Robust Residual Selection for Fault Detection

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Abstract— A number of residual generation methods have been developed for robust model-based fault detection and isolation (FDI). There have also been a number of offline (i.e., design-time) methods that focus on optimizing FDI performance (e.g., trading off detection performance versus cost). However, design-time algorithms are not tuned to optimize performance for different operating regions of system behavior. To do this, would need to define online measures of sensitivity and robustness, and use them to select the best residual set online as system behavior transitions between operating regions. In this paper we develop a quantitative measure of residual performance, called the detectability ratio that applies to additive and multiplicative uncertainties when determining the best residual set in different operating regions. We discuss this methodology and demonstrate its effectiveness using a case study.

I. INTRODUCTION

Model based diagnosis methods use analytical redundancy in the system model for fault detection and isolation. Analytic residual generation and analysis methods are typically used for fault detection, and analysis of non zero residuals forms the basis for fault isolation in the system [1]. However, the robustness and accuracy of residual detection methods can be affected by uncertainties, such as parameter uncertainty, structural uncertainty, measurement noise, and disturbances generated by the environment that affect system behavior. Structural uncertainty is attributed to inaccuracies in the system model because of the lack of complete knowledge of the real processes that make up the system, or approximations that may be made to simplify the computational aspects of the model. Parameter uncertainties arise because of practical limitations in making a sufficient number of measurements to accurately estimate the parameters, and the fact that system parameters may vary with operating conditions that are hard to quantify mathematically. Other sources of uncertainties in system models and behavior prediction can be attributed to computational issues, such as the use of finite difference methods as an approximation for solving partial differential equations, and the need to find approximate solutions for stiff systems to avoid computational complexity in online analysis [2], [3].

It is possible to reduce uncertainties in residual analysis by developing more accurate and complex models but there are no practical solutions to eliminating uncertainty in the analyses. Therefore, uncertainties must be accounted for in the design of any supervisory system. There are two general approaches to deal with uncertainty in supervisory systems. The first approach, *active fault detection*, identifies uncertainties and applies an auxiliary input to decouple them

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from the residuals. These approaches use different methods such as Bayesian filters, optimization methods, geometric control approaches, and robust control methods to perform partial or complete decoupling. They work well if there are few uncertainties in the system (see, for example, [1], [4], [5] and [6]). The second method, *passive fault detection*, develops robust decision making methods that utilize an adaptive threshold to achieve robustness [7], [8]. Performance measures that tradeoff accuracy versus false alarm rates are often used to derive the threshold values. Passive and active methods used for robust fault detection are reviewed in [9]. In relation to previous work, the methodology described in this paper quantifies a performance measure for each residual in the presence of uncertainties. Online analytic optimization methods may then be employed to select the most robust set of residuals for each operating region.

A fault f, is detectable if there is at least one residual that is sensitive to this fault. However, in most systems where a sufficient number of measurements can be made, there are usually multiple residual candidates that are sensitive to the fault. For a model with no uncertainties, all residuals linked to a fault will perform equally well, since deviations from zero can only be attributed to faults. However, when uncertainties are present, a non zero residual may be attributed to fault and/or uncertainty effects. Moreover, the sensitivity of residuals to faults and uncertainty effects can vary in different operating regions of system behavior. Therefore, it is possible that one residual has better detectability performance for a fault in one operating region, but another residual has better performance in another region of operation. Ideally, a FDI system should switch between residuals to optimize detectability performance over multiple operating regions.

Djeziri, et al. [7] present a method for handling parameter uncertainties in the model by deriving adaptive thresholds to avoid false alarms. The uncertainties in the parameters affect the prediction of system dynamics, and this is accounted for in the residuals. Therefore, the approach quantifies the diagnosability performance of the different residuals.

For time-discrete descriptor models with additive faults and noise, Eriksson, et al. [10] propose a diagnosability measure which can be used for quantifying diagnosability performance. This measure is used for online selection of the best set of residuals for different conditions [11]. For time-continuous dynamic systems with multiplicative faults and uncertainties, the diagnosability performance varies over time and depends on the operating regions and system dynamics. With respect to previous work, a contribution here is a measure for quantifying the performance of residuals in different operating regions by using sensitivity analysis [12].

Our goal in this paper is to develop a quantitative measure of diagnosability performance for residuals in the presence of

faults and uncertainties. By using a quantitative measure of performance that takes into account sensitivity to faults over that of uncertainties, appropriate residuals can be selected for different operating regions of system behavior. This can be used for offline analyses [13], but the focus in this paper is on online test selection for fault detection across different operating regions of system behavior.

The outline of this paper is as follows. In Section II, we present a formal definition of the problem, and develop a running example to illustrate the methodology that we develop. Section III reviews the theoretical background of the work and Section IV introduces the quantitative measure for the residual performance, called the *detectability ratio*. The applicability of the method is presented in Section V through a case study. Section VI concludes the paper and discusses future work.

II. PROBLEM FORMULATION

A. Model of the system

In this paper, we restrict our analyses to linear time-invariant dynamic systems. Though the system parameters for such a model are constant, their values may not be accurately determinable because of lack of complete knowledge about the system and the inability to make sufficient measurements to estimate the parameters. Their values may also be a function of the operating conditions, and change as the system degrades. The state space representation of such a system model may be described as:

$$\dot{x} = A(f, \delta)x + B_u(f, \delta)u + b_f(f, \delta)$$
$$y = C(f, \delta)x + d_f(f, \delta), \tag{1}$$

where x is the state vector, u is the input vector, y is the output vector, f represents the vector of possible faults, and δ represents the uncertainty and disturbance vector in the system. As shown in Fig. 1, parameter uncertainties have multiplicative effects on system behavior, whereas disturbances have additive effects. Faults can have additive (e.g., sensor and actuator faults) and multiplicative (parameter faults) effects. Each component parameter in the system may
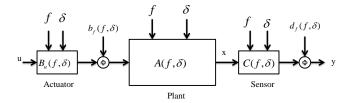

Fig. 1. Linear dynamic system with multiplicative and additive faults and uncertainties.

vary from its nominally specified value due to uncertainties and fault occurrences. This can be represented as: $C_n(1 + \delta_n)(1 + f_n)$, where C_n is the presumed nominal value of the component parameter, and δ_n and f_n represent uncertainties and faults associated with the component. It is also assumed that each uncertainty δ_n magnitude has a known upper bound Δ_n and each fault f_n magnitude has a lower bound F_n:

$$|\delta_n| \leq \Delta_n \qquad |f_n| \geq F_n. \tag{2}$$

B. Running example

As a running example for this paper, consider the following dynamic system

$$
\begin{aligned}
\dot{x}_1 &= -(1 + \delta_1)x_1 + u_1 + f & y_1 &= (1 + \delta_3)x_1 \\
\dot{x}_2 &= x_1 - 2(1 + \delta_2)x_2 + u_2 & y_2 &= (1 + \delta_4)x_2,
\end{aligned} \tag{3}
$$

where x_1 and x_2 are the system state variables, u_1 and u_2 represent the inputs to the system, y_1 and y_2 are the system measurements, δ_1 and δ_2 represent system uncertainties, δ_3 and δ_4 represent model uncertainties in the sensors, and f is an actuator fault. Fig. 2 illustrates the system model in block diagram form.
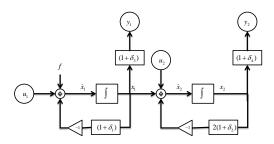

Fig. 2. Simple example diagram.

III. BACKGROUND

In this section, we briefly present the model-based diagnosis and sensitivity analysis approaches.

A. Model-based Fault Detection

A residual (also called an Analytic Redundancy Relation (ARR)) is a function of a subset of observables of the system (y and u in equation (1)). Under nominal conditions it has a value 0, but may report non zero values in the presence of faults, noise and uncertainties in the system [1]. In the example, u_1, u_2, y_1 and y_2 represent the observed variables from which sets of residuals can be created. These variables are presented as circles in Fig. 2.

A possible residual to detect fault f, can be derived from the relation between u_1 and y_1 (see equation (3)). Using a derivative causality formulation, the residual can be expressed as:

$$r_1 = \dot{y}_1 + y_1 - u_1. \tag{4}$$

Another residual sensitive to fault f, can be derived from the relation between u_1, u_2, and y_2:

$$r_2 = \ddot{y}_2 + 3\dot{y}_2 + 2y_2 - u_1 - u_2 - \dot{u}_2. \tag{5}$$

When uncertainty is not a factor, both r_1 or r_2 are equally effective in detecting f since any deviation from zero can only be explained by the fault. The only difference in this case would be the rise time of each residual, i.e., how fast they react to the fault. But when there is uncertainty in the system, the performance of the two residuals may differ significantly. Consider the simple example when $\delta_3 \neq 0$, and $\delta_1 = \delta_2 = \delta_4 = 0$. In this case, r_2 is preferred because it does not use the first measurement y_1, which is affected by the uncertainty δ_3. On the other hand, if $\delta_1 = \delta_2 = \delta_3 = 0$ and $\delta_4 \neq 0$, then r_1 is the better choice, since r_1 does not depend

on y_2, which is affected by δ_4. Thus, the performance of a residual depends on how sensitive it is to a fault as compared to uncertainties that also affect the residual. We build on this simplistic example to develop a sensitivity analysis method for evaluating residuals.

B. Sensitivity Analysis of ODEs

Sensitivity analysis [12] provides the basis for evaluating the effect of faults and uncertainties on residuals of dynamic systems. Consider a general dynamic system model of the form:

$$g(\dot{x}, x, u, \theta) = 0 \qquad h(y, x, \theta) = 0, \qquad (6)$$

where $x \in \mathbb{R}^{l_x}$ are state variables, $u \in \mathbb{R}^{l_u}$ are actuator signals, $y \in \mathbb{R}^{l_y}$ are measurement signals, and $\theta \in \mathbb{R}^{l_\theta}$ are model parameters.

Sensitivity analysis helps to evaluate how model behaviors are affected by variations in model parameters θ [14]. When performing sensitivity analysis, the derivative of (6) with respect to each parameter θ_k is computed as:

$$\frac{\partial g}{\partial \dot{x}} \dot{p}_{\theta_k} + \frac{\partial g}{\partial x} p_{\theta_k} + \frac{\partial g}{\partial u} \underbrace{\frac{\partial u}{\partial \theta_k}}_{=0} + \frac{\partial g}{\partial \theta_k} = 0$$

$$\frac{\partial h}{\partial y} q_{\theta_k} + \frac{\partial h}{\partial x} p_{\theta_k} + \frac{\partial h}{\partial \theta_k} = 0, \qquad (7)$$

where $p_{\theta_k} = \frac{\partial x}{\partial \theta_k}$ and $q_{\theta_k} = \frac{\partial y}{\partial \theta_k}$ represent the sensitivity of state and measurement variables to a deviation in parameter θ_k. $\dot{p}_{\theta_k} = \frac{\partial \dot{x}}{\partial \theta_k}$ and it is assumed that u is an external control signal which does not depend on θ_k, implying $\frac{\partial u}{\partial \theta_k} = 0$.

IV. QUANTITATIVE ANALYSIS OF RESIDUAL DETECTABILITY PERFORMANCE

In this section, we define a quantitative diagnosability measure using the sensitivity analysis formulation. A small example is used to show how to evaluate the performance of different residuals.

A. Detectability Analysis

A residual, $r(y, u) = 0$, captures nominal system behavior. Ideally, each residual should be zero in the fault-free case and residuals sensitive to a fault become nonzero when it occurs. Due to uncertainties and noise, the residual may deviate from zero even in the fault-free case, and this complicates the fault detection task. To design the fault detection scheme to have a low false alarm rate, a threshold must be derived that separates the uncertainties from the faults. Depending on the relationship between fault and uncertainty magnitudes, the likelihood of detecting a fault of a specified magnitude can vary. If we can quantify this relation, we can use it to select the best residual for detecting individual faults.

A common approach for evaluating detectability performance is to compare the ratio between the signal to be selected and the disturbance, for example the signal-to-noise ratio (SNR). In this case the key factor is the fault magnitude to noise ratio [10]. Also when designing robust residual generators using the H_∞ norm, the ratio between faults and disturbances is optimized [15]. To achieve good fault detectability performance with a low risk of false alarms,

the ratio of the fault magnitude to parameter uncertainties should be large. A larger ratio corresponds to a fault that is easier to detect since its impact relative to uncertainties will be larger.

B. Detectability Ratio

Since the effect of uncertainties and faults in the residuals are unknown, we estimate them using sensitivity analysis. It is assumed that the fault magnitude and uncertainties are small. So a first order linear approximation of the effect on a residual with respect to f and δ is justified, and given by:

$$r(y, u) \approx \frac{\partial r}{\partial f} f + \sum_{i=1}^{N} \frac{\partial r}{\partial \delta_i} \delta_i, \qquad (8)$$

where the partial derivatives are given by:

$$\frac{\partial r}{\partial f} = \frac{\partial r}{\partial y} \frac{\partial y}{\partial f} = \frac{\partial r}{\partial y} q_f, \qquad \frac{\partial r}{\partial \delta_i} = \frac{\partial r}{\partial y} \frac{\partial y}{\partial \delta_i} = \frac{\partial r}{\partial y} q_{\delta_i}.$$

The parameters q_f and q_{δ_i} represent the sensitivity of the sensor signals to the fault and the uncertainties, respectively. Faults and uncertainties are assumed to not affect the input u. Note that the term $\frac{\partial r}{\partial y}$ is not a function of fault and uncertainties, and the effects of faults and uncertainties on the residuals are through the sensor signals.

Since the actual magnitudes of the fault and uncertainties are unknown, the maximum values of the magnitude of uncertainties and minimum magnitudes of the fault are used to calculate the worst case scenario of the difficulty in detecting a fault. A quantitative measure of detectability performance is defined as follows.

Definition 1: (Detectability Ratio) Given the dynamic system in equation (1) the detectability ratio of fault f for residual r in the presence of uncertainties $\delta_i : i = 1, 2, ...N$ is defined as:

$$D(f|r) = \frac{\left| \frac{\partial r}{\partial f} F_{min} \right|}{\left| \frac{\partial r}{\partial f} F_{min} \right| + \sum_{i=1}^{N} \left| \frac{\partial r}{\partial \delta_i} \Delta_i \right|}, \qquad (9)$$

where Δ_i is the absolute value of the upper bound on the magnitude of uncertainty δ_i and F_{min} is lower bound on the magnitude of fault, f. If $\left| \frac{\partial r}{\partial f} F_{min} \right| = 0$, then $D(f|r) = 0$.

The detectability ratio lies in the interval $[0, 1]$ where 0 corresponds to situation that the fault will not affect the residual. The value 1 implies there are no uncertainties affecting the residual. If the effect of a fault is larger than the total effects of the uncertainties, i.e., $D(f|r) > 0.5$, which means that the fault can be reliably detectable without error. Note that the detectability ratio is not constant but depends on the state and operating region of the system. Higher detectability ratios imply lower thresholds can be chosen to avoid missing alarms, and fault detection time can be reduced.

1) System sensitivity to uncertainties and fault: Consider the system (1) with N uncertainties $\delta_i : i = 1, 2, .., N$ and a possible fault f in the system. For each of the uncertainties δ_i we define $p_{\delta i} = \frac{\partial x}{\partial \delta_i}$ and $q_{\delta i} = \frac{\partial y}{\partial \delta_i}$. Also for fault f we have $p_f = \frac{\partial x}{\partial f}$ and $q_f = \frac{\partial y}{\partial f}$. Therefore,

$$p_{\delta i}^{\cdot} = Ap_{\delta i} + \frac{\partial A}{\partial \delta_i}x + \frac{\partial B_u}{\partial \delta_i}u + \frac{\partial B_f}{\partial \delta_i}$$
$$q_{\delta i} = Cp_{\delta i} + \frac{\partial C}{\partial \delta_i}x + \frac{\partial D_f}{\partial \delta_i}, \tag{10}$$

$$\dot{p}_f = Ap_f + \frac{\partial A}{\partial f}x + \frac{\partial B}{\partial f}u + \frac{\partial B_f}{\partial f}$$
$$q_f = Cp_f + \frac{\partial C}{\partial f}x + \frac{\partial D_f}{\partial f}. \tag{11}$$

Note that to perform sensitivity analysis for the system in equation (1), partial derivatives of each of the matrices $A(f,\delta)$, $B_u(f,\delta)$, $C(f,\delta)$ and vectors $b_f(f,\delta)$ and $d_f(f,\delta)$ with respect to fault f and uncertainty δ must be computable and bounded. Moreover, since the state variables x appeared in equation (10) and equation (11) the linear system represented in equation (1) should be stable and observable.

As an example we compute $p_{\delta 1} = \frac{\partial x}{\partial \delta_1}$, $q_{\delta 1} = \frac{\partial y}{\partial \delta_1}$, $p_{\delta 2} = \frac{\partial x}{\partial \delta_2}$ and $q_{\delta 2} = \frac{\partial y}{\partial \delta_2}$ for the running example in equation (3) as:

$$\begin{aligned}
\dot{p}_{\delta 1_1} &= -p_{\delta 1_1} - x_1 & \dot{p}_{\delta 2_1} &= -p_{\delta 2_1} \\
\dot{p}_{\delta 1_2} &= p_{\delta 1_1} - 2p_{\delta 1_2} & \dot{p}_{\delta 2_2} &= p_{\delta 2_1} - 2p_{\delta 2_2} - 2x_2 \\
q_{\delta 1_1} &= p_{\delta 1_1} & q_{\delta 2_1} &= p_{\delta 2_1} \\
q_{\delta 1_2} &= p_{\delta 1_2} & q_{\delta 2_2} &= p_{\delta 2_2}.
\end{aligned} \tag{12}$$

For sensor uncertainties δ_3 and δ_4 we have:

$$\begin{aligned}
\dot{p}_{\delta 3_1} &= -p_{\delta 3_1} & \dot{p}_{\delta 4_1} &= -p_{\delta 4_1} \\
\dot{p}_{\delta 3_2} &= p_{\delta 3_1} - 2p_{\delta 3_2} & \dot{p}_{\delta 4_2} &= p_{\delta 4_1} - 2p_{\delta 2_2} \\
q_{\delta 3_1} &= p_{\delta 3_1} + x_1 & q_{\delta 4_1} &= p_{\delta 4_1} \\
q_{\delta 3_2} &= p_{\delta 3_2} & q_{\delta 4_2} &= p_{\delta 4_2} + x_2,
\end{aligned} \tag{13}$$

where $p_{\delta 3} = \frac{\partial x}{\partial \delta_3}$, $q_{\delta 3} = \frac{\partial y}{\partial \delta_3}$, $p_{\delta 4} = \frac{\partial x}{\partial \delta_4}$ and $q_{\delta 4} = \frac{\partial y}{\partial \delta_4}$. We can see that state variables x_1 and x_2 serve as the inputs in equations (12) and (13), and, therefore, the sensitivity of the system to the uncertainties is a function of the trajectories of x_1 and x_2. Since the system in equation (1) is observable, x_1 and x_2 or their estimates are computable at each operating point and we have no problems in computing the sensitivity of the system to the uncertainties. Sensitivity computations to f is computed as:

$$\begin{aligned}
\dot{p}_{f_1} &= -p_{f_1} + 1 & \dot{p}_{f_2} &= p_{f_1} - 2p_{f_2} \\
q_{f_1} &= p_{f_1} & q_{f_2} &= p_{f_2}.
\end{aligned} \tag{14}$$

Since the fault is additive the states do not appear in equation (14), the sensitivity of residual r_1 to fault f is not a function of the operating region.

Even though the sensitivity of states to faults and uncertainties are not part of the real system dynamics and we only need to compute them to analyze the performance of our residuals, it is important to prove the boundedness of these dynamic variables. Unboundedness of these variables increase computational expenses over time and eventually can stop the computation process and supervisory system operation. The stability of equations (10) and (11) is discussed in the following theorem.

Theorem 1: Given a stable linear system (1), sensitivity of states and outputs to uncertainties and faults represented by equations (10) and (11) are stable.

Proof: Consider equations (10) and (11). We can rewrite $p_{\delta i}^{\cdot}$ and $\dot{p}_f$ as:

$$p_{\delta i}^{\cdot} = Ap_{\delta i} + u_i, \tag{15}$$
$$\dot{p}_f = Ap_f + u_f, \tag{16}$$

where $u_i = \frac{\partial A}{\partial \delta_i}x + \frac{\partial B_u}{\partial \delta_i}u + \frac{\partial B_f}{\partial \delta_i}$ and $u_f = \frac{\partial A}{\partial f}x + \frac{\partial B}{\partial f}u + \frac{\partial B_f}{\partial f}$. Since the linear system (1) is stable, x is bounded and consequently u_i and u_f are bounded and equations (15) and (16) represent linear systems with stable system matrix A and bounded inputs. Therefore, $p_{\delta i}$ and p_f are stable. Considering stability of x, $q_{\delta i}$ and q_f are stable too. ∎

2) Residual sensitivity to uncertainties and fault: Using equations (10) and (11) we can derive sensitivity of residual $r(y,u)$ to fault f and uncertainty δ_i as:

$$\frac{\partial r}{\partial \delta_i} = \frac{\partial r}{\partial y}\frac{\partial y}{\partial \delta i} = \frac{\partial r}{\partial y}q_{\delta i}, \tag{17}$$
$$\frac{\partial r}{\partial f} = \frac{\partial r}{\partial y}\frac{\partial y}{\partial f} = \frac{\partial r}{\partial y}q_f. \tag{18}$$

Note that it is assumed that $\frac{\partial u}{\partial f} = 0$ and $\frac{\partial u}{\partial \delta i} = 0$. Consider the running example (3) and equations (12), (13) and (14). We compute $\frac{\partial r_1}{\partial \delta_1}$, $\frac{\partial r_1}{\partial \delta_2}$ and $\frac{\partial r_1}{\partial f}$ as:

$$\frac{\partial r_1}{\partial \delta_1} = -x_1 \quad \frac{\partial r_1}{\partial \delta_2} = 0 \quad \frac{\partial r_1}{\partial \delta_3} = u_1 \quad \frac{\partial r_1}{\partial \delta_4} = 0 \quad \frac{\partial r_1}{\partial f} = 1. \tag{19}$$

r_1 is not sensitive to δ_2 and δ_4, its sensitivity to δ_1 is a function of the state variable x_1 and its sensitivity to δ_3 is a function of the input u_1. Also independent of the operating point, its sensitivity to fault f is always 1. Considering $\mid \delta_i \mid \leq \Delta_i$ for $i = 1,2,3,4$ and F_m as the minimum absolute value of f that we wish to detect, the detectability ratio of f by r_1 is:

$$D(f|r_1) = \frac{F_m}{F_m + \Delta_1 \mid x_1 \mid + \Delta_3 \mid u_1 \mid}. \tag{20}$$

Considering r_2 we have:

$$\begin{aligned}
\frac{\partial r_2}{\partial \delta_1} &= -x_1 & \frac{\partial r_2}{\partial \delta_2} &= -2x_1 + 2x_2 - 2u_2 \\
\frac{\partial r_2}{\partial \delta_3} &= 0 & \frac{\partial r_2}{\partial \delta_4} &= u_1 + u_2 + \dot{u}_2 & \frac{\partial r_2}{\partial f} &= 1.
\end{aligned} \tag{21}$$

We can see that both of the residuals have the same sensitivity to fault f but their sensitivity to uncertainties are different. The detectability ratio of f by r_2 is given by equation (22).

Fig. 3 represents two trajectories of the system corresponding to u_1 and u_2 for the running example (3). In this case fault $f = 1$ occurs at time $t = 120s$, $\delta_1 = -0.095$, $\delta_2 = 0.095$, $\delta_3 = 0.095$ and $\delta_4 = 0.04$. Given $\Delta_1 = \Delta_2 = \Delta_3 = 0.1$, $\Delta_4 = 0.05$ and $F_m = 0.5$, the absolute value of residuals r_1 and r_2 and the corresponding detectability ratios are presented in Fig. 4. The fault occurs at $t = 120s$, therefore, the residuals should both be zero for $t < 120$ in the

$$D(f|r_2) = \frac{F_m}{F_m + \Delta_1 \mid x_1 \mid + \Delta_2 \mid -2x_1 + 2x_2 - 2u_2 \mid + \Delta_4 \mid u_1 + u_2 + \dot{u}_2 \mid}. \tag{22}$$

Fig. 3. Inputs of the system.

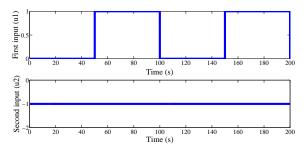Fig. 5. Two residuals and their combination.

Fig. 6. Two tanks system configuration.

ideal case. But because of uncertainties the value of residuals varies according to the state and input signal variations. An interesting observation is that the detectability ratio for each residual in this time interval is maximum when the residual is close to zero, and decreases as the residual deviates from zero. So we can say a residual with higher detectability ratio is less likely to produce false alarms. After $t = 120s$, when there is a fault in the system, the residual with higher amplitude (r_1 in this example) has a higher detectability ratio as well. So we can say the residual with higher detectability ratio is more likely to report the fault and also because of higher sensitivity it can detect smaller fault magnitudes.
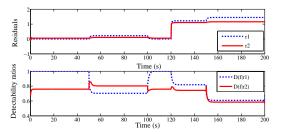

Fig. 4. Residuals and their corresponding detectability ratios.

The main idea of this paper is to track the detectability ratio online, and choose the best residual for each of the operating regions of the system. If we use the following hybrid residual for the system we will have a residual with maximum robustness to the uncertainties and sensitivity to the fault,
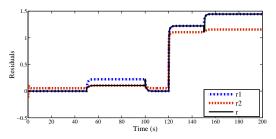
$$r = \begin{cases} r_1 & \text{if } D(f|r_1) \geq D(f|r_2) \\ r_2 & \text{otherwise.} \end{cases} \tag{23}$$

Fig. 5 represents r_1, r_2 and r. As we can see r obviously has a better performance rather than r_1 and r_2.

V. CASE STUDY

In order to show how detectability ratio can be applied to choose the best residual in real systems with uncertainties, we analyze a two tank system shown in Fig. 6.

The system consists of an input flow source (S) with volumetric flow rate u, two tanks (T_1 and T_2) and two valves (V_1 and V_2). Two sensors measure the pressure of the first and the second tank (p_1 and p_2). The first tank's nominal capacity is C_1 and the second one's is C_2. Valves T_1 and T_2 have nominal resistances of R_1 and R_2, respectively. Assume that there is δ_1 uncertainty in C_1, δ_2 uncertainty in C_2, a possible fault f in R_1 and δ_3 is the uncertainty in R_2. Considering p_1 and p_2 as the system state variables the system state space model is shown in equation (24), y_1 and y_2 represent the measurements. Equation (25) represents a simple proportional controller to regulate the pressure in the first tank.

$$u = K(r(t) - y_1), \tag{25}$$

where K is the controller gain and $r(t)$ is the desire trajectory of the pressure in C_1. To detect fault f two residuals r_1 and r_2 are designed.

$$\begin{aligned} r_1 &: \dot{y}_1 - \frac{K(r(t) - y_1)}{C_1} + \frac{y_1 - y_2}{R_1}, \\ r_2 &: \dot{y}_2 - \frac{y_1 - y_2}{R_1 C_2} + \frac{y_2}{R_2 C_2}. \end{aligned} \tag{26}$$

R_1 appears in both of the residuals and if there is no uncertainty in the model either r_1 or r_2 can be used to detect f. In the presence of uncertainty each residual is affected differently, and the sensitivity is also a function of the operating region of the system. In order to evaluate the performance of each residual as a function of the operating region, we compute the detectability ratio in each region. Step function $u(t)$ is considered as the desired trajectory for the first tank pressure. Three uncertainties $\delta_1 = -.09$, $\delta_2 = 0.08$ and $\delta_3 = 0.145$, are considered. Also it is assumed that we know, $\Delta_1 = \Delta_2 = 0.1$, $\Delta_3 = 0.2$ and $F_m = 1$. Fault $f = 1$ is considered as a fault in R_1 and it

$$\begin{pmatrix} \dot{p}_1 \\ \dot{p}_2 \end{pmatrix} = \begin{pmatrix} \frac{-K}{C_1(1+\delta_1)} + \frac{-1}{R_1(1+f)C_1(1+\delta_1)} & \frac{1}{R_1(1+f)C_1(1+\delta_1)} \\ \frac{1}{R_1(1+f)C_2(1+\delta_2)} & \frac{-1}{R_1(1+f)C_2(1+\delta_2)} + \frac{-1}{R_2(1+\delta_3)C_2(1+\delta_2)} \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} + \begin{pmatrix} \frac{K}{C_1(1+\delta_1)} \\ 0 \end{pmatrix} r(t),$$
$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}. \tag{24}$$
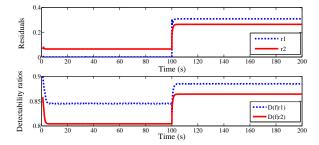

Fig. 7. Residuals and the corresponding detectability ratios for two tank system.

occurs at $t = 100s$. The absolute value of the residuals r_1 and r_2 and the detectability ratio of each is computed and plotted in Fig. 7. We can see from this figure that residual r_1 has a lower amplitude than r_2 before fault occurs and higher value after the fault happens. So this residual shows better performance to detect f and is less likely to cause false alarms when there is no fault in the system. We can see this in the detectability ratio of this residual, which is always higher than detectability ratio of r_2. The better performance of r_1 over r_2 is not surprising at all. Analyzing r_1, we see that the only source of uncertainty in this residual is uncertainty in C_1, and C_1 is multiplied by $(r(t) - p_1)$ in the numerator, which is the tracking error and is expected to be a small quantity. But on the other hand in r_2, R_2 and C_2 are two sources of uncertainty and the terms in the numerators are not expected to be a small quantity in this trajectory.

In the cases that the trajectory is known, we can do all the analysis offline and find the best residual at design time. Offline residual selection [13] can save computational expenses associated with online residual selection. Because we do not need to compute the sensitivity of state and residuals to the fault and uncertainties at each operation point. It is important especially when the systems has many state variables and parameters with different sources of uncertainty. Moreover, in the offline method the system does not need to be observable because we run the analysis only for the given trajectories, we do not need to measure or estimate the state variables. For systems with fast dynamic we believe that it is best to set a threshold in the switching process to avoid chattering between the residuals.

VI. Conclusions

The detectability ratio is defined as a quantitative measure of the residual detectability performance. It provides a measure to find the residual which is most sensitive to the fault and robust to the uncertainties at different regions of system operation. Two different examples are presented to evaluate and demonstrate the effectiveness of this approach. For online residual selection the detectability ratio is used to select the best residual for different operating points. In general, it is possible to achieve better detection by using more than one residual. For future work, we will propose a framework to consider all the potential residuals and design an optimal residual with the highest sensitivity taking into account that different faults may occur, and robustness to the uncertainties may change in different operating regions of system behavior. We can also extend the detectability analysis to isolability analysis that involves a number of potential single faults that can occur in the system.

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