Estimation of Fuel Equivalence Factor from a Wheel Loaders Driving Cycle*

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Abstract—For the case with repetitive driving cycles for a wheel loader the driving cycles parameters affects on the optimal control of the wheel loader is studied. It is clearly seen that average efficiency is not enough and also incorporating driving cycles parameter such as average power, maximum power and amount of recuperative energy will lead to an better estimate on the fuel equivalence factor W.

I. INTRODUCTION

For a hybrid vehicle, it is well known that optimal control of the energy management system depends on the driving cycle [1], [2], [3], [4], where the driving cycle is a speed profile over time. In this paper the power demand of a wheel loader is studied. The driving cycle of a wheel loader is often repetitive in its nature, normally it contains transport, filling the bucket with material and emptying the bucket on for example a hauler. Therefore it is investigated how repetitive power demands and different driving cycle parameters affect the optimal control and how to design an equivalent consumption minimization strategy, ECMS, from a given repetitive power demand. Using Dynamic Programming [5], DP, it is possible to calculate the fuel equivalence factor for the ECMS [6]. However, the DP is non-causal and cannot be used online unless the future driving cycle is known in advance, which rarely is the case. Thus a method that estimates the optimal control given historical data is needed. Repetitive cycles gives possibility for easier parameterization of the driving cycle and a modeling of lambda with dependency of fewer parameters.

II. MODELING

The modeling consists of two parts. The first part is the component modeling where the different components such as engine, electric machine and energy storage system are modeled and explained in more detail. The second part is the modeling of test cycles that will be used to examine the driving cycle parameters affect on the optimal control. The test cycles are modeled in such a way it resembles normal operations for a wheel loader. The setup is that of a simplified series hybrid which can be seen in Fig. 1. The input to the system is an electrical load. The electrical power need to be matched by the energy storage system (ESS) and

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the electric machine (generator/motor) (EM). The electric machine is coupled to an internal combustion engine (ICE). The components will be explained in more detail in the next section.



Fig. 1. Overview of the simplified series hybrid with an internal combustion engine (ICE), an electric machine (EM), an energy storage system (ESS) and also the electrical load

A. Component Modeling

The internal combustion engine, ICE, that is used in this paper is a diesel engine with a maximum power of 250 kW. The ICE is modeled as a Willan's model [7] and it is similar to the engine model that has been used in [8]. The output power from the ICE, P_{ice} , is a function of efficiency, fuel power, P_{fuel} , and losses which are dependent on the angular velocity of the ICE according to (1a)

$$P_{ICE} = \eta_{ICE}(T, \omega)P_{fuel} + P_{ICE,loss}(\omega)$$
(1a)

$$P_{EM} = \eta_{EM}(T, \omega)P_{ICE} + P_{EM,loss}(\omega)$$
(1b)

The electric machine, EM, have also been modeled as a Willan's model. The size of the EM is adapted to the ICE so that the EM does not limit the output power from the ICE-EM-path. The efficiency of the ICE and EM depends on the operating point, torque and rotational speed, of the axle between the ICE and EM (1), the efficiency for different operating points can be seen in Fig. 5. The energy storage system, ESS, is in this paper a battery and is modeled as an internal resistance model [4]

$$U_{oc} - IR = U \tag{2a}$$

$$U_{oc} = \kappa_2 q(t) + \kappa_1 \tag{2b}$$

$$q(t) = \frac{SOE}{ESS_{max}} \tag{2c}$$

$$P_{bat} = UI \tag{2d}$$

where U_{oc} is the open circuit voltage of the battery, I is the current drained or fed to the battery (depending on the direction of power in the battery), R is the internal resistance which cause ohmic losses when the battery is used, U is the voltage of the battery, q(t) is the state of charge, *SOC*, which is a percentage value of the total energy that the battery is currently holding, κ_1 and κ_2 can be used if one want to model that U_{oc} is depending on the SOC. In this paper $\kappa_2 = 0$ and κ_1 is the open circuit voltage. State of charge and state of energy, *SOE*, are closely related by (2c). The battery input or output power P_{bat} (depending on charging or discharging of the battery) is (2d).

B. Test Cycle Modeling

Usually a test cycle or a driving cycle is a time series of velocities that the vehicle has to track. The forces on the wheels that the vehicle has to overcome origins from rolling resistance losses, aerodynamic friction losses, inertial forces, uphill/downhill driving force and disturbances which are all the effects which have not been included in the first terms and affects the vehicle [4]. In this paper a series hybrid is analyzed and the input to the system according to Fig. 1 is an electrical load. The test cycle is therefore consisting of a power cycle which is a time series of power, P_{dem} , that the vehicle has to overcome by the two energy paths $(P_{bat} + P_{EM} = P_{dem})$. The power cycles are based on real measurements of a wheel loader to capture the essential properties of the power demand. To test how different driving cycle parameters affect the optimal control of the wheel loader an idealized test cycle is modeled. The test cycle can be seen in Fig. 2.



Fig. 2. Approximation of the power demand of a wheel loader during transport, bucket filling and bucket emptying. The tuning parameters for the test cycles are P_{max} , P_{min} , P_{avg} and T where the midlevel is calculated as $P_{mid} = \frac{4P_{avg} - P_{max} - P_{min}}{2}$

The motivation of the different levels of power demand correspond to the normal usage of a wheel loader, for example transport, bucket filling and bucket emptying. The levels can be adjusted to achieve different average power, peak power and minimum power levels in order to resemble different operations for example working in different materials such as sand, gravel, or shot rock. The parameters for the test cycle is average power P_{avg} , maximum power P_{max} corresponding to the upper level in the figure, minuium power P_{min} corresponding to the lower level and finally the length, T, of each of the levels in Fig. 2. Hence the cycle time is 4T and the test cycle consists of repetition of these to a length of 360 seconds. The midlevel, P_{mid} , is adjusted so that the average power P_{avg} is obtained $(P_{mid} = \frac{4P_{avg} - P_{max} - P_{min}}{2})$ for any changes in maximum or minimum power. This will be used when the test cases below are constructed to see how the cycle parameters affects the optimal control of the vehicle.

Four different variations of the test cycle in Fig. 2 will be used in this paper.

1) Test case 1: Varying average power P_{avg} for a test cycle with constant power $(P_{avg} = P_{max} = P_{min} = P_{mid})$.

2) Test case 2: Varying the maximum power, P_{max} , while holding the rest of the parameters constant ($P_{min} = -40kW$, $P_{avg} = 40kW$, T = 6s).

3) Test case 3: Varying the average power, P_{avg} , while the rest of the parameters are set to constant values ($P_{max} = 120kW$, $P_{min} = -40kW$, T = 6s).

4) Test case 4: Varying the cycle time or frequency by changing the level time T. The other parameters are constants $(P_{max} = 120kW, P_{min} = -40kW, P_{min} = 40kW)$.

III. OPTIMIZATION PROBLEM

The optimization criterion in this paper is the fuel power of the wheel loader which is strongly related to the fuel consumption. Given that the control signal u affects the power split between the ICE-EM-path and the ESS-path, how should the wheel loader be operated to get the lowest fuel consumption. This is of course with some constraints which are related to the physical aspects of the system.

The minimization problem with constraints for a test cycle of length t_f is

$$\underset{u}{\operatorname{argmin}} J = \int_{0}^{t_{f}} P_{fuel}(T, \omega) \ dt \tag{3a}$$

$$\frac{dSOE}{dt} = P_{ESS} \tag{3b}$$

$$P_{bat} + P_{EM} = P_{dem} \tag{3c}$$

$$T_{min}(T, \omega) \le T \le T_{max}(T, \omega)$$
 (3d)

$$\omega_{\min}(T,\omega) \le \omega \le \omega_{\max}(T,\omega)$$
 (3e)

$$I_{min} \le I \le I_{max} \tag{3f}$$

$$SOE_{min} \le SOE \le SOE_{max}$$
 (3g)

$$SOE(t=0) = SOE(t=t_f)$$
(3h)

where J is the cost function or fuel energy consumed during the test cycle, P_{fuel} is the fuel power that the ICE uses at each instant. P_{bat} and P_{EM} is the output power from the battery and electric machine, respectively and P_{dem} is the demanded or input power. Equations (3d)-(3g) are constraints related to the torque and angular velocity of the ICE and EM and also constraints which regards the battery. The battery current is limited and also the state of energy, SOE, window is limited due to lifetime issues of the battery. Finally (3h) refer to that the solution should be charge sustaining, the battery should have the same state of energy before as after the test cycle.

A. Dynamic Programming

Dynamic Programming, DP, is an optimal control approach which uses a priori data to calculate the solution for (3) [5], [9]. Due to the non-causal nature of the algoritm one cannot use the algorithm online unless the future driving cycle is known, which rarely is the case. In DP the statespace is discretized by a grid and the cost function is evaluated to calculate the arc cost to go from one state to another in time. The path that has the lowest cost is optimal. This is done backwards starting at time $t = t_f$ and then propagates backwards in time to the starting point of the optimization problem (3). The solution by Dynamic Programming is global and it will be used to calculate the Lagrange multiplier for the different test cases which were presented previously in this paper.

B. Minimum principle

From (3a) and (3b) the Hamiltonian H [10] is

$$H = P_{fuel} + \lambda P_{ESS} \tag{4}$$

where λ is the Lagrange multiplier or the Adjoined variable.

The Lagrange multiplier can be calculated offline with the help of DP. Along the optimal trajectory (SOE trajectory which is the only state variable) λ^0 can be calculated as

$$\lambda^{0}(SOE, t) = \frac{\partial J^{0}(SOE, t)}{\partial SOE}$$
(5)

where J^0 is the optimal cost-to-go function [6]. An approximation to (5) is W

$$W = \lambda_{approx} = \frac{\Delta J}{\Delta SOE} = \frac{J^{*+1} - J^{*-1}}{SOE^{*+1} - SOE^{*-1}} \tag{6}$$

where J^* and SOE^* are the optimal cost-to-go function and the optimal *SOE*-trajectory, respectively. The $J^{*\pm 1}$ is the cost-to-go function for a trajectory intersecting $SOE^{*\pm 1} =$ $SOE^* \pm \Delta SOE$ instead where ΔSOE is the smallest step in the grid of the state-space in DP. Notice that $J^{*+1} \leq J^{*-1}$ due to the higher SOE in J^{*+1} than in J^{*-1} . This mean that λ and also W is negative in our setting and it is due to positive P_{ESS} means charging of the ESS. The variable W is called the fuel equivalence factor or just equivalence factor and is an approximation of the Lagrange multiplier. The reason for the name equivalence factor is the fact that W is in each instant a measure on how much energy storage usage will influence the total drive cycle fuel consumption and therefore a (equivalence) factor between battery power and fuel power.

IV. RESULTS

In this section simulation results will be presented. For the four different test cases presented previously an approximation of λ namely the equivalence factor W which was introduced in the previous section will be used to examine how the test cycles parameters affects the optimal control of the wheel loader. For the assumption that the *SOE*-level do not affect the voltage U and that the ESS boundaries (3g) is not hit the equivalence factor W is a constant but how do this constant level change for changes in the drive cycle characteristics.

A. Simulation Results

For constant power levels which correspond to the first test case the results for varying the constant power level can be seen in Fig. 3 where the equivalence factor W and also the discharge and charge equivalence factor $s_{dis} \approx s_{chg} \approx -\frac{1}{\eta_{ICE}\eta_{EM}}$ which is mentioned in [4] of the components (ICE and EM) are showned (assuming $\eta_e \approx 1$). It is seen that the equivalence factor W is correlated with power demand (almost proportionally). Using power demand to approximate the equivalence factor will give a better estimate than an approximation of system efficiency.



Fig. 3. Varying constant power with $P_{avg} = P_{min} = P_{max} \in (5, 10, ..., 140) kW$ and T = 6s. For each of the points a test cycle have been generated and the equivalence factor have been calculated from the test cycle

Since the equivalence factor is a measure on how much battery usage cost related to fuel for the complete cycle it is important to know how the components will be used. For the second test case when average power and minimum power are fixed ($P_{avg} = 40kW$ and $P_{min} = -40kW$) and the peak power is varied which can be seen in Fig. 4, the equivalence factor is nearly constant for low peak power but for around $P_{max} \approx 160$ kW the battery energy tends to be cheaper which is indicated by the increase (notice the sign) of the equivalence factor. This means that the average power is not solely decisive driving cycle parameter that affects the equivalence factor.

In this case average power does not give all information since engine in the optimal solution is not operated in its sweet spot. This can be seen in Fig. 5 where the efficiency for different operating points of the ICE-EM-path is shown. The dotted black line correspond to the optimal trajectory (best efficiency) for varying power demand on the electric side of the EM. The stars in the figure correspond to the operating points for the test cycle with $P_{max} = 70kW$ and the dots in Fig. 5 are for the $P_{max} = 200kW$ -case. Since the engine generator track the demand to some extent the peak cycle power will also give an estimate on at what efficiencies the components will be operated and hence give an estimation of the equivalence factor. The reason that the operating points



Fig. 4. Varying maximum power with $P_{avg} = 40kW$, $P_{min} = -40kW$, T = 6s and $P_{max} \in (70, 75, ..., 200) kW$. For each of the generated test cycles the corresponding W have been calculated

is not closer to the sweet spot than they are is due to that the losses in the ESS would counteract any increase in efficiency for the ICE-EM-path.



Fig. 5. Efficiency for different operating points (ω,T) for the electric side of the EM. Dashed line correspond to the best efficiency for varying power demand. The stars correspond to operating points for $P_{max} = 70kW$ and the dots correspond to operating points for $P_{max} = 200kW$, respectively

The third test case is conducted by varying the average power when holding $P_{max} = 120$ kW and $P_{min} = -40$ kW. By changing the average power and hence the P_{mid} -level in Fig. 2 the amount available recuperate energy (7a)

$$E_{recup} = \int P_{dem} dt, \ P_{dem} < 0 \tag{7a}$$

$$E_{pos} = \int P_{dem} dt, \ P_{dem} \ge 0 \tag{7b}$$

will be the same but the ratio between positive energy in the test cycle E_{pos} (7b) and E_{recup} , $\frac{E_{pos}}{E_{recup}}$ is increased. The results can be seen in Fig. 6. In the figure one can see the correlation between average power and equivalence factor, the bias compared to Fig. 3 is due to the amount of E_{recup} that can be used in a better way in the latter case. This is supported by Fig. 7 where the case with no recuperation with varying

average power is shown. With no recuperation available, cost for using the battery is higher and the equivalence factor is decreased.



Fig. 6. Varying maximum power with $P_{max} = 120kW$, $P_{min} = -40kW$, T = 6s and $P_{avg} \in (20, 25, ..., 80) kW$. For each of the generated test cycles the corresponding W have been calculated



Fig. 7. Varying maximum power with $P_{max} = 120kW$, $P_{min} = 20kW$, T = 6s and $P_{avg} \in (35, 40, ..., 100) kW$. For each of the generated test cycles the corresponding W have been calculated

It is shown that cycles with different average efficiencies but different quotients of $\frac{E_{pos}}{E_{recup}}$ give different equivalence factors. Both $\frac{E_{pos}}{E_{recup}}$ and the efficiency regions of the components have to be combined to better estimate the equivalence factor.

Change in cycle time or frequency does not matter since the models are static. However, hitting a boundary in state of energy (3g) will highly influence the equivalence factor. For instance, during a period of high level of available regenerative power the cost for using the battery becomes lower. This is shown in Fig. 8 for which T = 45s.

The reason for this is that a gain in energy in the ESS is of less or no use to lower the fuel consumption at the end due to the ESS will be recharged to its maximum capacity independent of the current state of charge. This behavior is seen at time t = 135s in Fig. 8 where the ESS will be



Fig. 8. $P_{max} = 120kW$, $P_{min} = -40kW$, $P_{avg} = 40kW$ and T = 45s. Figure shows what happens to the equivalence factor when one hit the boundary on the SOE-window (3g)

recharged to maximum capacity and a gain or a decrease in SOE will not affect the fuel consumption at the end. The reason for the non-zero value of W at around time t = 160 and t = 350s is due to the calculations of W in (6) and that the DP-solution wait until the last possible point to recharge the ESS to full capacity and this results in that W will be non-zero in that region.

B. Estimation horisont

Due to the repetitive nature of the wheel loaders driving cycle a full length estimation horisont is not needed. To estimate the equivalence factor on historical data and a fixed estimation horisont as the length of the cycle time 4T is not good idea in this case. The reason for this is that even if the driving cycle parameters are fixed for the test cycle the corresponding estimates on these parameters would change due to the fixed estimation horisont when it travels in time in the test cycle. A fixed estimation horisont like in [9] would not be a good idea in this case. A better solution would to try to detect the cycle time and the repetition of these in the test cycle by for example pattern detection techniques and then use the last found pattern to estimates the parameters more accurately.

The results that have been showed in this section can guide one to an initial estimate of the fuel equivalence factor W. However, the results presented here is for the ideal case and in an online application the equivalence factor W would have to adapt to the current driving and therefore it can be used as an initial guess to adaptive approaches like [11].

V. CONCLUSIONS

For the studied case with repetitive driving cycles it is shown that the optimal solution depends on cycle characteristics such as average power demand, peak power demand, efficiency characteristics of the components, and the amount of energy available for recuperation. For constant power the magnitude of the fuel equivalence factor is positive correlated with power demand. The slope for varying power demand is also dependent on the efficiency of the components. To study the dependencies between the equivalence factor and the driving cycles parameters in more depth could involve that the voltage would depend on the current SOE and its affect on the optimal control.

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