A MINIMAL POLYNOMIAL BASIS SOLUTION TO RESIDUAL GENERATION FOR FAULT DIAGNOSIS IN LINEAR SYSTEMS

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Abstract: A fundamental part of a fault diagnosis system is the residual generator. Here a new method, the *minimal polynomial basis approach*, for design of residual generators for linear systems, is presented. The residual generation problem is transformed into a problem of finding polynomial bases for null-spaces of polynomial matrices. This is a standard problem in established linear systems theory, which means that numerically efficient computational tools are generally available. It is shown that the minimal polynomial basis approach can find all possible residual generators, including those of minimal McMillan degree, and the solution has a minimal parameterization. It is shown that some other well known design methods, do not have these properties.

Keywords: fault detection, diagnosis, polynomial methods, decoupling, disturbance rejection

1. INTRODUCTION

The task of fault diagnosis is to, from known signals, i.e. measurements and control signals, detect and locate any faults acting on the system being supervised. A fundamental part of a *model based* diagnosis system is the *residual generator*. The residual generator filters the known signals and generates a signal, the *residual*, that should be "small" (ideally 0) in the fault-free case and "large" when a fault is acting on the system. In a general system, not only the control signal u influence the system, but also disturbances d and the faults f that we wish to detect.

This work is a study of *linear* residual generation for *linear* systems with no model uncertainties. The diagnosis method considered is *structured residuals* (Gertler, 1991). Then the design of a residual generator becomes a decoupling problem. Further, only perfect decoupling of the disturbances is considered, and the issue of *approximate decoupling* associated with e.g. robust diagnosis is not considered here.

A general linear residual generator can be written

$$r = Q(s)\begin{pmatrix} y \\ u \end{pmatrix} \tag{1}$$

i.e. $Q(s)$ is a multi-dimensional transfer matrix with known signals y and u as inputs and a *residual* as output.

A number of design methods for designing linear residual generators, have been proposed in literature, see for example (Patton and Kangethe, 1989; Wünnenberg, 1990; White and Speyer, 1987; Massoumnia *et al.*, 1989; Nikoukhah, 1994; Chow and Willsky, 1984; Nyberg and Nielsen, 1997). All these methods are methods to design the transfer matrix $Q(s)$. Three natural questions that have not gained very much attention are the following:

- Does the method find all possible residual generators?
- Does the method find residual generators of minimal McMillan degree?
- Does the solution represent a minimal parameterization or is it over parameterized?

These three questions are naturally handled by formulating the residual generation problem in the standard framework of polynomial matrices. The outcome of this is a new method, *the minimal polynomial basis approach*, presented in Section 3, which is an intuitive solution to the residual generator problem. With this approach, it is shown that the decoupling problem is transformed into finding a minimal basis for a null-space of a polynomial matrix.

It will be shown that the minimal polynomial basis approach can find all possible residual generators, including the ones of minimal McMillan degree, and the solution has a minimal parameterization. In Section 5 it will be shown that some other well known methods do not have all these properties. A clear advantage with the minimal polynomial basis approach is also that design tools (Henrion *et al.*, 1997) are already available. This is since it is based on established linear systems theory. The algorithms that are used are well studied and have good numerical properties.

2. POLYNOMIAL BASES AND SPACES

This paper relies on established theory on polynomial matrices, polynomial/rational vector spaces, and polynomial bases for these spaces (Kailath, 1980; Forney, 1975; Chen, 1984). The main notions used, are presented in this section.

The row-degree of a row vector of polynomials is defined as the largest polynomial degree in the row-vector. In this paper, *polynomial bases* and *orders* of polynomial bases are of special interest. A basis is here represented by a polynomial matrix where the rows are the basis vectors. The order of a polynomial basis is defined in (Kailath, 1980) as

Definition 1. (Order of a polynomial basis). Let the rows of $F(s)$ form a basis for a vector space $\mathcal{F}$. Let μ_i be the row-degrees of $F(s)$. The order of $F(s)$ is defined as $\sum \mu_i$.

A *minimal polynomial basis* for $\mathcal{F}$ is then any basis that minimizes this order.

A property of a minimal polynomial basis is that it is *irreducible* (Theorem 6.5-10 in (Kailath, 1980)). A matrix $F(s)$ is irreducible if and only if $F(s)$ has full rank for all s. An irreducible basis has a nice property that is described in the following theorem:

Theorem 1. ((Kailath, 1980) p.401). If the rows of $F(s)$ is an irreducible polynomial basis for a space $\mathcal{F}$, then all polynomial row vectors $f(s) \in \mathcal{F}$ can be written $f(s) = \phi(s)F(s)$ where $\phi(s)$ is a polynomial row vector.

3. THE MINIMAL POLYNOMIAL BASIS APPROACH

This section introduces the minimal polynomial basis approach to the design of linear residual generators. The approach does not adopt an observer view, e.g. like the unknown input observer and eigenstructure assignment design methods. This is because the primary issues of this paper is to handle minimality and completeness of solution which are more easily addressed in a polynomial basis framework. All derivations are performed in the continuous case but the corresponding results for the time-discrete case can be obtained by substituting s by z and *improper* by *non-causal*.

3.1 *Problem Formulation*

The systems studied in this work are assumed to be on the form

$$y = G(s)u + H(s)d + L(s)f \tag{2}$$

where $y(t)$ is measurements, $u(t)$ is known inputs to the system, $d(t)$ is unknown disturbances including the set of faults we wish to decouple, and $f(t)$ is the rest of the faults. The filter $Q(s)$ in (1) is a residual generator if and only if $r(t) = 0$ for all $d(t)$ and $u(t)$ when $f(t) = 0$. To be able to detect faults, it is also required that $r(t) \neq 0$ when $f(t) \neq 0$.

Inserting (2) into (1) gives

$$r = Q(s) \begin{bmatrix} G(s) & H(s) \\ I & 0 \end{bmatrix} \begin{bmatrix} u \\ d \end{bmatrix} + Q(s) \begin{bmatrix} L(s) \\ 0 \end{bmatrix} f$$

To make $r(t) = 0$ when $f(t) = 0$, it is required that disturbances and the control signal are *decoupled*, i.e. for $Q(s)$ to be a residual generator, it must hold that

$$Q(s) \begin{bmatrix} G(s) & H(s) \\ I & 0 \end{bmatrix} = 0$$

This implies that $Q(s)$ must belong to the left null-space of

$$M(s) = \begin{bmatrix} G(s) & H(s) \\ I & 0 \end{bmatrix} \tag{3}$$

This null-space is denoted $\mathcal{N}_L(M(s))$. The matrix $Q(s)$ need to fulfill two requirements: belong to the left null-space of $M(s)$ *and* have good fault sensitivity properties. If, in a first step of the design, *all* $Q(s)$ that fulfills the first requirement is found, then a $Q(s)$ with good fault sensitivity properties can be selected. Thus, in a first step of the design of the residual generator $Q(s)$ we need not consider f or $L(s)$. The problem is then to find *all* rational $Q(s) \in \mathcal{N}_L(M(s))$. Of special interest are the residual generators with least McMillan degree, i.e. the number of states in a minimal realization.

This can be done by finding a minimal basis for the rational vector-space $\mathcal{N}_L(M(s))$. A minimal basis for a rational vector-space is a *polynomial* basis (Forney, 1975). In Section 4, a *computationally simple*, *efficient*, and *numerically stable* method, to find a *polynomial* basis for the left null-space of $M(s)$ is presented. The obtained basis is denoted $N_M(s)$. It is noteworthy that, by inspection of (3), it holds that the dimension (number of rows) of $N_M(s)$ is less than or equal to m (with equality when there are no disturbances).

3.2 *Forming the Residual Generator*

When a polynomial basis $N_M(s)$ have been obtained, the second and final step in the residual generator design is to shape fault-to-residual responses as described next.

The minimal polynomial basis $N_M(s)$ is irreducible and then, according to Theorem 1, all decoupling *polynomial* vectors $F(s)$ can be parameterized as

$$F(s) = \phi(s)N_M(s) \tag{4}$$

where $\phi(s)$ is a polynomial vector of suitable dimensions. This parameterization vector $\phi(s)$ can e.g. be used to shape the fault-to-residual response or simply to select one row in $N_M(s)$. Since $N_M(s)$ is a basis, the parameterization vector $\phi(s)$ have minimal number of elements, i.e. a minimal parameterization.

When a decoupling polynomial vector $F(s)$ has been selected for implementation to form a residual generator, it must be made realizable since a polynomial vector is improper and thus not realizable. A realizable rational transfer function $Q(s)$, i.e. the residual generator, can be found as

$$Q(s) = d_F^{-1}(s)F(s)$$

where the polynomial $d_F(s)$ has greater or equal degree compared to the row-degree of $F(s)$. The degree constraint is the only constraint on $d_F(s)$. This means that the dynamics, i.e. poles, of the residual generator $Q(s)$ can be chosen freely. This also means that the minimal order of a realization of a decoupling filter is determined by the row-degrees of the *minimal* polynomial basis $N_M(s)$.

4. METHODS TO FIND A MINIMAL POLYNOMIAL BASIS TO $\mathcal{N}_L(M(s))$

The problem of finding a minimal polynomial basis to the left null-space of the rational matrix $M(s)$ can be solved by a transformation to a problem of finding a minimal polynomial basis to the left null space of a polynomial matrix. This transformation can be done in several different ways. In this section, two possibilities are demonstrated, where one is used if the model is given on transfer function form and the other if the model is given in state-space form. Also included is a description on how to compute a basis for the null-space of a polynomial matrix.

The motivation for this transformation to a polynomial problem, is that there exists well established theory (Kailath, 1980) regarding polynomial matrices. In addition, the generally available Polynomial Toolbox (Henrion *et al.*, 1997) for MATLAB contains an extensive set of tools for numerical handling of polynomial matrices.

4.1 *Frequency Domain Solution*

One way of transforming the rational problem to a polynomial problem is to perform a right MFD on $M(s)$, i.e.

$$M(s) = \widetilde{M}_1(s)\widetilde{D}^{-1}(s) \tag{5}$$

One simple example is

$$M(s) = \widetilde{M}_1(s)d^{-1}(s)$$

where $d(s)$ is the least common multiple of all denominators. By finding a polynomial basis for the left null-space of the *polynomial* matrix $\widetilde{M}_1(s)$, a basis is found also for the left null-space of $M(s)$. No solutions are missed because $\widetilde{D}(s)$ (e.g. $d(s)$) is of full normal rank. Thus the problem of finding a minimal polynomial basis to $\mathcal{N}_L(M(s))$ has been transformed into finding a minimal polynomial basis to $\mathcal{N}_L(\widetilde{M}_1(s))$.

4.2 *State-Space Solution*

Assume that the system is described in state-space form,

$$\dot{x}(t) = Ax(t) + B_u u(t) + B_d d(t) \tag{6a}$$

$$y(t) = Cx(t) + D_u u(t) + D_d d(t) \tag{6b}$$

where x is the n-dimensional state. Then it is convienient to use the *system matrix* in state-space form (Rosenbrock, 1970) to find the left null-space to $M(s)$. The system matrix has been used before in the context of fault diagnosis, see e.g. (Nikoukhah, 1994; Magni and Mouyon, 1994). Denote the system matrix $M_s(s)$, describing the system with disturbances as inputs:

$$M_s(s) = \begin{bmatrix} C & D_d \\ -(sI - A) & B_d \end{bmatrix}$$

Lets define the matrix P as

$$P = \begin{bmatrix} I & -D_u \\ 0 & -B_u \end{bmatrix}$$

Then the following theorem gives a direct method on how to find a minimal polynomial basis to $\mathcal{N}_L(M(s))$ via the system matrix.

Theorem 2. Let $V(s)$ be a minimal polynomial basis for $\mathcal{N}_L(M_s(s))$ and let the pair $\{A, [B_u\ B_d]\}$ be controllable. Then $W(s) = V(s)P$ is a minimal polynomial basis for $\mathcal{N}_L(M(s))$.

The proof of this theorem can be found in (Frisk and Nyberg, 1998). In conclusion, as in the previous section, the problem of finding a minimal polynomial basis to $\mathcal{N}_L(M(s))$ has been transformed into finding a minimal polynomial basis to a polynomial matrix, in this case the system matrix $M_s(s)$.

4.3 *No Disturbance Case*

If there are no disturbances, i.e. $H(s) = 0$, the matrix $M(s)$ gets a simpler structure

$$M_{nd}(s) = \begin{bmatrix} G(s) \\ I \end{bmatrix} \tag{7}$$

A minimal polynomial basis for the left null-space of $M_{nd}(s)$ is particularly simple due to the special structure and a minimal basis is then given directly by the following theorem:

Theorem 3. ((Kailath, 1980)). If $G(s)$ is a proper transfer matrix and $\bar{D}_G(s)$, $\bar{N}_G(s)$ form an irreducible left MFD, i.e. $\bar{N}_G(s)$ and $\bar{D}_G(s)$ are left co-prime and $G(s) = \bar{D}_G^{-1}(s)\bar{N}_G(s)$. Then,

$$N_M(s) = [\bar{D}_G(s) \ -\bar{N}_G(s)] \tag{8}$$

forms a minimal basis for the left null-space of the matrix

$$M(s) = \begin{bmatrix} G(s) \\ I \end{bmatrix}$$

Here, the *dimension* of the null-space is m, i.e. the number of measurements, and the *order* of the minimal basis is given by the following theorem:

Theorem 4. The set of observability indices of a transfer function $G(s)$ is equal to the set of row degrees of $\bar{D}_G(s)$ in any row-reduced irreducible left MFD $G(s) = \bar{D}_G^{-1}(s)\bar{N}_G(s)$.

A proof of the dual problem, controllability indices, can be found in (Chen, 1984) (p. 284).

Thus, a minimal polynomial basis for matrix $M_{nd}(s)$ is given by a left MFD of $G(s)$ and the order of the basis is the sum of the observability indices of $G(s)$.

The result (8) implies that finding the left null-space of the rational transfer matrix (3), in the general case with disturbances included, can be reduced to finding the left null-space of the rational matrix

$$\widetilde{M}_2(s) = \bar{D}_G(s)H(s) \tag{9}$$

In other words, this is an alternative to the use of the matrix $\widetilde{M}_1(s)$ in (5). This view closely connects with the so called frequency domain methods, which are further examined in Section 5.

4.4 *Finding a Minimal Polynomial Basis for the null-space of a General Polynomial Matrix*

For the general case, including disturbances, the only remaining problem is how to find a minimal polynomial basis to a polynomial matrix. This is a well-known problem in the general literature on linear systems. At least two different algorithms exists. The first is based on the *Hermite form* (Kailath, 1980) and a second algorithm is based on the *polynomial echelon form* (Kailath, 1980). Both methods are implemented in the Polynomial Toolbox (Henrion *et al.*, 1997) for MATLAB and a detailed description can be found in (Frisk and Nyberg, 1998).

The two algorithms have very different numerical properties. Although the algorithm based on Hermite form is easy to understand, it has poor numerical properties. However the algorithm based on polynomial echelon form is both fast and numerically stable and should therefore be the preferred choice.

5. RELATION TO OTHER RESIDUAL GENERATOR DESIGN METHODS

This section discusses the relation between the minimal polynomial basis approach and two other design methods for linear residual generation. Also the relation to the concept of parity functions, although not a design method, is covered. It is interesting to find that the questions of minimality and completeness of solution is not at all obvious for other design methods for residual generation.

5.1 *Parity Equations*

Several interpretations of the terms *parity equations* (or parity relations) and *parity functions* exist in the fault diagnosis literature. To clarify the meaning here, we use the terms *polynomial parity equation* and *polynomial parity functions*, which are the type of parity equations/functions defined in (Chow and Willsky, 1984).

The definition of polynomial parity functions becomes:

Definition 2. (Polynomial Parity Function). A polynomial parity function is a function $h(u(t), y(t))$ that can be written as

$$h(u(s), y(s)) = A(s)y(s) + B(s)u(s)$$

where $A(s)$ and $B(s)$ are polynomial vectors (or matrices if multidimensional parity functions are considered) in s. The value of the function is zero if no faults are present.

A *polynomial parity equation* is then basically a polynomial parity function set to zero, i.e. $h(u(s), y(s)) = 0$.

Remark: Parity equations that are not polynomial are often mentioned in the literature, e.g. ARMA parity equation (Gertler, 1991), dynamic parity relations (Gertler and Monajemy, 1995). In accordance with standard mathematical notion, these should be called *rational parity equations*. A *rational parity function* is then identical with a linear residual generator.

Parity equations/functions are in our view not a design method; it is solely an equation/function with specific properties. Nevertheless there is a strong relationship between minimal polynomial approach and polynomial parity functions. For any choice of $\phi(s)$ in (4), $F(s)$ will be a parity function. Thus the minimal polynomial basis approach to residual generator design can be seen as a design method for polynomial parity functions.

5.2 *The Chow-Willsky Scheme*

Another method for constructing polynomial parity functions was presented in (Chow and Willsky, 1984). This method is usually referred to as the Chow-Willsky scheme.

It has been shown in (Nyberg and Nielsen, 1997) that for some systems, the Chow-Willsky scheme can not generate all possible polynomial parity functions. This is the case when there is dynamics controllable from the faults but not from inputs or disturbances. This further implies that the Chow-Willsky scheme can not generate a polynomial basis for the left null-space of $M(s)$ defined in (3). However (Nyberg and Nielsen, 1997) presents a modified version of the Chow-Willsky scheme and it is shown that this modified version is able to generate all possible polynomial parity equations. This modified version is here referred to as the *universal* Chow-Willsky scheme. Now the question is if this universal Chow-Willsky scheme can generate a polynomial basis for $\mathcal{N}_L(M(s))$.

Since the Chow-Willsky scheme is well known, only a short description is given here. Designing polynomial parity functions with the universal Chow-Willsky scheme comes down to finding the null-space to a constant real matrix $[R_\rho\ H_\rho]$, where

$$R_\rho = [C^T\ A^TC^T\ \dots\ A^{\rho T}C^T]^T$$

and H_ρ is a lower triangular Toeplitz matrix describing the propagation of the disturbances through the system. Compared to the problem of finding the left null-space of $M(s)$ in (3), this is a much simpler problem since only constant matrices are involved. Let the matrix W be a basis for the left null-space of $[R_\rho\ H_\rho]$ and let $F_{CW}(s)$ denote the polynomial matrix $W\,[\Psi_y(s)\ \ -Q\Psi_u(s)]$, where

$$\Psi_y(s) = [I_m\ sI_m\ \dots\ s^\rho I]^T \quad \Psi_u(s) = [I_k\ sI_k\ \dots\ s^\rho I]^T$$

and Q is a lower triangular Toeplitz matrix describing the propagation of the inputs through the system.

To investigate if $F_{CW}(s)$ becomes a polynomial basis for the left null-space of $M(s)$, the following theorem (reformulated) from (Nyberg and Nielsen, 1997) is useful:

Theorem 5. Consider an $M(s)$ as in (3). For each vector $f(s) \in \mathcal{N}_L(M(s))$ and $\deg f(s) = \rho$ there is a vector w such that $f(s) = w\,[\Psi_y(s)\ \ -Q\Psi_u(s)]$ and $w\,[R_\rho\ H_\rho] = 0$.

Theorem 5 implies that for some ρ, $F_{CW}(s)$ will span the left null-space of $M(s)$. However, the number of rows of $F_{CW}(s)$ is in general larger than m, which is the maximal dimension of $\mathcal{N}_L(M(s))$ (see Section 3.1). The matrix $F_{CW}(s)$ is therefore not a basis for $\mathcal{N}_L(M(s))$; it represents an over parameterized solution. In conclusion, all this means that the Chow-Willsky scheme as stated in for example (Chow and Willsky, 1984) or (Nyberg and Nielsen, 1997), will not generate a minimal polynomial basis for $\mathcal{N}_L(M(s))$.

From a numerical perspective, the Chow-Willsky scheme is not as good as the minimal polynomial basis approach. The reason is that, for anything but small ρ, the matrix $[R_\rho\ H_\rho]$ will have high powers of A. It is likely that this results in that $[R_\rho\ H_\rho]$ becomes ill-conditioned. Thus to find the left null-space of $[R_\rho\ H_\rho]$ can imply severe numerical problems. The minimal polynomial basis approach does not have these problems of high power of A or any other term. This difference is highlighted in (Frisk, 1998), where both the Chow-Willsky scheme and the minimal polynomial basis approach are applied to the problem of designing polynomial parity functions for a turbo-jet aircraft-engine. The Chow-Willsky scheme fails because of numerical problems, while the minimal polynomial basis approach manage to generate a basis for all parity functions.

5.3 *Frequency Domain Approaches*

A number of design methods described in literature are called *frequency domain methods* where the residual generators are designed with the help of different transfer matrix factorization techniques. Examples are (Frank and Ding, 1994) for the general case with disturbances and (Ding and Frank, 1990; Viswanadham *et al.*, 1987) in the non-disturbance case. The methods can be summarized as methods where the residual generator is parameterized as

$$\begin{aligned} r &= R(s)[\tilde{D}(s)\ \ -\tilde{N}(s)]\begin{pmatrix} y \\ u \end{pmatrix} \\ &= R(s)(\tilde{D}(s)y - \tilde{N}(s)u) \end{aligned} \tag{10}$$

where $\tilde{D}(s)$ and $\tilde{N}(s)$ form a left co-prime factorization of $G(s)$ over $\mathcal{RH}_\infty$, i.e. the space of stable real-rational transfer matrices. Note the close relationship with Equation (8) where the factorization is performed over polynomial matrices instead of over $\mathcal{RH}_\infty$.

Inserting (2) into Equation (10) and as before assuming $f = 0$, gives

$$r = R(s)\tilde{D}(s)H(s)d$$

Therefore to achieve disturbance decoupling, the parameterization transfer matrix $R(s)$, must be belong to the left null-space of $\tilde{D}(s)H(s)$, i.e.

$$R(s)\tilde{D}(s)H(s) = 0$$

Here, note the close connection with $\widetilde{M}_2(s)$ in (9). This solution however does not generally generate a residual generator with minimal McMillan degree. In (Ding and Frank, 1990) and (Frank and Ding, 1994), the co-prime factorization is performed via a minimal state-space realization of the complete system, including the disturbances as in Equation (6). This results in $\tilde{D}(s)$ and $\tilde{N}(s)$ of McMillan degree n that, in the general case, is larger than the lowest possible McMillan degree of a disturbance decoupling residual generator. Thus, to find a basis of lowest order that spans all residual generators $Q(s) = R(s)[\tilde{D}(s) \ - \tilde{N}(s)]$, extra care is required since "excess" states need to be canceled. Note that the polynomial basis approach on the other hand, has no need for cancelations and is in this sense more elegant.

6. CONCLUSIONS

Design of residual generators to achieve perfect decoupling in linear systems is considered. The goal has been to develop a design method and four issues have been addressed, namely that the method (1) is able to generate *all* possible residual generators, (2) explicitly gives the solutions with minimal McMillan degree, (3) results in a minimal parameterization of the solutions, i.e. all residual generators, and (4) has good numerical properties.

The residual generator design problem is formulated with standard notions from linear algebra and linear systems theory such as polynomial bases for rational vector spaces and it is shown that the design problem can be seen as the problem of finding *polynomial* matrices in the left null-space of a rational matrix $M(s)$. Within this framework, the completeness of solution, i.e. issue (1) above, and minimality, i.e. issues (2) and (3), are naturally handled by the concept of *minimal polynomial bases*.

Finding a minimal polynomial basis for a null-space is a well-known problem and there exists computationally simple, efficient, and numerically stable algorithms, i.e. issue (4), to generate the bases. In addition, generally available implementations of these algorithms exists.

The question of minimality and completeness of solution is not obvious for other design methods. Relations to two well known methods are discussed and it is shown that they generally do not generate minimal solutions, i.e. it is not possible to generate minimal polynomial bases with these methods.

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