A generalized fault isolability matrix for improved fault diagnosability analysis

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***Abstract*— A generalized fault isolability matrix is proposed for quantitative analysis of fault isolability properties. The original fault isolability matrix gives information about which faults that are isolable from each other. However, other relavant isolability properties are not visible which can be important, for example, information regarding alternative fault hypotheses and multiple-fault isolability. The result of the analysis can be presented in the same compact form as the existing fault isolability matrix which makes it simple to visualize. As a case study, a model of an internal combustion engine is analyzed and two different solutions to the test selection problem are compared.**

I. INTRODUCTION

In model-based diagnosis, two common measures to evaluate diagnosability properties of the model of a system to be monitored, are fault detectability and isolability. Single-fault isolability analysis can be presented compactly using a fault isolability matrix. For systems with few candidate faults, this analysis can be extended to multiple-fault isolability by extending the number of rows and columns in the fault signature matrix. However, presenting multiple-fault isolability properties in a compact form is a non-trivial problem since, for example, the number of multiple-faults grows exponentially with number of faults.

Analyzing multiple-fault isolability has been considered in, for example, [4] and [7]. In these previous works, multiple-fault isolability properties are presented using a graph-based structure called a lattice. The lattice representation gives full information about multiple-fault isolability. However, depending on the system properties the size of the lattice grows fast with increasing number of candidate faults. With respect to these previous works, a quantitative fault isolability analysis of structural models is proposed which is not as detailed as the results in previous works but can be represented in a more compact form which does not suffer from complexity issues.

The general diagnosis system architecture considered in this work uses a set of residuals to detect different faults. Based on which residuals have triggered, a fault isolation algorithm computes diagnosis candidates that represent different sets of faults that can explain the triggered residuals. In the ideal case, each residual should be sensitive to only one fault. Then, fault detection and isolation of both single-faults and multiple-faults would be trivial. However, in many applications each residual will be sensitive to more than one fault. This means that some type of fault isolation logic is necessary [1]. Therefore, analyzing both single-fault and multiple-fault isolability properties is an important tool when designing the diagnosis system, for example, when performing sensor selection [8] or test selection [9].

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TABLE I

FAULT SIGNATURE MATRIX OF THE SOLUTION SET SELECTED USING THE REFERENCE TEST SELECTION ALGORITHM.

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}
r_1			X				X		X	X	X
r_2			X	X		X		X	X	X	X
r_3		X		X	X		X		X	X	X
r_4	X	X	X		X	X	X	X		X	X
r_5		X		X	X	X		X			X
r_6	X	X	X	X			X	X	X		
r_7	X			X	X	X			X	X	

TABLE II

FAULT SIGNATURE MATRIX OF THE SOLUTION SET SELECTED USING THE PROPOSED TEST SELECTION ALGORITHM.

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}
r_1			X				X		X	X	X
r_2	X							X			
r_3			X		X			X	X	X	X
r_4			X	X	X	X		X		X	
r_5		X		X	X	X			X		X
r_6		X	X			X	X	X			X
r_7	X	X		X			X		X	X	

Residual selection is an important step in the diagnosis system design to fulfill fault detectability and isolability requirements. Even for systems with low redundancy, the total number of candidate residuals can be too large to implement all of them in the diagnosis system. Thus, the goal is to find a minimal set of residuals that fulfills a set of fault isolability requirements. However, this is an NP-complete problem and different heuristic search strategies have been proposed, see for example, [6] and [9].

Consider the two fault signature matrices presented in Table I and Table II, which will also be considered later in the case study. An X at position (i, j) in the fault signature matrix corresponds to that residual r_i is sensitive to fault f_j. A fault isolability matrix for each of the sets will show that all single faults are isolable from each other, meaning that single-fault isolability of the two sets are equivalent. However, the residuals in Table II are generally sensitive to fewer faults, i.e., the matrix is more sparse, meaning that more fault hypotheses can be rejected with fewer triggered residuals. This type of quantitative comparison is not possible using the existing fault isolability matrix.

TABLE III

TWO EXAMPLES OF FAULT SIGNATURE MATRICES WITH THE SAME SINGLE-FAULT ISOLABILITY.

(a) Residual set 1

	f_1	f_2	f_3	f_4
r_1	X	X	X	
r_2	X	X		X
r_3	X		X	X
r_4		X	X	X

(b) Residual set 2

	f_1	f_2	f_3	f_4
r_1	X			
r_2		X		
r_3			X	
r_4				X

The main contribution is a quantitative method for fault isolability analysis given a structural model of the system, or a set of residuals. The result is presented as a generalized version of the existing fault isolability matrix. The proposed method gives more detailed information regarding fault isolability accuracy, and also multiple-fault isolability, by analyzing properties of all diagnosis candidates that can explain a faulty scenario. The second contribution is a test selection strategy which tries to minimize the solution set and at the same time make the fault signature matrix sparse.

II. AN INTRODUCTORY EXAMPLE

Before stating the problem, the following example is used as a motivation. A small system is analyzed which can be affected by four possible faults, f_1, f_2, f_3, and f_4. Two different sets of residuals have been selected and the corresponding two fault signature matrices are shown in Table III. The two sets of residuals represent two extreme cases when all single-faults are isolable, i.e., when each residual in the set is either sensitive to all but one fault or only one fault. The single-fault isolability performances are equal for the two residual sets which can be presented using an fault isolability matrix as shown in Table IV. An X at position (i, j) in the fault isolability matrix represents that fault f_i is not isolable from fault f_j. If all faults are isolable from each other, there will only be Xs in the diagonal.

In residual set 1, only one fault is rejected by each residual. This means that little information is given about which fault has occurred when a residual triggers since most faults can explain the triggered residual. Using residual set 1, it is necessary that three of the residuals trigger before the true fault can be located. Residual set 2 in this sense is optimal since there is only one single-fault that can explain that a residual has triggered, meaning that fault isolation is trivial.

If two or more faults occur in the system, residual set 1 will not be able to isolate the faults since all residuals would trigger, given any pair of faults. It is only possible to isolate multiple-faults using residual set 2 since each fault of faults will trigger a unique set of residuals. Also, from a multiple-fault isolation perspective, the right fault signature matrix is optimal since any combination of faults can be isolated. However, this type of information regarding multiple-fault isolability cannot be obtained from the fault isolability analysis in Table IV.

TABLE IV

SINGLE-FAULT ISOLABILITY MATRIX FOR EACH OF THE TWO SETS OF RESIDUALS DESCRIBED IN TABLE III.

	f_1	f_2	f_3	f_4
f_1	X			
f_2		X		
f_3			X	
f_4				X

III. PROBLEM STATEMENT

The main problem is how to visualize the type of multiple-fault isolability properties that were discussed in the previous section. The analysis should be applicable, either for a given set of residuals or for a given model, and the result should be represented in a compact form that is easy to visualize. The graph-based methods in [4] and [7] can visualize all multiple-fault isolability properties. However, for a system with many faults, these methods will suffer of complexity issues due to that the number of fault combinations grows exponentially with number of faults. Therefore, a compact representation of multiple-fault isolability properties is still interesting even though the information is conservative, since it gives more information than, for example, the existing single-fault isolability analysis. As an application, a test selection algorithm has been developed and the proposed fault isolability analysis method is used to compare the solution of the proposed algorithm with an existing test selection algorithm.

Here, the consistency-based fault isolation algorithm presented in [2] is used to compute diagnosis candidates. It is assumed that residuals are designed with negligible false-alarm rate but a positive missed-detection rate. This means that only triggered residuals are taken into consideration in the fault isolation logic, i.e., the exoneration assumption is not valid [3].

IV. BACKGROUND

Let $\mathcal{F} = \{f_1, f_2, \ldots, f_{n_f}\}$ be a set of faults that can occur in the system. To describe the system state, the term *fault mode* is used. A fault mode F is used to denote a subset of faults $F \subseteq \mathcal{F}$ present in the system, i.e., a fault mode can represent one or several faults but also the fault-free case NF (No Fault). It is assumed that there exists a set of n_r residual candidates, $\mathcal{R} = \{r_1, r_2, \ldots, r_{n_r}\}$, where each residual r_i is sensitive to a subset of faults $\mathcal{F}(r_i) \subseteq \mathcal{F}$, denoted the *test support* of r_i [4].

A diagnosis candidate d_k denotes a set of faults $d_k \subseteq \mathcal{F}$ which can explain a set of triggered residuals. Let $D = \{d_1, d_2, \ldots, d_{n_d}\}$ denote a set of feasible diagnosis candidates. Some properties of the fault isolation algorithm [2] and computed diagnosis candidates are summarized as follows. Assume that d_k is a diagnosis candidate. Then, if a new residual r_l triggers:

1) d_k is still a diagnosis candidate if $\exists f_i \in d_k$ such that $f_i \in \mathcal{F}(r_l)$.
2) if d_k is no longer a diagnosis candidate, then all $d_k \cup \{f_i\}$ for each $f_i \in \mathcal{F}(r_l)$ are diagnosis candidates.

3) if no subset $d_l \subset d_k$ is a diagnosis candidate then d_k is a minimal diagnosis candidate.

The fault isolation algorithm described in [2] computes all minimal diagnosis candidates. Note that the set of all minimal diagnosis candidates represents all diagnosis candidates since all supersets of faults are also diagnosis candidates. The list describes the principle of how the fault isolation algorithm updates the set of minimal diagnosis candidates when a new residual triggers. Note that the fault isolation algorithm works sequentially and updates the set of diagnosis candidates when new residuals trigger.

For multiple-fault isolability analysis, the following definition of isolability between two fault modes is used.

Definition 1: A fault mode F_i is isolable from another fault mode F_j if it is possible to generate a residual r sensitive to at least one of the faults in F_i but no faults in F_j.

The cardinality of a set S is denoted $|S|$ and is equal to the number of elements in the set. Then, the following definition is used to define if a fault mode is isolable from all other fault modes of same cardinality, i.e., the number of faults in the fault mode.

Definition 2: A fault mode F is *uniquely isolable* if the true diagnosis candidate, i.e., the diagnosis candidate $d_F = F$, has lower cardinality compared to all other diagnosis candidates, i.e.,

$$|d| > |d_F|, \forall d \in D \setminus d_F. \quad (1)$$

V. A GENERALIZED FAULT ISOLABILITY ANALYSIS

A generalization of the single-fault isolability matrix is proposed here by analyzing the cardinality of computed minimal diagnoses. To motivate the analysis method the following property of minimal diagnosis candidates is utilized.

Proposition 1: Let D_{old} denote a set of minimal diagnosis candidates and D_{new} is the new set after a test has triggered. Then, for each $d_k \in D_{\text{new}}$ there exists a $d_l \in D_{\text{old}}$ such that $d_l \subseteq d_k$.

The proposition follows from the properties of the consistency-based fault isolation algorithm summarized in Section IV.

Consider the single-fault case f_i and the computed minimal diagnoses given that all residuals sensitive to f_i have triggered. If the single fault f_i is uniquely isolable all minimal diagnosis candidates except the true diagnosis candidate, $\{f_j\}$, will have cardinality two or higher. If there is another minimal diagnosis candidate of the same cardinality, the true fault mode is not uniquely isolable. This means that all other faults will either be in a minimal diagnosis candidate of higher cardinality than the true diagnosis or not in any minimal diagnosis candidate.

Consider the double-faults $\{f_i, f_j\}$. Using Proposition 1, if the minimal cardinality of all minimal diagnosis candidate except the true one has cardinality three or higher for each single-fault, f_i and f_j, the double-fault mode $F = \{f_i, f_j\}$ is also uniquely isolable. The same is true for multiple-faults of higher cardinality and this result is summarized in the following proposition.

TABLE V

COMPUTED MINIMAL DIAGNOSES WHEN EACH SINGLE-FAULT OCCURS GIVEN THE TWO EXAMPLE RESIDUAL SETS IN TABLE III.

fault	residual set 1	residual set 2
f_1	$\{f_1\}, \{f_2, f_3\}, \{f_2, f_4\}, \{f_3, f_4\}$	$\{f_1\}$
f_2	$\{f_2\}, \{f_1, f_3\}, \{f_1, f_4\}, \{f_3, f_4\}$	$\{f_2\}$
f_3	$\{f_3\}, \{f_1, f_2\}, \{f_1, f_4\}, \{f_2, f_4\}$	$\{f_3\}$
f_4	$\{f_4\}, \{f_1, f_2\}, \{f_1, f_3\}, \{f_2, f_3\}$	$\{f_4\}$

TABLE VI

TWO EXAMPLES OF FAULT SIGNATURE MATRICIES WITH THE SAME SINGLE-FAULT ISOLABILITY.

	f_1	f_2	f_3	f_4
f_1	1	2	2	2
f_2	2	1	2	2
f_3	2	2	1	2
f_4	2	2	2	1

	f_1	f_2	f_3	f_4
f_1	1			
f_2		1		
f_3			1	
f_4				1

Proposition 2: Consider a fault mode $F = \{f_1, \ldots, f_k\}$ of cardinality k and let D_i denote all minimal diagnosis candidates given the single fault f_i. Then, fault mode F is uniquely isolable if the cardinality of all $d \in D_i \setminus \{f_i\}$ are greater than k, for all $f_i \in F$.

The proof of Proposition 2 follows from Proposition 1. One useful result of Proposition 2 is that analyzing the single-fault case also gives information about multiple-fault isolability properties. This information can be summarized in an $\mathbb{R}^{n_f \times n_f}$ matrix where the number at position (i, j) is the minimal cardinality of any minimal diagnosis candidate including f_j when all residuals sensitive to f_i have triggered. An empty position (i, j) corresponds to that fault f_j is not included in any minimal diagnosis candidate.

Again, consider the introductory example and the two sets of residuals in Table III. In the case when all residuals are sensitive to all faults but one, the corresponding computed minimal diagnoses for each single-fault in the ideal case, i.e., when all residuals sensitive to the fault trigger, are shown in Table V. If two or more faults occur, residual set 1 will return minimal diagnosis candidates including all pair of faults. For the case of residual set 2, fault isolation is trivial and the only minimal diagnosis candidate the true diagnosis candidate.

The results from the fault isolability analysis of the two residual sets are shown in Table VI. All single-faults are uniquely isolable since there are only ones in the diagonal. However, when considering double-faults, the results of the two residual sets become different. All non-diagonal positions have the value two given residual set 1 and are empty given residual set 2. This shows that residual set 1 cannot uniquely isolate double-faults, nor multiple-faults of higher cardinality. Residual set 1 can isolate multiple-faults of any cardinality which is visible in Table VI since all non-diagonal elements are empty.

As a third example, consider a new set of residuals with fault signature matrix in Table VII. The first four residuals in the set are sufficient for single-fault isolability and the corresponding proposed generalized fault isolability matrix is shown to the left in Table VIII. The minimal diagnosis

TABLE VII

FAULT SIGNATURE MATRIX GIVEN RESIDUAL SET 3 WITH THE SAME SINGLE-FAULT ISOLABILITY AS THE PREVIOUS RESIDUAL SETS IN THE INTRODUCTORY EXAMPLES.

	f_1	f_2	f_3	f_4
r_1	X	X		
r_2	X		X	
r_3		X		X
r_4			X	X
r_5	X			X
r_6		X	X	

TABLE VIII

TWO EXAMPLES OF FAULT SIGNATURE MATRICES WITH THE SAME SINGLE-FAULT ISOLABILITY.

	f_1	f_2	f_3	f_4
f_1	1	2	2	
f_2	2	1		2
f_3	2		1	2
f_4		2	2	1

	f_1	f_2	f_3	f_4
f_1	1	3	3	3
f_2	3	1	3	3
f_3	3	3	1	3
f_4	3	3	3	1

candidates for each single-fault case are also shown in Table VII. Similar to the performance of residual set 1, it is only possible to uniquely isolate single-faults since the lowest non-diagonal number in each row is still two. However if two faults occurs, the number of minimal diagnosis candidates of cardinality two will be lower compared to using residual set 1 which is visible when comparing the number of minimal diagnosis candidates in Table IX with Table V.

If considering the whole residual set 3, i.e., all six residuals, the corresponding generalized fault isolability matrix is shown to the right in Table VIII. Compared to the left matrix, all non-diagonal positions have the value three which means that all fault modes up to, at least, double-faults can be uniquely isolated.

Some observations of the properties of the generalized fault isolability matrix can be summarized in the following bullets:

- A fault f_i is isolable from another fault f_j if the position (i, j) is empty or have a value greater than 1.
- A fault f_i is uniquely isolable if the positions (i, j), for all $f_j \in \mathcal{F}$ where $j \neq i$, are empty or have a value greater than 1.
- An empty position (i, j) means that f_j will not be part of any minimal diagnosis candidate if fault f_i occurs.
- A fault mode F is uniquely isolable if the positions (i, j), for all $f_i \in F$ and $i \neq j$, are empty or have values greater than $|F|$.

The examples show that the proposed generalized fault isolability matrix extends the single-fault isolability matrix and gives more information of fault isolability performance including multiple-fault isolability. Note that the analysis can be performed for a given model by analyzing the set of Minimal Test Equation Sets (MTES) generated from the model [4]. In short, an MTES represents the maximum number of over-determined equations that is sensitive to a minimal set of faults and can be used to generate residuals. An important property is that the set of MTES describes

TABLE IX

COMPUTED MINIMAL DIAGNOSES WHEN EACH SINGLE-FAULT OCCURS GIVEN WHOLE RESIDUAL SET 3 IN TABLE VII AND WHEN ONLY CONSIDERING THE FIRST FOUR RESIDUALS $\{r_1, r_2, r_3, r_4\}$.

fault	$\{r_1, r_2, r_3, r_4\}$	residual set 3
f_1	$\{f_1\}, \{f_2, f_3\}$	$\{f_1\}, \{f_2, f_3, f_4\}$
f_2	$\{f_2\}, \{f_1, f_4\}$	$\{f_2\}, \{f_1, f_3, f_4\}$
f_3	$\{f_3\}, \{f_1, f_4\}$	$\{f_3\}, \{f_1, f_2, f_4\}$
f_4	$\{f_4\}, \{f_2, f_3\}$	$\{f_4\}, \{f_1, f_2, f_3\}$

maximum multiple-fault isolability given by the model [4].

VI. TEST SELECTION

To illustrate how the proposed analysis method can be used, a test selection algorithm is developed where the solution is compared to the solution of an existing test selection strategy. Here, only single-fault isolability requirements are taken into consideration when formulating the test selection problem.

The goal is to find a minimal set of residuals $R \subseteq \mathcal{R}$ that fulfills a set of single-fault isolability requirements. For each single-fault isolability requirement, i.e., be able to isolate f_i from f_j, there is a subset of the candidate residuals that can fulfill that requirement denoted $\mathcal{I}_{i,j} \subseteq \mathcal{R}$. Thus, the test selection problem can be formulated as a minimal hitting set problem [9] as

$$\min_{R \subseteq \mathcal{R}} \quad |R| \qquad \text{s. t.} \quad R \cap \mathcal{I}_{i,j} \neq \emptyset, \forall f_i, f_j \in \mathcal{F} \tag{2}$$

Note that only ideal residual performance is considered here. However, quantitative detection performance of the residual candidates can be taken into consideration in the test selection problem by defining each set $\mathcal{I}_{i,j}$ such that all residuals $r_k \in \mathcal{I}_{i,j}$ fulfill some given quantitative performance requirement for isolating f_i from f_j.

Since the minimal hitting set problem (2) is NP-complete, there are several heuristic search strategies proposed to solve the test selection problem. Here, a modification is proposed of the greedy search algorithm presented in [9]. In [9], the greedy algorithm adds a new residual to the solution set R until all requirements are fulfilled. The residual selected in each step is the residual r^* which maximizes the number of new fulfilled isolability requirements, i.e.,

$$r^* = \arg \max_{r \in \mathcal{R}} \text{Number of new } \mathcal{I}_{i,j} \text{ fulfilled.} \tag{3}$$

To analyze the properties of this test selection strategy, note that the number of single-fault isolability requirements that can be fulfilled by a residual r sensitive to $m = |\mathcal{F}(r)|$ faults is $m(n_f - m)$. Maximizing with respect to m gives that

$$\frac{d}{dm} m(n_f - m) = n_f - 2m = 0 \Rightarrow m = \begin{cases} \frac{n_f}{2} & \text{if } n_f \text{ even} \\ \frac{n_f \pm 1}{2} & \text{if } n_f \text{ odd.} \end{cases} \tag{4}$$

This indicates that the solution found by [9] will not try to find a solution similar to the right fault signature matrix in Table III, in the general case. Residuals sensitive to more faults will have a higher utility since they will fulfill more isolability requirements. However, If some residuals require a longer time to detect the fault, and if each residual is sensitive to a large set of faults, it will take a long time before anything can be said about the detected fault.

To reduce the number of diagnosis candidates, and improved multiple-fault isolability, residuals should be sensitive to few faults. Then, when a residual triggers, only a few number of faults can explain the triggered residual, which will make it easier to draw some conclusions about the true fault even before it is uniquely isolated. By modifying the utility function (3), it is possible to prioritize residuals sensitive to few faults without significantly increasing the number of residuals in the solution set.

The proposed modification of the utility function is such that the residual selected in each step r^* should maximize the following utility function:

$$r^* = \arg\max_{r \in \mathcal{R}} \frac{\text{Number of new } \mathcal{I}_{i,j} \text{ fulfilled}}{n_f m(r)}. \tag{5}$$

where $m(r)$ is used to emphasize that m depends on r. The main difference between the proposed utility function and (3) is the normalization factor $n_f m(r)$. Since the number of single-fault isolability requirements that can be fulfilled by a residual sensitive to m faults is $m(n_f - m)$, the utility function can be written as

$$\frac{m(n_f - m)}{n_f m} = 1 - \frac{m}{n_f} \tag{6}$$

which is maximized when m is minimized. The modified utility function should result in a selected set of residuals with a more sparse fault signature matrix.

VII. Evaluation

To evaluate the proposed fault isolability analysis and the proposed test selection algorithm, a model of the air-flow in a internal combustion engine is used as a case study. The model of the engine contains 96 equations, 90 unknown variables, 12 known signals, 14 states, and 11 faults, including sensor faults, leakages and air filter clogging. The degree of redundancy of the system is six.

A. *fault isolability analysis of engine model*

The computed generalized fault isolability matrix of the engine model is presented in Table X. There are only ones in the diagonal which shows that all single-faults are uniquely isolable. All other positions in the table have values larger than or equal to four. This means that any set of multiple-faults up to at least cardinality three are uniquely isolable given the model. Note that these results are restrictive and there might be some fault modes with multiple-faults of cardinality four and larger that could be uniquely isolable. However, then a more detailed isolability analysis for each fault mode is necessary.

TABLE X

Multiple-fault isolability analysis of the IC engine model.

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}
f_1	1	5	6	5	5	5	6	5	6	6	6
f_2	5	1	5	5	5	5	5	5	5	5	5
f_3	6	5	1	5	4	4	4	6	4	5	5
f_4	5	5	5	1	5	5	5	5	5	5	5
f_5	5	5	4	5	1	4	4	5	4	5	5
f_6	5	5	4	5	4	1	4	5	4	5	5
f_7	6	5	4	5	4	4	1	6	4	5	5
f_8	5	5	6	5	5	5	6	1	6	6	6
f_9	6	5	4	5	4	4	4	6	1	5	5
f_{10}	6	5	5	5	5	5	5	6	5	1	5
f_{11}	6	5	5	5	5	5	5	6	5	5	1

B. *Fault isolability analysis of residual sets*

The proposed test selection algorithm is compared to the greedy search [9] which is used as a reference algorithm. A Monte Carlo analysis is performed where the order of the candidate residuals is permuted randomly to analyze the influence of the order on the solution. One set of residual candidates are used in the evaluation based on all minimally overdetermined sets of equations (MSO) given the model [5]. Each MSO can be used to design residual candidates which here result in 101925 residual candidates.

The solution from the reference algorithm is presented in Table I and the solution from the proposed algorithm in Table II. The solution of the reference algorithm is able to isolate all single-faults using seven test quantities. However, the fault signature matrix have in total 47 Xs, while the solution of the proposed algorithm have 37 Xs, i.e., a 20% reduction. The reduced number of Xs will have a positive impact on fault isolability performance which is visible when comparing the generalized fault isolability matrices.

The result from the analysis of the solutions of the reference algorithm is shown in Table XI and the proposed algorithm in Table XII, respectively. The solution found by the two algorithms fulfill the single-fault isolability requirements. However, the solution of the proposed test selection algorithm has some attractive fault isolability properties. Several positions in Table XII are empty and there are more positions with value three compared to Table XI. Also, note that the proposed test selection algorithm appears to find solutions with better multiple-fault isolability performance without taking multiple-fault isolability requirements into consideration.

C. *Robustness analysis of the test selection algorithm*

Since the solution of the proposed test selection algorithm might depend on the order of the set of residual candidates a robustness analysis of the solution is necessary. A Monte Carlo study is performed where the set of residual candidates is permuted in random order. In the Monte Carlo analysis, the total number of Xs in the faults signature matrix and the number of residuals in the solution are evaluated. The results of the analysis are presented in Fig. 1 and Fig. 2 showing that the proposed method in average gives a more sparse fault signature matrix even though the solution set more often contains an extra residual.

TABLE XI

GENERALIZED FAULT ISOLABILITY ANALYSIS BASED ON THE SOLUTION OF THE REFERENCE TEST SELECTION ALGORITHM.

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}
f_1	1	2	2	2	2	2	2	2	2	2	2
f_2	2	1	2	2	2	2	2	2	2	2	2
f_3	2	2	1	2	2	2	2	2	2	2	2
f_4	2	2	2	1	2	2	2	2	2	2	2
f_5	2	2	2	2	1	2	2	2	2	2	2
f_6	2	2	2	2	2	1	2	2	2	2	2
f_7	2	2	2	2	2	2	1	2	2	2	2
f_8	2	2	2	2	2	2	2	1	2	2	2
f_9	2	2	2	2	2	2	2	2	1	2	2
f_{10}	2	2	2	2	2	2	2	2	2	1	2
f_{11}	3	2	2	2	2	2	2	2	2	2	1

TABLE XII

GENERALIZED FAULT ISOLABILITY ANALYSIS BASED ON THE SOLUTION OF THE PROPOSED TEST SELECTION ALGORITHM.

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}
f_1	1	2		2			2	2	2	2	
f_2	2	1	2	2	2	2	2	2	2	2	2
f_3		2	1	2	2	2	2	2	2	2	2
f_4	2	2	2	1	2	2		2	2	2	2
f_5		2	2	2	1	2		2	2	2	2
f_6		2	2	2	2	1	2	2	2	2	2
f_7	2	2	2	2		2	1		2	2	2
f_8	2	3	2	3	3	3	3	1	3	3	3
f_9	2	2	2	2	2	2	2	3	1	2	2
f_{10}	2	2	2	2	2	2	2	2	2	1	2
f_{11}		2	2	2	2	3	2	2	2	2	1

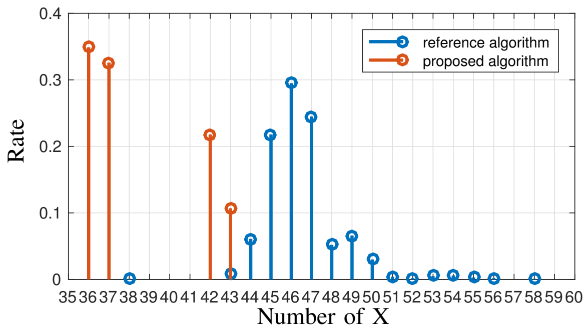

Fig. 1. Monte Carlo evaluation of the number of Xs in the fault signature matrix for the two test selection algorithms.

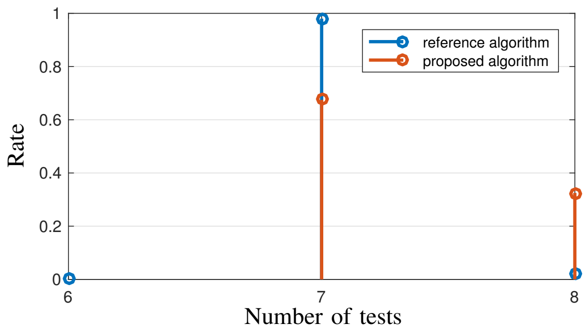

Fig. 2. Monte Carlo evaluation of how many residuals selected by the two test selection algorithms.

VIII. CONCLUSIONS

The proposed fault isolability analysis method gives a compact overview of some multiple-fault isolability properties given a system or a set of residuals. The results can be represented as a generalization of the fault isolability matrix. Further analysis is necessary to better understand the properties of the generalized fault isolability matrix. The generalized fault isolability matrix can be used for a more quantitative comparison of isolability performance between, for example, different sets of sensors or residuals, compared to the standard fault isolation matrix. The proposed greedy test selection strategy tries to find a minimal residual set which also has a sparse fault signature matrix. Analysis of the case study illustrates the advantages of the proposed test selection method regarding reducing the number of computed diagnosis candidates and improved multiple-fault isolability. The case study also shows that the proposed methods can be applied in combination with structural methods to analyze complex models which also makes it applicable early in the diagnosis system design process.

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