F_i\theta)$ and $p_{\text{NF}} = p(\tau, \bar{0})$ corresponds to the no fault case.

Now, the measure for quantitative diagnosability performance can be defined by using the Kullback-Leibler divergence, $K(p\|q) = \int_{-\infty}^{\infty} p(\nu)\log\frac{p(\nu)}{q(\nu)}d\nu$, see Kullback and Leibler (1951), as follows.

Definition 2. (Distinguishability). Given a sliding window model (2), distinguishability $\mathcal{D}_{i,j}(\theta)$ of a fault f_i with a given fault time profile θ from a fault mode f_j is defined as

$$\mathcal{D}_{i,j}(\theta) = \min_{p^j \in \mathcal{Z}_{f_j}} K\left(p^i_\theta \| p^j\right) \quad (6)$$

where the set $\mathcal{Z}_{f_j}$ is defined in Definition 1. ■

The definition of distinguishability fulfills the following two propositions.

Proposition 1. Given a window model (2), a fault $f_i = \theta \in \Theta_i$ is isolable from a fault mode f_j if and only if

$$\mathcal{D}_{i,j}(\theta) > 0 \quad (7)$$

■

Proposition 2. If $\bar{0}$ is a boundary point of Θ_j, where $\Theta_j = \mathbb{R}^n \setminus \{\bar{0}\}$, for a fault mode f_j then

$$\mathcal{D}_{i,j}(\theta) \le \mathcal{D}_{i,\text{NF}}(\theta). \quad (8)$$

■

It is assumed, without loss of generality, that

$$\text{cov}\left(\mathcal{N}_H N e\right) = I \quad (9)$$

where I is the identity matrix, see Eriksson et al. (2011b). Since the noise in (2) is Gaussian distributed, distinguishability can be computed explicitly given the following theorem.

Theorem 3. Distinguishability for a sliding window model (2) with Gaussian distributed stochastic vector e, under assumption (9), is given by

$$\mathcal{D}_{i,j}(\theta) = \frac{1}{2}\|\mathcal{N}_{(H\ F_j)}F_i\theta\|^2 \quad (10)$$

where the rows of $\mathcal{N}_{(H\ F_j)}$ is an orthonormal basis for the left null space of (H, F_j). ■

3.2 Relation of residual generators

To design a residual generator r isolating faults from fault mode f_j, multiply (2) from the left with $\gamma\mathcal{N}_{(H\ F_j)}$ where γ is a row-vector to obtain

$$r = \gamma\mathcal{N}_{(H\ F_j)}Lz = \gamma\mathcal{N}_{(H\ F_j)}Ff + \gamma\mathcal{N}_{(H\ F_j)}Ne. \quad (11)$$

The residual generator (11) is a scalar model in the same form as (2). Therefore, distinguishability can be computed for the residual generator, which is denoted $\mathcal{D}^\gamma_{i,j}(\theta)$, where the superscript γ is used to distinguish from when computing distinguishability for the model. The connection between distinguishability and FNR of r is given by the following theorem, which also gives an alternative way of computing distinguishability for a scalar model.

Theorem 4. A residual generator (11), for a model (2) where e is Gaussian distributed under assumption (3), is also Gaussian distributed $\mathcal{N}(\lambda(\theta), \sigma^2)$ and

$$\mathcal{D}^\gamma_{i,j}(\theta) = \frac{1}{2}\left(\frac{\lambda(\theta)}{\sigma}\right)^2$$

where θ is the fault time profile of a fault f_i, and $\lambda(\theta)/\sigma$ is the fault to noise ratio with respect to fault f_i in (11).

An important connection between $\mathcal{D}^\gamma_{i,j}(\theta)$ and $\mathcal{D}_{i,j}(\theta)$ is given by the inequality described in the following theorem.

Theorem 5. For a model (2) under assumption (9), an upper bound for $\mathcal{D}^\gamma_{i,j}(\theta)$ in (11) is given by

$$\mathcal{D}^\gamma_{i,j}(\theta) \le \mathcal{D}_{i,j}(\theta)$$

with equality if and only if γ and $\mathcal{N}_{(H\ F_j)}F_i\theta$ are parallel. ■

Theorem 5 shows that $\mathcal{D}_{i,j}(\theta)$ gives an upper limit of the FNR which can be achieved by any residual generator (11). Note that if a fault $f_i = \theta$ is isolable from a fault mode f_j, then the theorem shows how to design a residual generator with maximum FNR by choosing $\gamma = (\mathcal{N}_{(H\ F_j)}F_i\theta)^T$. This will be used when automatically generating new residual generators by the sequential test selection algorithm described in the following section.

4. GENERALIZATION OF DISTINGUISHABILITY

The definition of distinguishability in Section 3 only considers fault modes containing single faults. Here, it is also interesting to quantify how easy it is to isolate a fault f_a from several faults d_j. This corresponds to quantifying how easy it is to reject a minimal candidate d_j for a new minimal candidate $d_i = \{f_a\} \cup d_j$. In order to handle fault modes with multiple faults a generalization of the previous definition of distinguishability is presented here. Most results presented here can be derived in a similar way as for the single fault case in Section 3.

First, a generalization of the sets $\mathcal{Z}_{f_j}$ in Definition 1 is defined by considering combinations of different fault time profiles for each fault in the fault mode. Consider all faults present given a fault mode d_j and let $\bar{\theta} \in \Theta_{d_j}$ denote a matrix $\bar{\theta} = (\theta_1, \theta_2, \ldots, \theta_k)$ where each column corresponds to a fault time profile $f = \theta$ for each fault $f \in d_j$. Then, a generalization of Definition 1 can be made as follows.

Definition 3. Let $\mathcal{Z}_{d_j}$ denote the set of all pdf's, $p(\tau, \mu)$, for all combination of fault time profiles $\bar{\theta} \in \Theta_{d_j}$, describing τ which could be explained by the fault mode d_j,

$$\mathcal{Z}_{d_i} = \left\{p(\tau, \mathcal{N}_H F_{d_j}\bar{\theta})|\forall\bar{\theta} \in \Theta_{d_j}\right\} \quad (12)$$

where F_{d_j} contains the rows of F corresponding to the faults $f \in d_j$. ■

Recall that $p^a_\theta = p(\tau, \mathcal{N}_H F_a\theta)$. Since isolating a fault f_a from all faults d_j corresponds to rejecting d_j for $d_i = \{f_a\} \cup d_j$, a generalization of Definition 2 can be formulated as follows.

Definition 4. (Distinguishability of multiple faults). Given a sliding window model (2), distinguishability $\mathcal{D}_{d_i,d_j}(\theta)$ of a fault $f_a \notin d_j$ where $d_i = \{f_a\} \cup d_j$ with a fault time profile θ from a fault mode d_j with multiple faults is defined as

$$\mathcal{D}_{d_i,d_j}(\theta) = \min_{p^j \in \mathcal{Z}_{d_j}} K\left(p^a_\theta \| p^j\right) \quad (13)$$

where the set $\mathcal{Z}_{d_j}$ is defined in Definition 3. ■

In the Gaussian case it follows from Theorem 3 that (13) can be computed explicitly using

$$\mathcal{D}_{d_i,d_j}(\theta) = \frac{1}{2}\|\mathcal{N}_{(H\ F_{d_j})}F_a\theta\|^2 \tag{14}$$

where F_{d_j} are the rows of F corresponding to the faults in d_j.

Proposition 1 holds given the new definition of distinguishability and a generalization of Proposition 2 can be formulated as follows.

Proposition 6. Let $d_k \subset d_j$ and $f_a \notin d_k, d_j$. Then let $d_l = \{f_a\} \cup d_k$ and $d_i = \{f_a\} \cup d_j$, then the following inequality holds.

$$\mathcal{D}_{d_i,d_j}(\theta) \leq \mathcal{D}_{d_l,d_k}(\theta). \tag{15}$$

■

The proposition states that it performance always decreases when isolating a specific fault from an increasing set of faults. If we want to generate a residual generator which is sensitive to few faults then a residual generator with maximum FNR can never have higher FNR than if we would create an residual with maximum FNR which is sensitive to a super-set of faults.

To design a residual generator which is sensitive to a fault f_a but no fault $f \in d_j$ multiply (2) from the left with $\gamma\mathcal{N}_{(H\ F_{d_j})}$ to obtain

$$\gamma\mathcal{N}_{(H\ F_{d_j})}Lz = \gamma\mathcal{N}_{(H\ F_{d_j})}Ff + \gamma\mathcal{N}_{(H\ F_{d_j})}Ne \tag{16}$$

Theorem 4 and Theorem 5 can be generalized to the case of multiple faults. As follows from Theorem 5, a residual generator with maximum FNR isolating a fault $f_a = \theta$ from the faults d_j is designed by choosing $\gamma = (\mathcal{N}_{(H\ F_{d_j})}F_a\theta)^T$.

5. SEQUENTIAL TEST SELECTION

Here, an algorithm for sequential test selection is presented. The algorithm updates the set of active test quantities in the diagnosis algorithm given the present minimal candidates. The algorithm applies the results in Section 4 to generate residual generators and to quantify their diagnosability performance. Finally, the sequential test selection algorithm is evaluated using an academic example.

5.1 Principles

A diagnosis algorithm computes minimal candidates consistent with the alarmed test quantities, see Fig. 1. New test quantities are selected which are able to reject any present minimal candidates and fulfills a required minimum distinguishabiliy. Each time one or more test quantities alarms, D is updated and a new set of test quantities is selected.

The fault isolation algorithm used here is described in de Kleer and Williams (1987) and updates the set of minimal candidates each time new test quantities alarms. A rejected minimal candidate d_j is replaced by new minimal candidates d_i such that $d_i = \{f_a\} \cup d_j$ where $f_a \notin d_j$. If any of the new minimal candidates is a super-set of any other minimal candidate then the new candidate is removed. Note that it is assumed here, that the available test quantities are designed such that the risk of false alarms is minimized.

To be able to reject a minimal candidate d_j for another candidate $d_i = \{f_a\} \cup d_j$ requires a test quantity which is sensitive to f_a but not any $f \in d_j$, see de Kleer and Williams (1987).

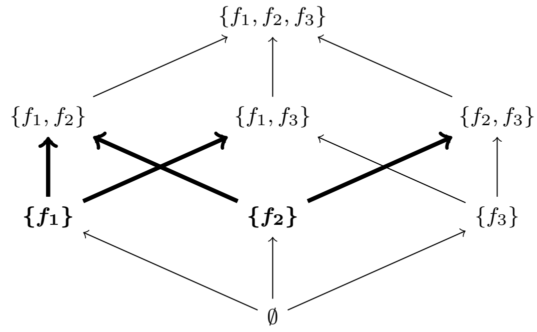

Fig. 3. A lattice set representation of all possible candidates for a system with three possible faults where $\emptyset$ corresponds to the fault-free case. If $D = \{\{f_1\}, \{f_2\}\}$ then tests, able to reject $\{f_1\}$ for $\{f_1, f_2\}$ and $\{f_1, f_3\}$ and able to reject $\{f_2\}$ for $\{f_1, f_2\}$ and $\{f_2, f_3\}$, are needed to improve D.

The following example shows how a lattice set representation of all possible candidates can be used to visualize what kind of isolablity performance that is required by the active set of test quantities.

Example 1. Consider a system with three possible faults f_1,f_2, and f_3. All possible candidates can be described using a lattice set, see Fig. 3. Each node is a candidate where all nodes at each level have the same cardinality. Each node have arrows going to all nodes at the next level which are super-sets of the first node. For example from node $\{f_1\}$ goes arrows to $\{f_1, f_2\}$ and $\{f_1, f_3\}$. The bottom node represents the fault-free case.

If the present minimal candidates are $\{f_1\}$ and $\{f_2\}$, then tests need to be activated with the capability of rejecting $\{f_1\}$ for each of the candidates $\{f_1, f_2\}$ and $\{f_1, f_3\}$, and rejecting $\{f_2\}$ for each of the candidates $\{f_1, f_2\}$ and $\{f_2, f_3\}$ to be able to refine the candidates. This is visualized in Fig. 3 by thicker arrows going from the present minimal candidates to new possible minimal candidates if a new test quantity alarms. ⋄

Sequentially updating the minimal candidates in the fault isolation algorithm requires an active set of test quantities able to reject each $d_j \in D$ for each $d_i = \{f_a\} \cup d_j$ where $f_a \notin d_j$. By using the results in Section 3, the set of active test quantities can be selected and generated for each minimal candidate $d_j \in D$ if there exists a test quantity in the set which have maximum distinguishability, or fulfills a minimum required distinguishability, when rejecting that minimal candidate d_j.

5.2 Algorithm

In the previous subsection a discussion was made regarding which types of test quantities to use depending on the present set of minimal candidates D. How well the present faults can be identified, depends on which test quantities $T \in \mathcal{T}$ that are active in the diagnosis algorithm.

Let $\mathcal{E}_{i,j}$ contain each test quantity $T \in \mathcal{T}$ which can reject d_j for d_i, where $d_i = \{f_a\} \cup d_j$ and $f_a \notin d_j$, and fulfills a minimum required distinguishability. If there exists at least one test which is able to reject a minimal candidate d_j for a new minimal candidate d_i with at least minimum required distinguishability then $\mathcal{E}_{i,j} \neq \emptyset$. Otherwise if $\mathcal{E}_{i,j} = \emptyset$, then d_j can not be rejected for d_i.

The sequential test selection algorithm is summarized in the following steps.

I Initialize the set of minimal candidates, $D = \emptyset$.
II Solve (1) given D to compute a new set of test quantities.
III Replace the previous set of active test quantities by the new set of test quantities.
IV Run the new set of test quantities until a test alarms.
V Update D given the alarmed test quantities.
VI Return to step II.

The optimization problem (1) is a minimum hitting set problem which is NP-hard, see Moret and Shapiro (1985), but there exists many heuristic search methods to find a solution, see for example de Kleer (2011) and references. The following example considers a small system and is used to describe how the sequential test selection algorithm updates the set of active test quantities as the minimal candidates are updated.

Example 2. Consider the system with three possible faults. Here, a diagnosis algorithm applying the sequential test selection algorithm is designed to detect and isolate faults while minimizing the number of active tests, i.e., $c(T) = 1$ in (1), $\forall T \in \mathcal{T}$. Assume that there are seven available test quantities, $\mathcal{T} = \{T_1, T_2, \ldots, T_7\}$ where the detectability performance of each test quantity and fault is quantified using distinguishability, see Table 1. First in the considered fault scenario, a fault f_1 enters the system and then later another fault f_3 also enters the system.

When choosing each set $\mathcal{E}_{d_i,d_j}$, the performance of each test quantity is chosen to fulfill a required minimum distinguishability. In this example, we choose that if the cardinality of d_i is 1 then distinguishability should be at least 2.0, if the cardinality of d_i is 2 then distinguishability should at least be 1.0, and if the cardinality of d_i is 3 then distinguishability should at least be 0.5. Based on the distinguishability of the test quantities given in Table 1, the sets will be

$$\begin{aligned}
&\mathcal{E}_{\{f_1\},\emptyset} = \{T_1, T_2, T_3\}, && \mathcal{E}_{\{f_2\},\emptyset} = \{T_1\},\\
&\mathcal{E}_{\{f_3\},\emptyset} = \{T_1, T_2\}, && \mathcal{E}_{\{f_1,f_2\},\{f_1\}} = \{T_4\},\\
&\mathcal{E}_{\{f_1,f_3\},\{f_1\}} = \{T_4\}, && \mathcal{E}_{\{f_1,f_2\},\{f_2\}} = \emptyset,\\
&\mathcal{E}_{\{f_2,f_3\},\{f_2\}} = \{T_5\}, && \mathcal{E}_{\{f_1,f_3\},\{f_3\}} = \{T_3\},\\
&\mathcal{E}_{\{f_2,f_3\},\{f_3\}} = \emptyset, && \mathcal{E}_{\{f_1,f_2,f_3\},\{f_1,f_2\}} = \{T_5\},\\
&\mathcal{E}_{\{f_1,f_2,f_3\},\{f_1,f_3\}} = \{T_7\}, && \mathcal{E}_{\{f_1,f_2,f_3\},\{f_2,f_3\}} = \{T_6\}.
\end{aligned} \quad (17)$$

Note that some sets are empty, for example $\mathcal{E}_{\{f_1,f_2\},\{f_2\}} = \emptyset$ because there are no test quantities which are sensitive to f_1 but not to f_2 with *sufficiently high* distinguishability.

When the diagnosis algorithm is initialized the only minimal candidate is the empty set, i.e., $D = \{d_0\}$ where $d_0 = \emptyset$. The first set of active test quantities is the solution to the problem

Table 1. The performance of each test is quantified using distinguishability. Distinguishability of a test T_i detecting a fault $f_j = \theta$ is written in position (i, j).

	f_1	f_2	f_3
T_1	2.5	2.4	2.0
T_2	2.2	1.5	2.3
T_3	2.0	0.8	0
T_4	0	1.3	1.5
T_5	0	0	1.2
T_6	0.5	0	0
T_7	0	0.6	0

$$\begin{aligned}
&\min \sum_{i=1}^{7} x_i \\
&s.t.\ x_1 + x_2 + x_3 \geq 1, && (\mathcal{E}_{\{f_1\},\emptyset})\\
&\quad x_1 \geq 1, && (\mathcal{E}_{\{f_2\},\emptyset})\\
&\quad x_1 + x_2 \geq 1, && (\mathcal{E}_{\{f_3\},\emptyset})\\
&\quad x_i = \{0, 1\}, && i = 1, 2, \ldots, 7.
\end{aligned}$$

The optimal solution is $x_1 = 1$, and $x_i = 0, \forall i \neq 1$, which means that only T_1 is activated. Then a fault f_1 enters the system and T_1 alarms. The new minimal candidates are $D = \{d_1, d_2, d_3\}$ where $d_1 = \{f_1\}$, $d_2 = \{f_2\}$, and $d_3 = \{f_3\}$. To update the set of test quantities, the following problem is solved

$$\begin{aligned}
&\min \sum_{i=1}^{7} x_i \\
&s.t.\ x_4 \geq 1, && (\mathcal{E}_{\{f_1,f_2\},\{f_1\}})\\
&\quad x_4 \geq 1, && (\mathcal{E}_{\{f_1,f_3\},\{f_1\}})\\
&\quad x_3 \geq 1, && (\mathcal{E}_{\{f_1,f_3\},\{f_3\}})\\
&\quad x_5 \geq 1, && (\mathcal{E}_{\{f_2,f_3\},\{f_2\}})\\
&\quad x_i = \{0, 1\}, && i = 1, 2, \ldots, 7.
\end{aligned} \quad (18)$$

Note that $\mathcal{E}_{\{f_1,f_2\},\{f_2\}}$ and $\mathcal{E}_{\{f_2,f_3\},\{f_3\}}$ are empty sets and therefore not included in (18). The new set of active test quantities are T_3, T_4, and T_5. Later T_3 alarms and the remaining minimal candidates are $D = \{d_1, d_2\}$. The corresponding new set of active test quantities is T_4 and T_5. Since only f_1 is present in the system and there is no test quantity which have sufficient performance to reject the other candidate $\{f_2\}$ for $\{f_1, f_2\}$, $\{f_1\}$ can not be isolated from $\{f_2\}$. Note that by using another selection criteria when defining each set $\mathcal{E}_{d_i,d_j}$ could let $\mathcal{E}_{\{f_1,f_2\},\{f_2\}}$ contain T_6 which is sensitive to f_1 but not f_2.

Later a fault f_3 also enters the system which first results in that T_4 alarms. The new minimal candidates becomes $D = \{d_2, d_4\}$ where $d_4 = \{f_1, f_3\}$. The corresponding new activated test quantities are T_5 and T_7. Later T_5 alarms which replaces d_2 with $d_5 = \{f_2, f_3\}$ and activates the test quantities T_6 and T_7. Finally T_6 alarms which removes d_5 since the new candidate $\{f_1, f_2, f_3\}$ is a super-set of d_4 which is the true and remaining candidate. ◇

The properties of each set in (17) can be visualized using the lattice in Fig. 3. If $\mathcal{E}_{d_i,d_j} = \emptyset$ then there exists no test quantity which have sufficiently high distinguishability to reject d_j for d_i, see Table 1. Each set $\mathcal{E}_{d_i,d_j}$ is related to an arrow from d_j to d_i, see Fig. 4. All non-empty sets $\mathcal{E}_{d_i,d_j}$ are represented by thicker arrows.

6. CASE STUDY: DC CIRCUIT

The sequential test selection algorithm presented in Section 5 is applied to a DC-circuit. The DC-circuit is a part of the diagnosis test bed ADAPT, see Poll et al. (2007). The analyzed system is similar to the circuit analyzed in Gorinevsky et al. (2009) except that some additional faults are included here while outputs and faults in batteries and loads are unknown and not considered.

6.1 System

The system contains 19 sensors measuring voltages and currents in the circuit and 12 sensors measuring the positions of the relays. The number of faults included in the model is 38,

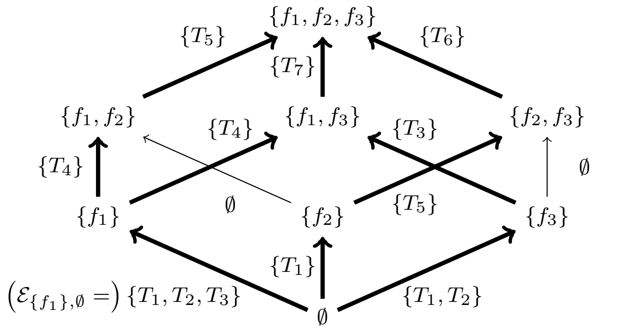

Fig. 4. A lattice set representation of all possible candidates for a system with three possible faults. The set at each arrow represents which test quantities that are able to reject d_j for d_i and fulfills a specified distinguishability criteria.

both relay faults and sensor faults. Not all faults are detectable because there are not enough sensors and some parts of the circuit are assumed unknown, e.g., the power consumption of the loads and the output from the batteries. Some faults are not fully isolable from each other because several faults affect the same part of the system.

A model of the DC-circuit has been developed by formulating the Sparse Tableau Analysis (STA) equations, see Gorinevsky et al. (2009). Model uncertainties have been included as i.i.d. Gaussian measurement noise and the faults are modeled as additive signals. The model of the DC-circuit is linear and static and is written in the form (2) where $n = 1$.

6.2 Diagnosis algorithm

The implemented diagnosis algorithm applies the result from Theorem 5, i.e., how to generate residuals with maximum FNR, to automatically design and generate residuals during run-time. One problem when sequentially updating the set of active test quantities is how to find residuals which have sufficiently high distinguishability given the present minimal candidates. When choosing which residual to generate the inequality (15) is used. Based on the inequality, residuals should be generated where as few faults are decoupled as possible while still being able to reject d_j for d_i to maximize distinguishability. Residuals with maximum distinguishability can be generated, using the result in Section 4, by considering each of the present minimal candidates. As the minimal candidates are updated, new tests with maximum distinguishability are automatically generated.

The distinguishability criteria when selecting the sets $\mathcal{E}_{d_i,d_j}$ are such that each set $\mathcal{E}_{d_i,d_j}$ contains the residual with highest distinguishability when rejecting d_j for d_i. Also, if there are more residuals able to reject d_j for d_i, they are included in $\mathcal{E}_{d_i,d_j}$ if they fulfill a minimum required distinguishability.

In this implementation, when a new set of active residuals is selected the residuals are activated when the next sample of data is received from the system. This implementation has been chosen to visualize how the number of active residuals changes each time one or more residuals alarms. Note that this way of implementation results in a delay in the fault isolation process because a fault could be isolated faster if new residuals are used directly on previous data samples, see Krysander et al. (2010). Such a change in the implementation can easily be made and do not affect the sequential test selection algorithm.

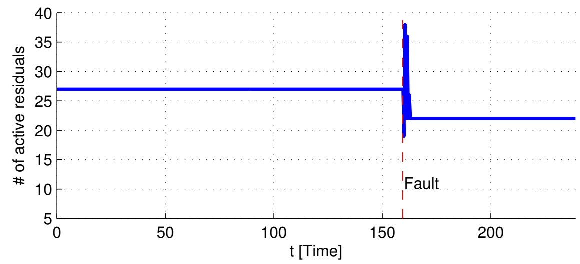

Fig. 5. The number of active residuals during a scenario where a relay gets stuck open, f_{10}, occurs at 159 s. The total number of active residuals during the scenario is 64.

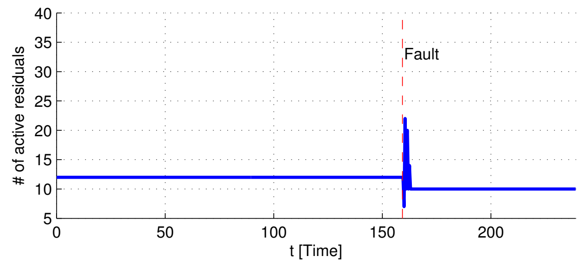

Fig. 6. The number of active residuals during a scenario where a relay gets stuck open, f_{10}, occurs at 159 s. The required minimum distinguishability for each residual is lower compared to the result in Fig. 5 resulting in fewer active residuals during the scenario. The total number of active residuals during the scenario is 41.

Here, the optimization problem (1) is solved by using a simple greedy test selection algorithm to find a solution. The greedy search algorithm iteratively picks the residual with maximum distinguishability in each set $\mathcal{E}_{d_i,d_j}$ if no other residual in the set is already selected.

6.3 Evaluation

The sequential test selection algorithm described in section 5.2 is evaluated using a number of different test scenarios. In the first scenario, a relay breaks, f_{10}, at 159 s. The number of active residuals at any time during the scenario is shown in Fig. 5. In the first evaluation, residuals are included in $\mathcal{E}_{d_i,d_j}$ if their corresponding distinguishability is higher than 0.5, which corresponds to $FNR = 1$ if the fault amplitude is 1. Note that if only the residuals which have maximum distinguishability in each set $\mathcal{E}_{d_i,d_j}$ is active from the start the number of used residuals would exceed 370.

The total number of used residuals is 64 but the maximum number of active residuals is mostly below 30, except right after the fault is detected where the number of active residuals rises to almost 40.

If the required distinguishability is chosen lower, for example 0.045 ($FNR = 0.3$), the number of active residuals used during the scenario is reduced which is shown in Fig. 6. The number of active residuals are less than 15 most of the time and less than 25 when the fault is detected. The total number of generated residuals during the whole scenario is 41.

The final minimal candidates for the two choices of minimum distinguishability are in this case equal and are shown in Fig. 7.

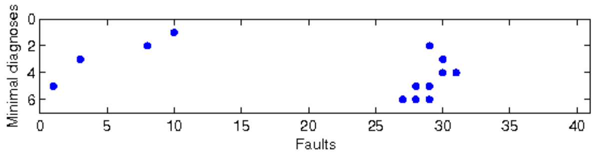

Fig. 7. The final minimal candidates for the first scenario given by the diagnosis algorithm, sorted by cardinality. The final candidates are in this case the same for both choices of minimum required distinguishability.

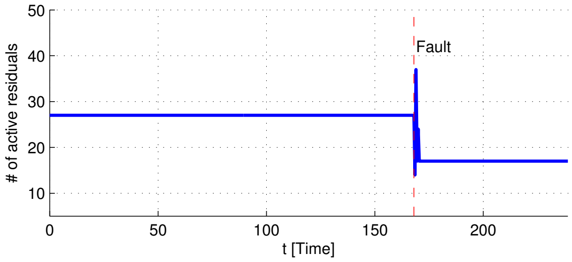

Fig. 8. The number of active residuals during a scenario where two sensors fail, f_{24} and f_{35}, occurs at 168 s. The total number of active residuals during the scenario is 54.

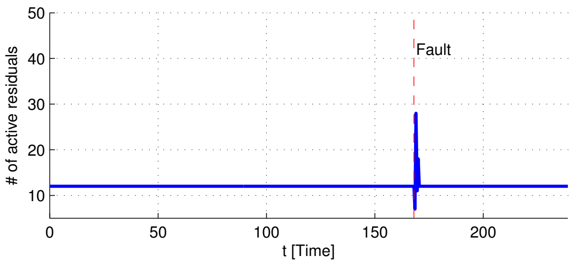

Fig. 9. The number of active residuals during a scenario where two sensors fail, f_{24} and f_{35}, occurs at 168 s. The required minimum distinguishability for each residual is lower compared to the result in Fig. 8 resulting in fewer active residuals during the scenario. The total number of active residuals during the scenario is 40.

Each row represent a minimal candidate and the only candidate of minimum cardinality, $\{f_{10}\}$, is also the correct isolated fault.

In the second scenario two sensor faults, one voltage sensor f_{35} and one current sensor f_{24}, occurs simultaneously. The diagnosis algorithm have been run twice, first with minimum required distinguishability chosen to be 0.5 and the second time to be 0.045. The number of active residuals during the first run is shown in Fig. 8. The total number of generated residuals is 54 and the number of active residuals is below 30 except right after the faults are detected.

In the second run the number of active residuals is reduced to 12 during the whole scenario except right after the faults are detected. The total number of generated residuals is 40.

The total number of minimal candidates is 121, see Fig. 10. The number of candidates with minimum cardinality equal to two are $\{f_{24}, f_{35}\}$ and $\{f_{23}, f_{35}\}$ where $\{f_{24}, f_{35}\}$ is the true candidate. The other candidate, $\{f_{23}, f_{35}\}$, can not be rejected because f_{23} and f_{24} are not isolable from each other.

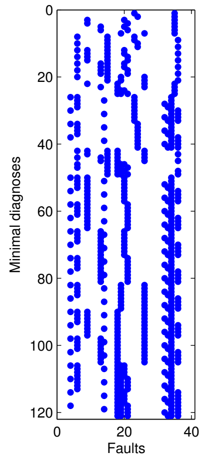

Fig. 10. The final minimal candidates given by the diagnosis algorithm, sorted by cardinality. In this case, both choices of required minimum distinguishability resulted in the same final candidates.

Choosing a lower required distinguishability when creating the sets $\mathcal{E}_{d_i,d_j}$ can result in solutions where the set of active test quantities has lower cardinality. A lower number of active test quantities is a trade off which will probably result in reduced diagnosability performance since the diagnosis algorithm will accept active test quantities with lower distinguishability when rejecting some minimal candidates. In this analysis, the actual faults in the measured data are relatively large compared to the fault amplitudes used when analyzing distinguishability. This could explain why the minimal candidates are equal in the two scenarios even though the required distinguishability is changed.

7. TUNING THE TEST SELECTION ALGORITHM

Here, some extra tuning steps which could be added to the test selection algorithm are discussed. The proposed algorithm is also related to test selection algorithms presented in other works. It is shown how the problem formulation in this work can be used to describe other test selection algorithms.

7.1 Off-line

If the available set of test quantities $\mathcal{T}$ is large and many of the sets $\mathcal{E}_{d_i,d_j}$ have high cardinality then it will be computationally expensive to find an optimal solution to (1). There might be several test quantities that will never be used because there are other test quantities that have better performance. This might be the case if tests quantities are generated automatically off-line or sequentially during run-time or if a developer have not been able to remove not so good redundant test quantities.

If the set of available test quantities $\mathcal{T}$ is generated off-line, the computational time for solving (1) on-line, and the memory needed for storing all test quantities, could be reduced by first solving (1) for all sets of possible minimal candidates off-line, i.e., $D = \{\text{all possible minimal candidates}\}$. By using only the set of test quantities in the solution as available test quantities $\mathcal{T}$ will remove unnecessary test quantities when the diagnosis algorithm is on-line.

Another approach to reduce the computational time for solving (1) could be to neglect minimal candidates of higher cardinality. If multiple faults of cardinality higher than $q > 0$ are assumed to be unlikely then only test quantities able to reject minimal candidates of lower cardinality need to be considered. In Svärd (2012), an algorithm for automatically generating residuals off-line for a non-linear system uses a greedy search algorithm to find a set of test quantities able to detect and isolate single faults. This corresponds to solving (1) for all minimal candidates d_j of maximum cardinality one. In Svärd (2012), only deterministic performance of test quantities are considered which is the same as letting $\mathcal{E}_{d_i,d_j}$ contain all test quantities able to reject d_j for d_i independent of quantitative performance.

7.2 On-line

In Krysander et al. (2010), the proposed sequential test selection algorithm solves the optimization problem (1) every time the minimal candidates are updated, but do not consider any quantitative diagnosability performance which is done in this work. Although, in Krysander et al. (2010), an extra constraint to the solution of (1) requires that any combination of faults should always be detectable if possible. For example, there could be situations where two faults could cancel out each other which would in such a case not be detectable. An example is a residual $r = f_1 + f_2$ which would be zero if $f_1 = 1, f_2 = -1$. Such cases could be assumed highly unlikely, and therefore neglected, but this extra constraint can also easily be included in the algorithm proposed in this paper.

One approach to save computational time is to let each set $\mathcal{E}_{d_i,d_j}$ only contain the best test quantity, i.e., the test quantity with highest distinguishability. Then, all sets $\mathcal{E}_{d_i,d_j}$ would either contain one element or no elements depending on whether there exists a test quantity able to reject d_j for d_i or not. Then the optimal solution to (1) is found by simply selecting the test quantity in each set $\mathcal{E}_{d_i,d_j}$ where $d_j \in D$. This assures that the diagnosis algorithm always uses the best set of tests given D at the cost of requiring more active tests used at the same time. Note that in such a case where each set $\mathcal{E}_{d_i,d_j}$ maximally contains one element, it is trivial to find the optimal solution.

7.3 Other measures of diagnosability performance

Here, distinguishability has been used to quantify diagnosability performance when selecting a set of test quantities with sufficient detectability and isolability performance. The proposed test selection algorithm is not limited to the use of distinguishability but could use any type of measure to quantified diagnosability performance to compare the performance of different sets of test quantities, see for example Wheeler (2011).

8. CONCLUSION

A sequential test selection algorithm to update an active set of test quantities given the present minimal candidates is proposed. A contribution here is the use of a measure for quantitative diagnosabiltiy performance, distinguishability, to choose which test quantities to be used to reject the present minimal candidates. A generalization of distinguishability is introduced in order to handle isolability from multiple faults.

The test selection algorithm proves useful when, for example, implementing a diagnosis algorithm where the set of tests are automatically selected and generated on-line. As an example, a diagnosis algorithm for a DC-circuit is evaluated which automatically generates new residuals with maximum distinguishability given the present minimal candidates. The example shows that the number of test quantities can be greatly reduced by using the test selection algorithm while fulfilling a required quantitative detectability and isolability performance.

The proposed sequential test selection algorithm is also compared and related to other works, including both off-line and on-line test selection algorithms. Different tuning parameters can be used to reduce computational time or change the properties of the solution. The proposed algorithm can be used for both on-line and off-line test selection algorithms.

9. ACKNOWLEDGMENT

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