Evaluating some Gain Scheduling Strategies in Diagnosis of a Tank System

Marcus Klein

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Abstract

In model-based diagnosis the problem of finding all the relations that can be used to detect and isolate different faults, is solved for linear systems, with e.g. "The Minimal Polynomial Basis Method". However, for nonlinear systems the situation is much more complicated. Here an approach will be taken using the linear method above together with gain scheduling. Linear residual generators are designed at a number of stationary points. The approach is based on using a nominal selector matrix, using null-space redesign dependent on the scheduling variable, and using a proposed optimization method. Two different gain scheduling strategies are applied to form the residual generators between design points, namely nearest neighbour approximation and linear interpolation. The approach is applied to a simple nonlinear system consisting of two coupled water tanks. The simulations show that the performance of the residual generators are good under steady state conditions. It is also shown that linear interpolation has better performance than nearest neighbour approximation, especially during transients.

Keywords: Model-based diagnosis; Gain scheduling; Minimal polynomial basis; Water-tank system;

Contents

Chapter 1

Introduction

Modern systems are made more and more autonomous and thereby more complex, requiring that the increasing number of sensors that the control system uses are not faulty. To avoid severe failures for the process, a fault must be detected and preferably isolated. This is especially important in safety critical applications, such as e.g. nuclear plants or an aircraft engine. Environmentally based requirements for diagnosis systems such as OBDII(On Board Diagnostics II) and EOBD (European On Board Diagnostics) on automotive engines has also increased both academic and industrial interest in diagnosis.

In model-based diagnosis a model of the system is used to find the relations that can be used to detect and isolate different faults. Therefore methods of finding these relations, called residuals, have to be found. For linear systems all these relations can be found by using a method called "Minimal Polynomial Basis Method" presented by E. Frisk and M. Nyberg (Frisk & Nyberg 1999). In many cases it is hard to find a model for a system that is linear in its whole operating range, but quite easy to find a nonlinear one. In many control applications *gain scheduling* has been successfully used together with linear control methods to control nonlinear systems. This approach will be applied to design of diagnosis systems, i.e. to use the linear method mentioned earlier at different operating points and then use a gain scheduling strategy in the diagnosis system for a nonlinear process. This approach will be applied to a simple nonlinear system consisting of two coupled water tanks.

Report overview

In Chapter 2 the gain scheduling strategies taken in this paper will be presented together with the implications it has in model-based diagnosis. Chapter 3 describes the modeling of the water tank system and the faults that can be introduced. Chapter 4 describes the design of the diagnosis system. Finally simulations of the water tank system are made and evaluated in Chapter 5, followed by Chapter 6 where conclusions from the simulations are drawn.

Chapter 2

Some Gain Scheduling Strategies

Gain scheduling has been used in control of nonlinear systems for a long time. In this paper this approach will be applied to diagnosis. It relies on linearization of the nonlinear system and design of diagnosis residuals for that specific linearization locally. Then these residuals are combined using a scheduling strategy into a nonlinear diagnosis system.

2.1 Model

A general state space description of a nonlinear system

$$\dot{x}(t) = f(x(t), u(t)) \tag{2.1a}$$

$$y(t) = h(x(t), u(t)) \tag{2.1b}$$

can be reformulated and parameterized as a Linear Parameter Varying (LPV) system (2.2) (Lagerberg 1996),

$$\dot{x}(t) = A(\alpha)x(t) + B(\alpha)u(t) \tag{2.2a}$$

$$y(t) = C(\alpha)x(t) + D(\alpha)u(t) \tag{2.2b}$$

$$\alpha = \alpha(x(t), u(t)) \tag{2.2c}$$

where we for frozen values of α have a linear system. A parameter/signal that indicate the operating conditions of the system is called/denoted *scheduling variable*.

In gain scheduling the linear matrices in (2.2) are not calculated for every value of α, rather for some specified values used with the scheduling variable and the chosen gain scheduling strategy to approximate the nonlinear model for every operating point.

2.2 Residuals

A residual can be defined as (Nyberg 1997)

Definition 2.1 (Residual) *A residual $r(t)$ is a scalar or vector which is 0 (or small) in the fault free case and $\neq 0$ (or large) when a fault has occured.*

The residuals are used to detect and to isolate faults, i.e. determine in which fault mode the system is. The residuals are generated by a residual generator (see Figure 2.1), which has the input u and the output y from the system as inputs. The residual evaluator is then used to determine which fault mode is present, which is also called a diagnosis.

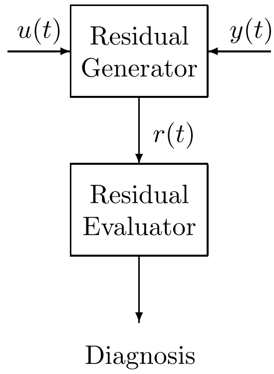

Figure 2.1: Internal structure of a diagnosis system.

Residual generation

A general linear residual generator can be written as:

$$r = H_{yr}(p)y + H_{ur}(p)u = Q(p)\begin{pmatrix} y \\ u \end{pmatrix} \tag{2.3}$$

where u is the input and y is the output of the system. p is the derivative operator. H_{yr} is the transfer function from y to r with appropriate dimension. H_{ur} and Q are defined in a similar way. The output y of the system is a function of the input u, the faults f that can occur and the disturbances d. The output can therefore be written as:

$$y = G_{uy}(p)u + G_{fy}(p)f + G_{dy}(p)d \tag{2.4}$$

Replacing y in (2.3) with (2.4) yields:

$$\begin{aligned} r &= (H_{yr}(p)G_{uy}(p) + H_{ur}(p))u + H_{yr}(p)G_{fy}(p)f + H_{yr}(p)G_{dy}(p)d = \\ &= Q(p)\begin{bmatrix} G_{uy}(p) & G_{dy}(p) \\ I & 0 \end{bmatrix}\begin{bmatrix} u \\ d \end{bmatrix} + Q(p)\begin{bmatrix} G_{fy}(p) \\ 0 \end{bmatrix} f = 0 \end{aligned} \tag{2.5}$$

According to Definition 2.1 the residual should in the fault free case be zero, i.e. Q should be chosen such that (2.6) holds.

$$Q(p)M(p) = Q(p)\begin{bmatrix} G_{uy}(p) & G_{dy}(p) \\ I & 0 \end{bmatrix} = 0 \tag{2.6}$$

To be able to detect faults the transfer function from every fault to the residual must be non-zero. If such a Q can be found then in an ideal world the residual would only depend on the faults f, but since not all disturbances and model uncertainties can be accounted for, here called $\acute{d}$, and therefore not decoupled, the residual can be expressed as:

$$r = G_{fr}(p)f + G_{\acute{d}r}(p)\acute{d} \tag{2.7}$$

In the design of a residual generator the aim is to make $G_{fr}(p)$ as large as possible and $G_{\acute{d}r}(p)$ as small as possible (in some sense).

A feasible method of finding all appropriate Q:s is *the Minimal Polynomial Basis Approach* as described in (Frisk & Nyberg 1999). The method finds all Q:s and parameterize them in a minimal way. The method can be summarized in two steps:

1. Find a minimal polynomial basis $N_M(s)$ for $\mathcal{N}_L(M(s))$, i.e. find the basis for the left null space of $M(s)$.

2. Choose the polynomial matrices $\phi(s)$ and $D_W(s)$ in

$$Q(s) = D_W^{-1}(s)\phi(s)N_M(s) = D_W^{-1}(s)W(s) \tag{2.8}$$

where $\phi(s)$ is a polynomial matrix used to pick out the appropriate residual generator from the basis $N_M(s)$. To make $Q(s)$ realizable $D(s)_W$ is a invertible polynomial matrix with row degree higher or equal to $\phi(s)N_M(s)$:s corresponding row degree (Kailath 1980). Since there are no other constraints on $D_W(s)$ than the degree, the poles of the residual generator $Q(s)$ can be chosen arbitrarily.

With this method all Q are found and they are parameterized in a minimal way. The resulting general linear residual generator r as in (2.3) can then be described as:

$$r = \frac{A(p)y(t) + B(p)u(t)}{C(p)} \tag{2.9a}$$

$$A(p) = a_n p^n + \cdots + a_1 p + a_0 \tag{2.9b}$$

with B and C defined in a similar way. The only constraint on C is that r should be realizable, i.e. the order of C must be equal or larger than the highest order of A and B.

2.3 Interpolation

A diagnosis system using only one linear residual generator for the entire operating range of a nonlinear system is bound to fail (or at least to have low performance). This since the linear model of the system that the design of the diagnosis system is based upon, does not catch the nonlinear behaviour of a nonlinear system. However, dividing the operating range into smaller regions, where every region has a specified design point would probably be better. The linear residual generators that are designed at specified operating points work well, when we are close to one of these operating points. But since the system is nonlinear, the residual generators will not work very well in between these operating points. One solution could be to find the residual generator at the present operating point on-line, but is not (for the time being) applicable due to the computational burden it brings along. Interpolation of residual generators designed off-line then seems feasible. The interpolation can be made in several different ways, but only two of them will be tried out here, namely *nearest neighbour* and *linear interpolation*. Another way to interpolate could of course be to use a polynomial approximation of higher order.

A linear residual generator is described as in (2.9a), i.e. for every design point h^j the residual generator r^j is described as:

$$r^j = \frac{A^j(p)y(t) + B^j(p)u(t)}{C^j(p)} \tag{2.10a}$$

$$A^j(p) = a_n^j p^n + \cdots + a_1^j p + a_0^j = \sum_{i=0}^{n} a_i^j p^i \tag{2.10b}$$

with B^j and C^j in a obvious fashion. Interpolation between the two parameters a_i^j and a_i^{j+1} then yields a_i (the parameter in (2.9b)) determined by the scheduling variable h and the chosen interpolation method.

Here we will neglect that the different residuals have different states when the interpolation is made. Therefore choosing the same C^j (in (2.10a)) for all design points h^j makes it the same to interpolate the residuals as to interpolate the coefficients a_i^j and b_i^j in (2.10b). This will be the approach used in the simulations.

One disadvantage is that we can not guarantee that (2.6) holds for every operating point, except for the points where the residual generators are designed, hereon called design points. I.e. we can not guarantee that Q will belong to $\mathcal{N}_L(M(s))$ for any operating point except for the design points and therefore we can not guarantee that the disturbances and the non-monitored faults are decoupled.

2.3.1 Gain Switching

One gain scheduling approach is to linearize at all design points, then design residual generators for all of them, and then just *switch* between them when moving around in the operation range, i.e. to use nearest neighbour approximation. The parameters a_i can be calculated as:

$$a_i = \beta^{j+1} a_i^{j+1} + \beta^j a_i^j \tag{2.11}$$

where

$$\beta^{j+1} = \begin{cases} 1 & \beta \geq 0.5 \\ 0 & \beta \leq 0.5 \end{cases}$$

$$\beta^j = \begin{cases} 0 & \beta \geq 0.5 \\ 1 & \beta \leq 0.5 \end{cases}$$

$$\beta = \frac{h - h^j}{h^{j+1} - h^j} \qquad \forall h \in [h^j, h^{j+1}[\tag{2.12a}$$

By calculating all the parameters in (2.9a) in this manner, we end up with a residual generator with gain switching as its gain scheduling strategy.

One problem with this approach is that at the design points the diagnosis system will work fine, but the diagnosis performance will decrease/degrade as we move further away from a design point. When we reach a switch point (a point where $\beta = 0.5$) another problem can occur, namely a jump (discontinuity) in the residual due to switching of parameters in the residual generator. This can be avoided if we can find a bump-less transfer function. For control systems this can be avoided if the controller has integral action (by adjusting the integral value at the switch point) or by using several controllers, where the inactive ones are in a stand-by mode and are forced to follow the control signal of the active controller (Lagerberg 1996). Another problem, called chatter, can occur when the present operating point moves back and forth over the switching point, causing the control (or diagnosis) system to switch back and forth. This can be avoided using hysteresis. These matters will however not be explored any further here.

2.3.2 Continuous Gain Scheduling

To avoid the problem with discontinuities in the residual when switching the parameters, continuous gain scheduling can be used. The parameters for the residual generator are then continuous, and the residuals become continuous over the entire operating range. Linear interpolation is an example of such an interpolation. A linear interpolation between the two parameters a_i^j and a_i^{j+1} would be

$$a_i = \beta a_i^{j+1} + (1 - \beta) a_i^j \tag{2.13}$$

where β is determined by the scheduling variable h as:

$$\beta = \frac{h - h^j}{h^{j+1} - h^j} \qquad \forall h \in [h^j, h^{j+1}[\tag{2.14}$$

By calculating all the parameters in (2.9a) in this manner, we end up with a residual generator with linear interpolation as its gain scheduling strategy.

Chapter 3

The Water Tank System

The different gain scheduling strategies are to be implemented and tested on a water tank system. Figure 3.1 shows a schematic picture of the process. The water tank system consists of two coupled tanks, where the water level of the upper tank is controlled by the pump. Each tank has a water level sensor and flow sensors are placed at different places. Leaks in the system can be introduced by opening the appropriate valves at two different places. Sensor and actuator faults can also be introduced.

3.1 Modeling (fault free case)

In this section a physical model of the water tank system is derived in the fault free case. Mass balance for one tank gives us:

$$A\frac{dh}{dt} = Q_{in} - Q_{out} \tag{3.1}$$

where A is the cross section, h is the water level and $Q_{in/out}$ is the flow in/out of the tank. The flow out of the tank can be described by Bernoulli's law

$$Q_{out} = a\sqrt{2gh} \tag{3.2}$$

where a is the cross section of the outlet hole and g is the acceleration of gravity.

The input to the process is v (input voltage to the pump [0-10 V]) and the outputs are y_1 and y_2 (voltage from the level measurement in tank 1 and 2 [0-10 V]), and y_3, y_4 (voltage from the flow sensors [0-10 V]). Assume that the flow generated by the pump is proportional to the applied voltage, i.e. $Q_{in} = k_a v$. The model can then be described as

$$\frac{dh_1}{dt} = \frac{k_a}{A_1}v - \frac{a_1}{A_1}\sqrt{2gh_1} \tag{3.3}$$

$$\frac{dh_2}{dt} = \frac{a_1}{A_2}\sqrt{2gh_1} - \frac{a_2}{A_2}\sqrt{2gh_2} \tag{3.4}$$

where subscript i corresponds to tank i. Assuming that the measured outputs are proportional to the true outputs, the outputs can be modeled as $y_i = k_i h_i; i = 1, 2$ and $y_i = k_i q_i; i = 3, 4$ respectively, where q_i is the flow through the flow sensor corresponding to output i, and k_i denotes the proportional constant for the level sensors and for the flow sensors respectively.

3.1.1 Stationary Points

The stationary points (h_1^0, h_2^0, v^0) for the water tank system are given by setting the derivatives in (3.3) equal to 0:

$$\frac{k_a}{A_1}v^0 = \frac{a_1}{A_1}\sqrt{2gh_1^0} \tag{3.5}$$

$$\frac{a_1}{A_2}\sqrt{2gh_1^0} = \frac{a_2}{A_2}\sqrt{2gh_2^0} \tag{3.6}$$

which gives the stationary points:

$$h_1^0 = \frac{k_a^2}{2ga_1^2}(v^0)^2 \tag{3.7}$$

$$h_2^0 = \frac{a_1^2}{a_2^2}h_1^0 = \frac{k_a^2}{2ga_2^2}(v^0)^2 \tag{3.8}$$

parameterized by e.g. the stationary water level h_1^0.

3.1.2 Linearization

The linearization at a stationary point (h_1^0, h_2^0, v^0) for $h_1^0 > 0$ is given by

$$\dot{x} = \begin{pmatrix} \frac{-a_1}{A_1}\sqrt{\frac{g}{2h_1^0}} & 0 \\ \frac{a_1}{A_2}\sqrt{\frac{g}{2h_1^0}} & \frac{-a_2}{A_2}\sqrt{\frac{g}{2h_2^0}} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \frac{k_a}{A_1} \\ 0 \end{pmatrix} u \tag{3.9}$$

$$y - y^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ a_1\sqrt{\frac{g}{2h_1^0}} & 0 \\ 0 & a_2\sqrt{\frac{g}{2h_2^0}} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} u \tag{3.10}$$

$$y^0 = \begin{pmatrix} h_1^0 \\ h_2^0 \\ a_1\sqrt{2gh_1^0} \\ a_2\sqrt{2gh_2^0} \end{pmatrix} \tag{3.11}$$

where the states $x_i = h_i - h_i^0$ and the input $u = v - v^0$. Introducing the constants $c_i = \frac{a_i}{A_i}\sqrt{2g}$ and $b_i = \frac{1}{A_i}$ and using that h_2^0 can be expressed in h_1^0 the linearization in (3.9)–(3.11) can be rewritten as:

$$\dot{x} = \begin{pmatrix} -\frac{c_1}{2\sqrt{h_1^0}} & 0 \\ \frac{b_2c_1}{2b_1\sqrt{h_1^0}} & -\frac{c_2}{2\sqrt{h_2^0}} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} k_ab_1 \\ 0 \end{pmatrix} u \tag{3.12}$$

$$y - y^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ \frac{c_1}{2b_1\sqrt{h_1^0}} & 0 \\ 0 & \frac{c_2^2b_1}{2c_1b_2^2\sqrt{h_1^0}} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} u \tag{3.13}$$

$$y^0 = \begin{pmatrix} h_1^0 \\ \frac{c_1^2b_2^2}{c_2^2b_1^2}h_1^0 \\ \frac{c_1}{b_1}\sqrt{h_1^0} \\ \frac{c_1}{b_1}\sqrt{h_1^0} \end{pmatrix} \tag{3.14}$$

The linearization in (3.12) is parameterized by the parameter h_1^0 and therefore the variable h_1 will be used as the scheduling variable in gain scheduling. Another choice could be v or h_2 since the linearization is also parameterized by these respectively. But the input v will not reflect the current state of the system, and therefore it would not be a good choice. Using h_2 instead of h_1 would be equivalent with this configuration of the water tanks.

3.2 Fault Modeling

Faults occuring in a system can often be divided into three groups, namely actuator faults, process faults and sensor faults, where a typical process fault can be a leak or a clog in the system. An easy way of modeling actuator faults f_a and sensor faults f_s are as additive faults on the input signal and the output signal respectively, i.e. :

$$y = G(p)(u + f_a) + f_s \tag{3.15}$$

where $G(p)$ is the model of the process. In this diagnosis system no process faults will be considered and therefore not modeled.

The faults (fault modes) listed in Table 3.1 can occur in the water tank system (compare Figure 3.1): Leaks can also be introduced to the process at

f_a	Actuator fault
f_{h1}	Sensor fault in the water level sensor in the upper tank (tank 1).
f_{h2}	Sensor fault in the water level sensor in the lower tank (tank 2).
f_{f1}	Fault in flow sensor 1 (flow out of tank 1).
f_{f2}	Fault in flow sensor 2 (flow out of tank 2).

Table 3.1: Faults that can be introduced to the water tank system.

different places, but this will not be utilized here. Equation 3.3 can then be rewritten to include the faults in Table 3.1 as:

$$\frac{dh_1}{dt} = k_a b_1(v + f_a) - c_1\sqrt{h_1} \tag{3.16}$$

$$\frac{dh_2}{dt} = \frac{b_2}{b_1}c_1\sqrt{h_1} - c_2\sqrt{h_2} \tag{3.17}$$

$$y = \begin{pmatrix} h_1 + f_{h1} \\ h_2 + f_{h2} \\ \frac{c_1}{b_1}\sqrt{h_1} + f_{f1} \\ \frac{c_2}{b_2}\sqrt{h_2} + f_{f2} \end{pmatrix} \tag{3.18}$$

This model will be used for simulation of the water tank system in Chapter 5.

3.3 Parameter Identification

To obtain the parameters in (3.12)–(3.14) for the tank system, some simple experiments have been performed on each tank, and the parameters are estimated from the data using the physical model and least square methods. The outcome of the parameter identification is summarized in Table 3.3. For convenience the

Parameter	Value	Standard deviation
c_1	0.0638	$6.28 \cdot 10^{-5}$
c_2	0.0878	$2.22 \cdot 10^{-4}$
b_1	0.0452	$5.73 \cdot 10^{-5}$
b_2	0.0419	$2.30 \cdot 10^{-5}$

Table 3.2: The values and standard deviations for the parameters in (3.16)–(3.18).

gain factor for the pump, k_a, as well as the gain factors for all the sensors, k_i, are assumed to be 1 in the simulations.

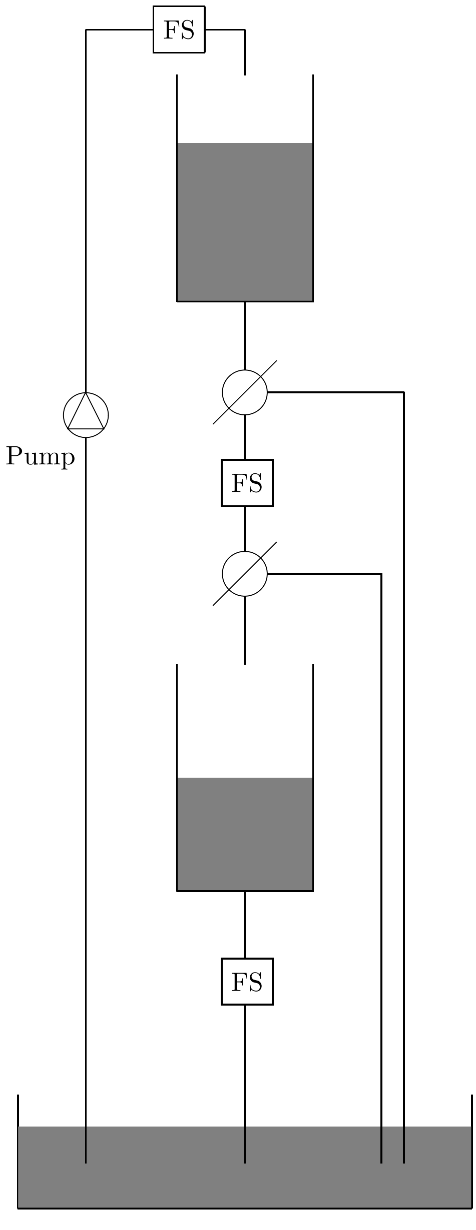

Figure 3.1: A schematic figure of the water tank system, where FS are flow sensors. Each tank also has a level sensor. The water level in tank 1 is controlled by the pump. Leaks in the system can be introduced by opening the valves.

Chapter 4

Diagnosis System

In this chapter the design of the diagnosis system for the coupled water tanks will be described.

4.1 Structure

To be able to isolate different faults, or fault modes, residuals that are sensitive to certain faults (called monitored faults) and insensitive to other faults (called non-monitored faults) are needed. To systematize this a residual structure is used, where for every residual the monitored faults are denoted with a '1' and the non-monitored faults are denoted with a '0'. How should the residual structure be chosen? According to (Gertler 1998) making the number of '0':s in each column as large as possible is preferable. For the water tank system with the five faults in Table 3.1 this would result in a 5×5 residual structure with three '0':s in each column. This would result in a null space of dimension 1, i.e. the design freedom is unnecessarily limited. Instead a diagonal residual structure is chosen and shown in Table 4.1. It is able to detect and to isolate all five faults. A problem is that it can not be guaranteed that the decoupling of the non-monitored fault is achieved when the actual operating point is not a design point. For this reason it seems reasonable to decouple as few faults as possible in each residual to make the residual more robust. By choosing

	f_{h1}	f_{h2}	f_{f1}	f_{f2}	f_a
r_1	0	1	1	1	1
r_2	1	0	1	1	1
r_3	1	1	0	1	1
r_4	1	1	1	0	1
r_5	1	1	1	1	0

Table 4.1: A diagonal residual structure.

a structure where only one fault has to be decoupled from a certain residual, a larger freedom in forming the residual is also given, compared to if more faults are to be decoupled in each residual. When no faults are decoupled, the dimension of the null-space is equal to the number of measurements in the non-disturbance case (Lagerberg 1998), i.e. 4 for the water tank system. Therefore by decoupling one fault per residual, we will end up with a basis for the null space with dimension 3. As an example the polynomial basis $N_M(s)$ for the design point $h_1^0 = 3$ with the fault f_{h1} decoupled (i.e. residual 1 in Table 4.1) is:

$$N_M(s) = \begin{bmatrix} 0 & 0.033+s & -0.042 & 0 & 0 \\ 0 & 0 & 0.016+s & 0 & -0.016 \\ 0 & -0.78 & 0 & 1 & 0 \end{bmatrix} \tag{4.1}$$

From the polynomial basis $N_M(s)$ in (4.1) all residual generators $Q(s)$ decoupling the fault f_{h1} at the design point $h_1 = 3$ can be found through

$$Q(s) = D_W^{-1}(s)\phi(s)N_M(s) = D_W^{-1}(s)W(s) \tag{4.2}$$

where $\phi(s)$ is a polynomial selection matrix used to select the appropriate residual generator from the basis $N_M(s)$. To make $Q(s)$ realizable $D(s)_W$ is a invertible polynomial matrix with row degree higher or equal to $\phi(s)N_M(s)$:s corresponding row degree (Kailath 1980). Since there are no other constraints on $D_W(s)$ than the degree, the poles of the residual generator $Q(s)$ can be chosen arbitrarily. Here the same low-pass filter $D_W(s) = s + 1$ will be used for all design points, i.e. $C^j = s + 1$ for all j (see (2.10a)). Constant faults are often important to detect and isolate, and therefore a low pass filter $s + 1$ is chosen.

4.2 Design of Residuals from the Null Basis $N_M(s)$

How should a residual from the null space $N_M(s)$ be chosen? By making the response from all the monitored faults f_j to the residual r_i as large as possible, not forgetting that the non-monitored faults should stay decoupled. I.e. the transfer function $G_{r_i f_j}$ should in some sense be maximized. In this approach only the constant gain from f_j to r_i will be used. Further only constant selection matrices $\phi(s)$ will be considered. Then finding the $\phi_i(s)$ in 4.3 gives our $Q(s)$.

Thus, the optimization criterion

$$\phi_i(s)^j = \arg\max_{\phi_i(s)} \min_k \|G_{r_i f_k}(0)\|_2 \tag{4.3}$$

will yield the selection matrix ϕ_i that maximizes the smallest DC-gain from any of the monitored faults k at the design point j. Using the constraint that every vector $\|\phi_i^j\| = 1$, we will for the water tank system end up with five residual generators for every design point h^j. When using the optimization criterion in (4.3) in every design point (i.e. designing locally) yields different ϕ_i^j for every design point h^j. This could be an disadvantage when the present operating point is not a design point, since it then can not be guaranteed that $Q \in \mathcal{N}_L(M(s))$. Choosing different ϕ_i^j (for j) could result in worse performance, than if the same ϕ_i^j had been chosen for all j, because ϕ_i^j and ϕ_i^{j+1} could represent different permutations leading to improper interpolation. Although in our simulations this problem has not occured, the same ϕ_i^j will be used for all j. Summing up, we are using a fix selection matrix based on a design point in the middle of the operating range, but there are different null-spaces $W^j(s)$ for the different operating points.

Local design at $h_1^0 = 3$ yields the polynomial matrix W^j:

$$W^j = \begin{bmatrix} 0 & -0.018 & 0.018 + 0.98s & 0.02 & -0.018 \\ -0.0082 + 0.45s & 0 & 0.02 & 0.02 + 0.53s & -0.02 \\ 0.0014 + 0.51s & 0.0021 + 0.47s & 0 & 0.017 & -0.023 \\ -0.0091 + 0.45s & 0.02 + 0.53s & 0.02 & 0 & -0.02 \\ -0.28 & -0.28 & 0.69 & 0.31 & 0 \end{bmatrix} \tag{4.4}$$

A typical component of $W^j(s)$ is $a_1^j s + a_0^j$ (compare (2.9a)). In the application studied here, the coefficients a_1^j are constant when the design point j is varied. On the other hand the absolute value of the coefficients a_0^j decreases monotonically with the design point j, and therefore also the scheduling variable. This is depicted in Figure 4.1, which shows the coefficient behaviour for the coefficient $W^j(1,2)$ for different values of the design point (scheduling variable). The non-

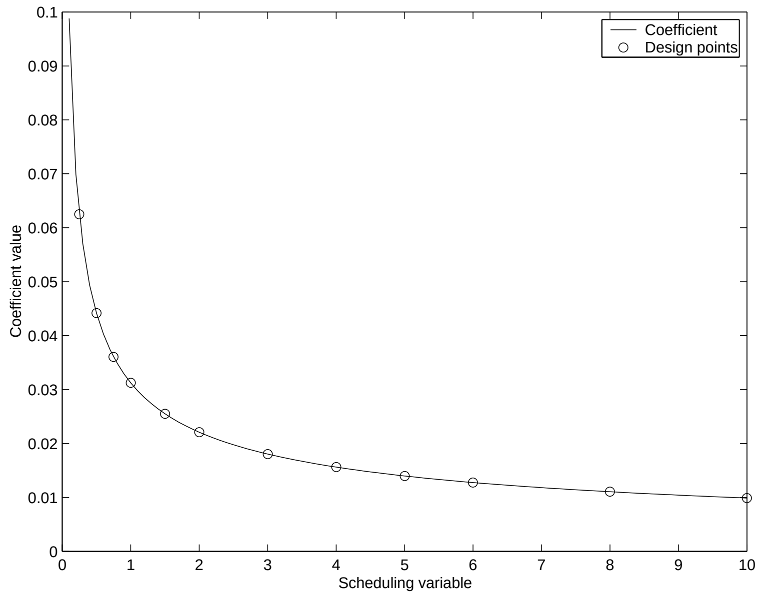

Figure 4.1: Typical coefficient dependence of the scheduling variable.

linearities are apparently bigger for small values of the scheduling variable h_1, so it seems reasonable to have a denser distribution of design points for lower h_1-values. Bearing this in mind, design points are chosen to be:

Design point j	Scheduling variable value h_1^{0j}
1	0.25
2	0.5
3	0.75
4	1
5	1.5
6	2
7	3
8	4
9	5
10	6
11	8
12	10

With the design points above and the interpolation strategies discussed in Section 2.3.1 and in Section 2.3.2 respectively, the coefficients dependence of the scheduling variable h_1 is shown in Figure 4.2 for the gain switching strategy and in Figure 4.3 for the linear interpolation strategy, together with the real dependence of the coefficients. Comparing Figure 4.2 and Figure 4.3, we can conclude that linear interpolation strategy is expected to work better than the gain switching strategy, since it captures the coefficients dependence of the scheduling variable better.

With $D_W(s) = s + 1$ as mentioned earlier, the monitored (and non-

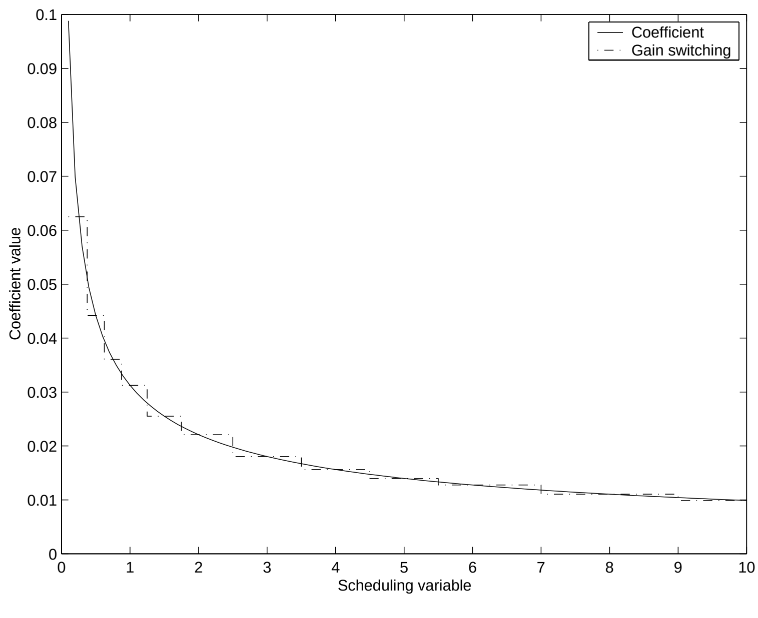

Figure 4.2: Typical coefficient dependence of the scheduling variable together with coefficient dependence when using the gain switching strategy.

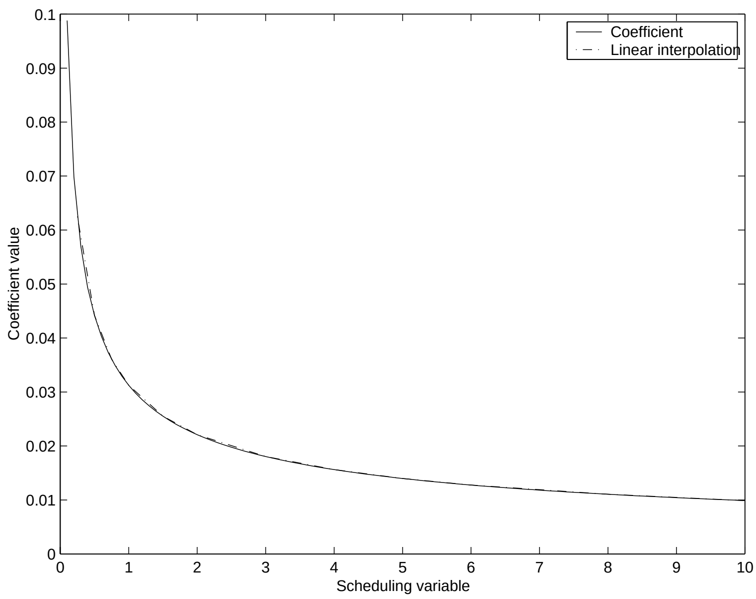

Figure 4.3: Typical coefficient dependence of the scheduling variable together with coefficient dependence when using the linear interpolation strategy.

monitored) faults response in the residuals can be calculated as

$$G_{rf} = Q \begin{bmatrix} G_{yf} \\ 0 \end{bmatrix} \tag{4.5}$$

shown in Figure 4.4 for the first residual r_1 where fault f_{h1} was decoupled in design point 7. The DC-gain from the monitored faults to residual r_1 are approximately the same, which was the goal of the optimization criteria in (4.3).

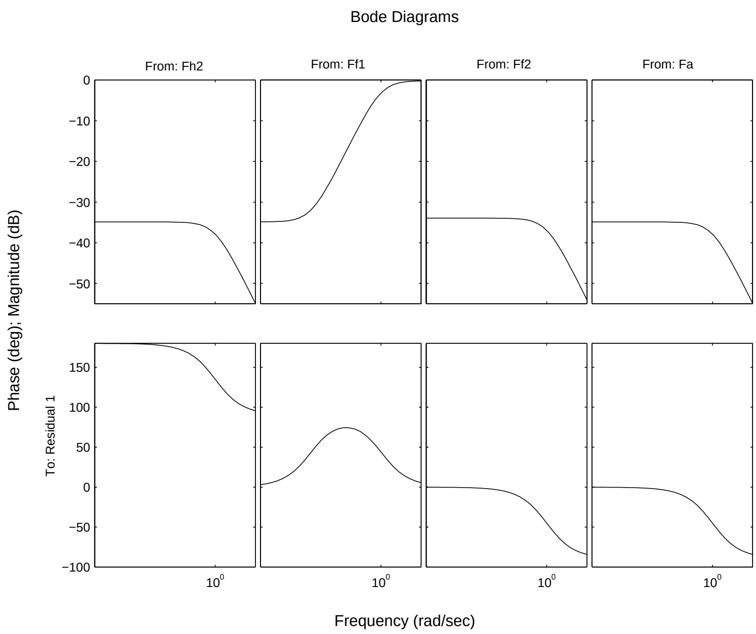

Figure 4.4: The monitored faults response in residual r_1 for design point 7.

Chapter 5

Simulations

In this chapter the gain scheduled diagnosis system will be tested through simulations of the water tank system. As a test case a 480 seconds long test cycle has been defined, shown in Figure 5.1.

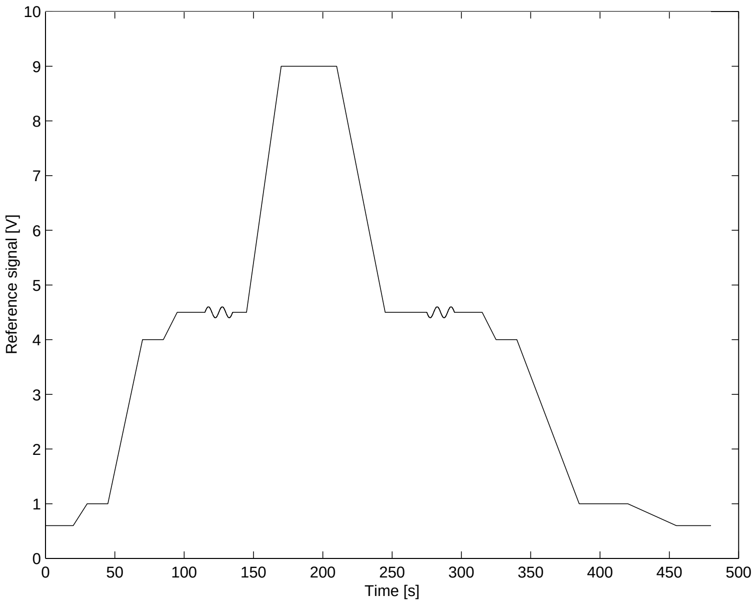

Figure 5.1: The reference signal for the test case.

5.1 Description of the test case

The test case can be divided into several interesting regions (Table 5.1), all of them showing a specific behaviour for the diagnosis system. The transients are less steep on the second half of the test cycle, when the tank levels are sinking, due to the slow dynamics when emptying the tanks. In the simulations the faults described in Table 3.1 can be introduced as steps at a specific time, here chosen

Test region	T [s]	Description of time interval
1	0–20	Actual operating point $h_1 \simeq 0.6$ between dp2 and dp3.
2	20–45	Slow transient behaviour, when moving from $h_1 = 0.6$ to $h_1 = 1$ (dp4).
3	45–85	Transient behaviour, when moving from $h_1 = 1$ to $h_1 = 4$ (dp4 to dp8)
4	85–115	Moving from $h_1 = 4$ to $h_1 = 4.5$, i.e. from dp8 to the switch point between dp8 and dp9.
5	115–135	Slow sinusoidal behaviour around the switch point $h_1 = 4.5$.
6	135–145	Steady state operation at operation point $h_1 = 4.5$.
7	145–170	Fast transient behaviour, when moving from $h_1 = 4.5$ to $h_1 = 9$, i.e. from the switch point between dp8 and dp9 to the switch point between dp11 and dp12.
8	170–210	Constant reference signal aiming for $h_1 = 9$.
9	210–275	Fast transient behaviour, when moving from $h_1 = 9$ to $h_1 = 4.5$, i.e. from the switch point between dp11 and dp12 to the switch point between dp8 and dp9.
10	275–295	Slow sinusoidal behaviour around the switch point $h_1 = 4.5$.
11	295–315	Steady state operation at operation point $h_1 = 4.5$.
12	315–340	Moving from $h_1 = 4.5$ to $h_1 = 4$, i.e. from the switch point between dp8 and dp9 to dp 8.
13	340–420	Transient behaviour, when moving from $h_1 = 4$ to $h_1 = 1$ (dp8 to dp4)
14	420–455	Slow transient behaviour, when moving from $h_1 = 1$ (dp4) to $h_1 = 0.6$.
15	455–480	Actual operating point $h_1 \simeq 0.6$ between dp2 and dp3.

Table 5.1: A description of the different regions for the test case in Figure 5.1. The acronym *dp* means design point.

to be $t_{step} = 15s$. The size of the faults are all chosen to be 0.5. The different regions in Table 5.1 represent different problems for the residual generator that can occur with the gain scheduling strategy. These problems are:

1. Slow transient behaviour, i.e. when the water level in tank 1 is changed slowly. (Test regions 2,4,12,14)

2. Fast transient behaviour, i.e. when the water level in tank 1 is changed quickly. (Test regions 3,7,9,13)

3. The chatter problem, described in Section 2.3.1. The scheduling variable is jumping back and forth around the switch point, causing the diagnosis system to switch residual generator back and forth. This will be a problem when gain switching is used as gain scheduling strategy, but not when linear interpolation is used. (Test regions 5,10)

4. Faults in the scheduling variable. (Test regions 1,2,8)

5. Decoupling when the actual operation point *is not* a design point. (Test regions 1,6,8,11,15)

6. Decoupling when the actual operation point *is* a design point. (Test regions are parts of the regions 2, 3 ,12 and 13.)

5.2 Results

Simulations are made with the test cycle defined above, using both gain switching and linear interpolation as gain scheduling strategy for all the cases of a single fault of size 0.5 occuring at $t = 15$ s and in the fault free case. The resulting residuals of these simulations are shown in Appendix A. Parts of these residuals are enlarged in the following to describe the problems above and in the last two cases another test cycle is used.

5.2.1 Transient behaviour

When the water tank system is exposed to a change in the reference signal, the value of the scheduling variable changes, moving the system to another operating point. This will cause a transient behaviour for the residual generator. Since the linearizations are made in equilibrium, they will not correspond to well with the nonlinear system during transients and this model error can cause residuals to signal even in the fault free case. This is exemplified in Figure 5.2, where residual 5 is shown in the fault free case with linear interpolation as the gain scheduling strategy. During the time intervals where transients are present (compare with Figure 5.1) the amplitude of the residual becomes considerably higher, making the evaluation of the residual harder (i.e. determine if it is

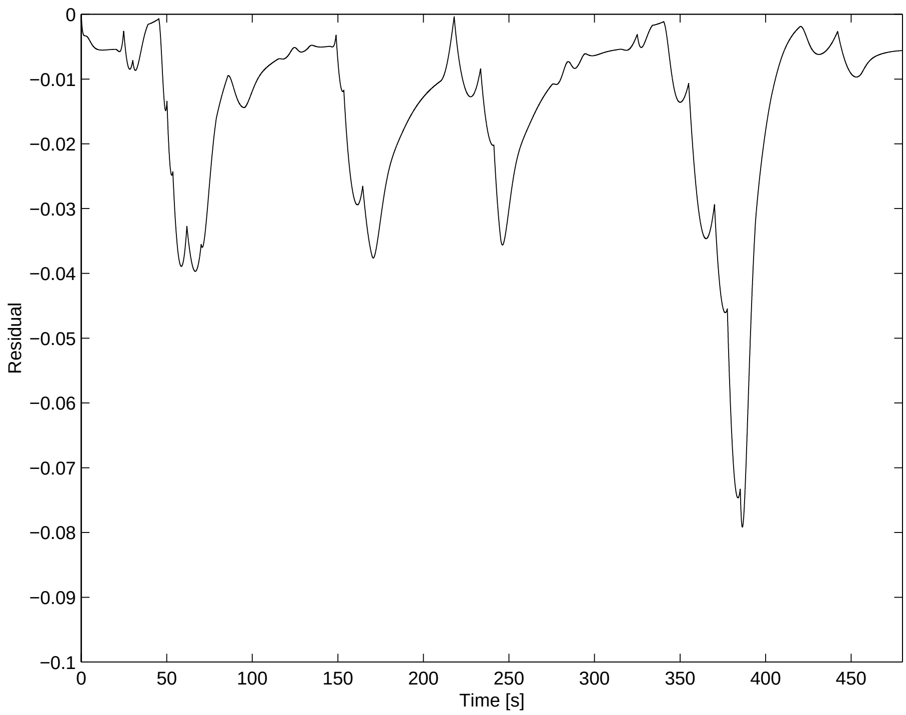

Figure 5.2: Residual 5 in the fault free case for the whole test cycle (0–480 s), when using linear interpolation as gain scheduling strategy. During transients the residual becomes bigger, due to the difference between the nonlinear model and the gain scheduled (linear) model.

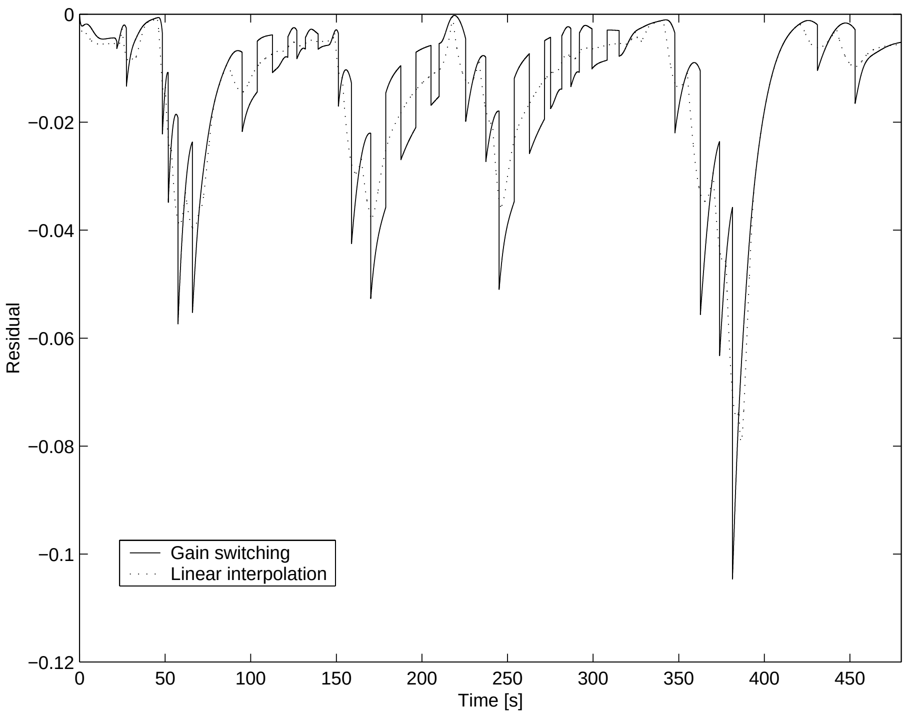

Figure 5.3: Residual 5 in the fault free case for the whole test cycle (0–480 s), when using linear interpolation and gain switching as gain scheduling strategy. During transients the residual becomes bigger, due to the difference between the nonlinear model and the gain scheduled (linear) model.

big or small). In Figure 5.3 the gain switched variant is also shown together with the linearly interpolated variant. The second method here proves to be better since the peak value during a transient is smaller for linear interpolation, than for gain switching. This suggests that linear interpolation should be used. (The reason for the residual being larger for the last transient is yet unknown.) During transients a weighting factor on the significance of the residual could be introduced, being small when the system is subject to transient behaviour in the scheduling variable. This is not investigated any further here.

5.2.2 Chatter

The chattering problem is now considered. Test region no 5 (from Table 5.1) with parts of the neighbouring intervals is used in Figure 5.4, where residual 1 is simulated using gain switching and linear interpolation in the fault free case. The gain switched type has large discontinuities and higher amplitude than the linearly interpolated type. For comparison purposes a constant fault $f_{f1} = 0.5$ is introduced (at $t = 15$ s) and the resulting residuals for both strategies are shown in Figure 5.5 together with the fault free case from Figure 5.4. The

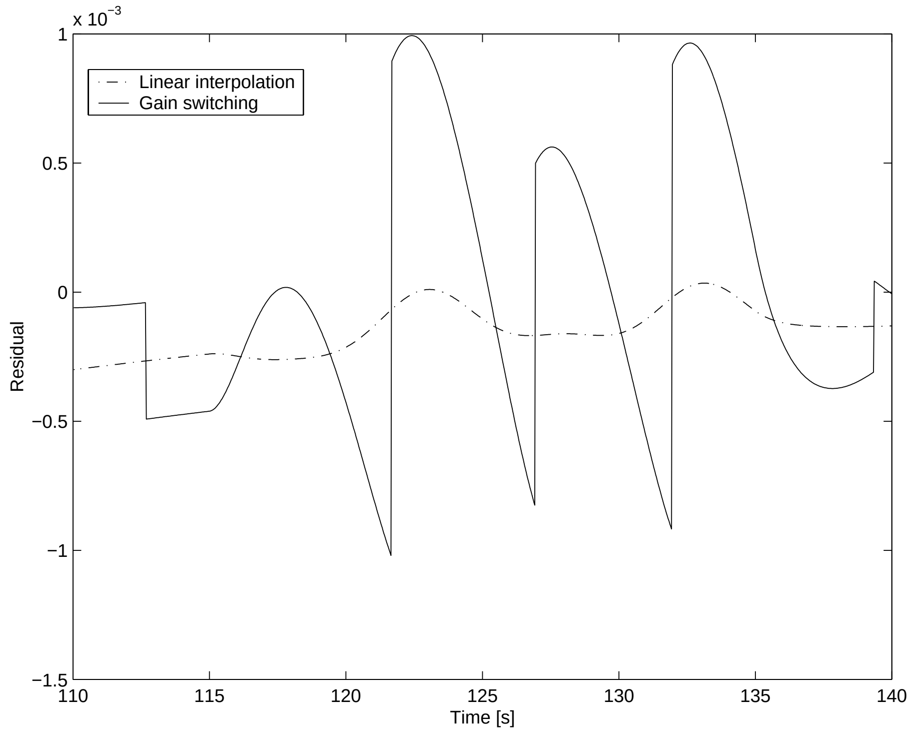

Figure 5.4: Residual 1 for the time interval 110–140 s, when using gain switching and linear interpolation as gain scheduling strategies in the fault free case. The gain switching yields as a more abruptly changing residual.

levels of the residuals subjected to the fault have a clearly larger amplitude, so evaluation would not be a problem here. But the ratio between the residual subject to f_{f1} and the residual in the fault free case is much larger in the linearly interpolated case, suggesting that smaller faults could be detected with the linearly interpolated case.

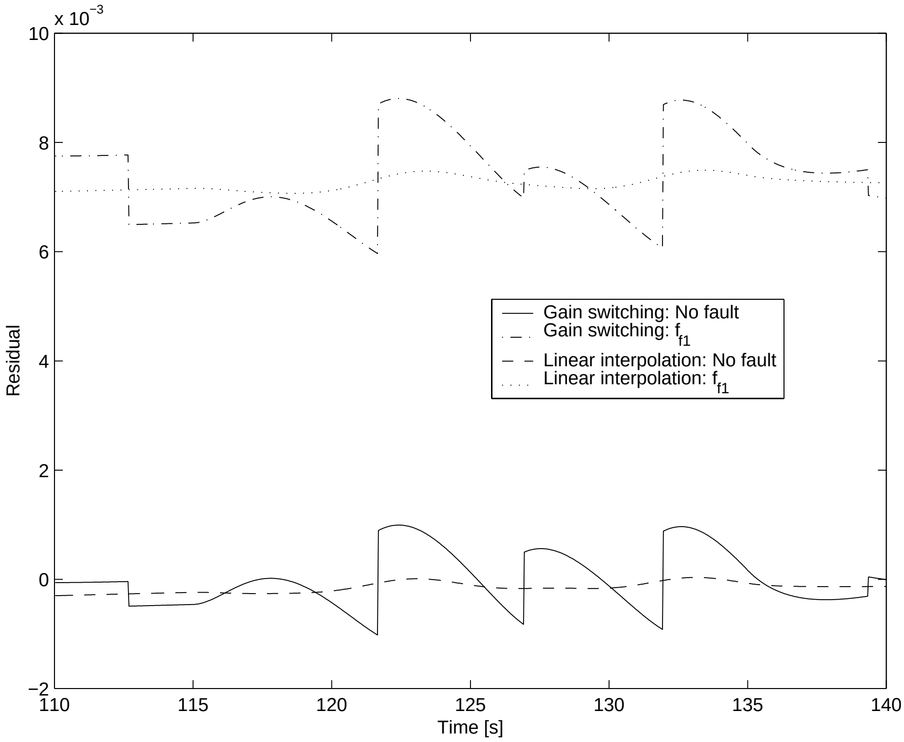

Figure 5.5: Residual 1 for the time interval 110–140 s, when using gain switching and linear interpolation as gain scheduling strategies, in both the fault free case and in the case with the fault $f_{f1} = 0.5$.

5.2.3 Decoupling Problem for a Fault in the Scheduling Variable

A so far neglected problem, is that the use of the measurement signal y_1 as the gain scheduled variable h_1 could severly degrade the performance of the diagnosis system. When the scheduling variable is subject to a fault, the consequence will be that the diagnosis system concludes that the water tank system is at a particular operating point, different from the actual operating point of the water tank system. That situation has occured in Figure 5.6, where residual 1 is shown using linear interpolation as gain scheduling strategy. The residual is shown in the fault free case, together with the residuals when the faults f_{h1} and f_{f1} respectively, are introduced at $t = 15$ s. According to the design of residual 1 (see Table 4.1) the fault f_{h1} should be decoupled, but apparently it is not. Comparing this to when the fault f_{f1} is introduced, the amplitude of the residual is smaller than for the case with the fault f_{h1}. Therefore evaluating the first residual as big and the second as small would therefore be very hard,

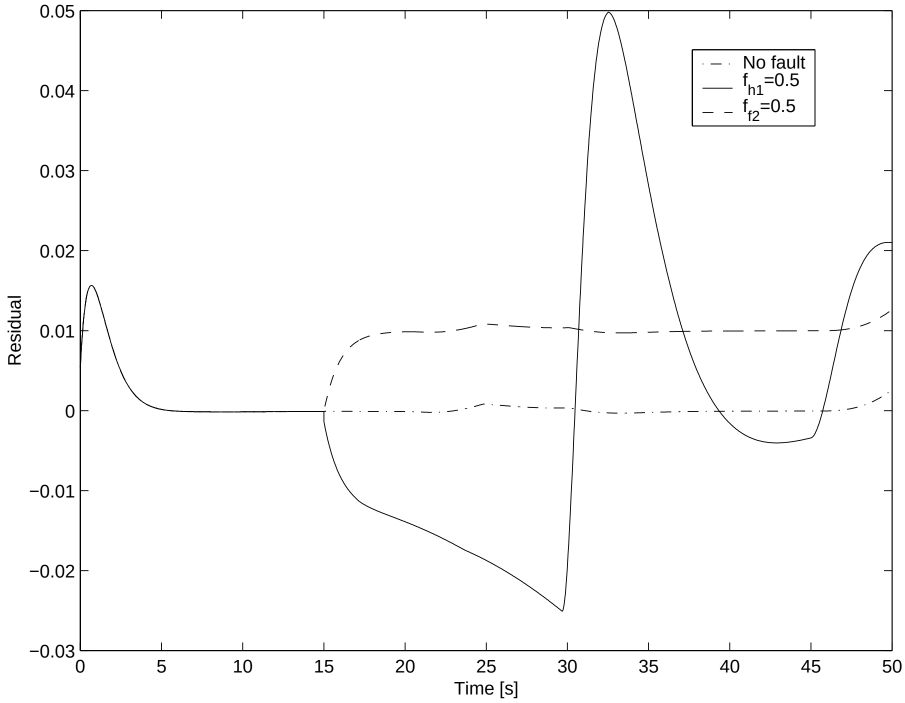

Figure 5.6: Residual 1 for the time interval 0–50 s, when using linear interpolation as gain scheduling strategy. The fault free case (dash-dotted line) is shown together with the cases with the fault $f_{h1} = 0.5$ (solid line) and with the fault $f_{f2} = 0.5$ (dashed line). The faults are introduced at $t = 15$ s. Although residual 1 should decouple f_{h1}, it does not. Comparing with the residual when f_{f2} is introduced, it can be concluded that f_{h1} is not decoupled.

which we want to do according to the definition of a residual (Definition 2.1).

This is a problem for the situation in Figure 5.6, but not for the situation in Figure 5.7, where the same faults are introduced at $t = 180$ s. Here residual

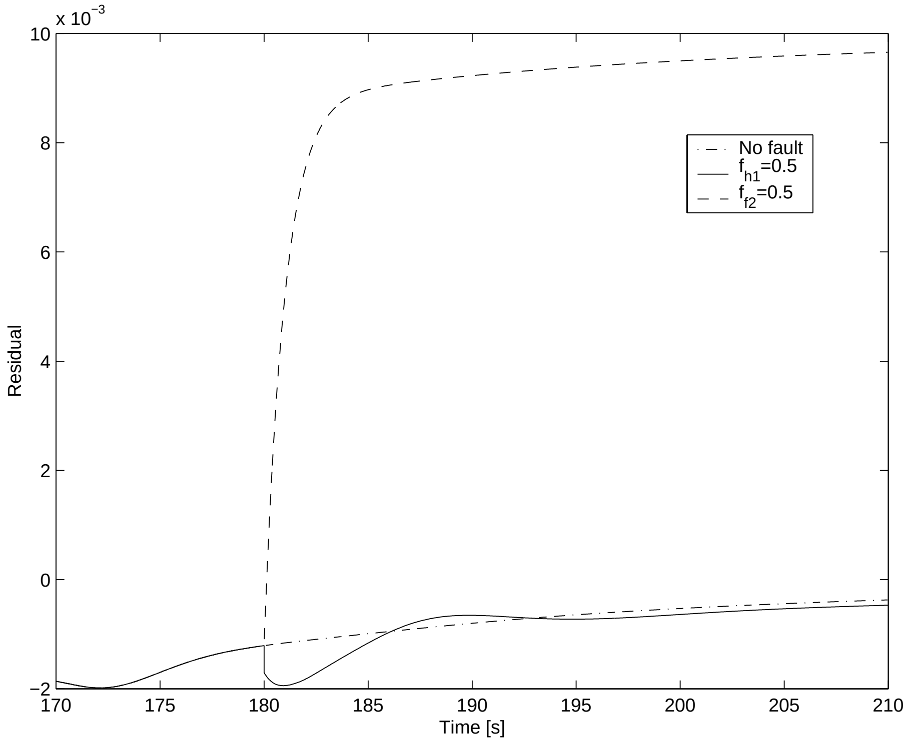

Figure 5.7: Residual 1 for the time interval 170–210 s, when using linear interpolation as gain scheduling strategy. The fault free case (dash-dotted line) is shown together with the cases with the fault $f_{h1} = 0.5$ (solid line) and with the fault $f_{f2} = 0.5$ (dashed line). The faults are introduced at $t = 180$ s. Comparing f_{h1} with the residual when f_{f2} is introduced, it can be concluded that f_{h1} is decoupled.

1 subject to the fault f_{f1} is much larger than residual 1 subject to the fault f_{h1}. The last one is approximately the same as the fault free case, suggesting that the fault f_{h1} is approximately decoupled. The better decoupling of the fault f_{h1} in test region 8 than in the combined test region 1 and 2, is expected since the coefficients in the residual generator change quicker for low water tank levels than for higher water tank levels, as shown in Figure 4.1. This suggests that for the low level case that the linearization at a certain design point has a smaller validity range than the high level case, which corresponds well to the observation that the nonlinear model is approximated better by a linear model in the high level case, than in the low level case.

To solve the problem with the decoupling of a fault in the scheduling variable, we could either put in an extra “1” in the residual structure in Table 4.1 and thereby limit the isolation abilities of the diagnosis system or we could use e.g.

an observer for h_1 not using measurement signal y_1, as the scheduling variable for (at least) residual 1. If the observer follows the real water level in tank 1 well, the fault f_{h1} would probably be decoupled in residual 1.

5.2.4 The cases (Operation Point $\neq$ Design Point) and (Operation Point = Design Point)

With a new test cycle (see Figure 5.8), where the reference signal is constant during longer periods than in the earlier defined test cycle (see Figure 5.1), the residuals are allowed to find their final value. In Figure 5.9 and in Figure 5.10 residual 1 is shown using gain switching and linear interpolation respectively. In the first time interval (0–60 s) the residual converges to a nonzero value (although small), showing that perfect decoupling is not achieved when the actual operating point is not a design point, or rather that we have a model fault. In the second time interval (70–130 s) the residual converges to zero, showing that perfect decoupling can be achieved, since the model now is perfect. This shows that one should not expect the residual generator to work as well between design points as at design points. The final values of the residuals in the first time interval are approximately the same for the gain switching and linear interpolation strategy. In the transient region (60–70 s) we once again

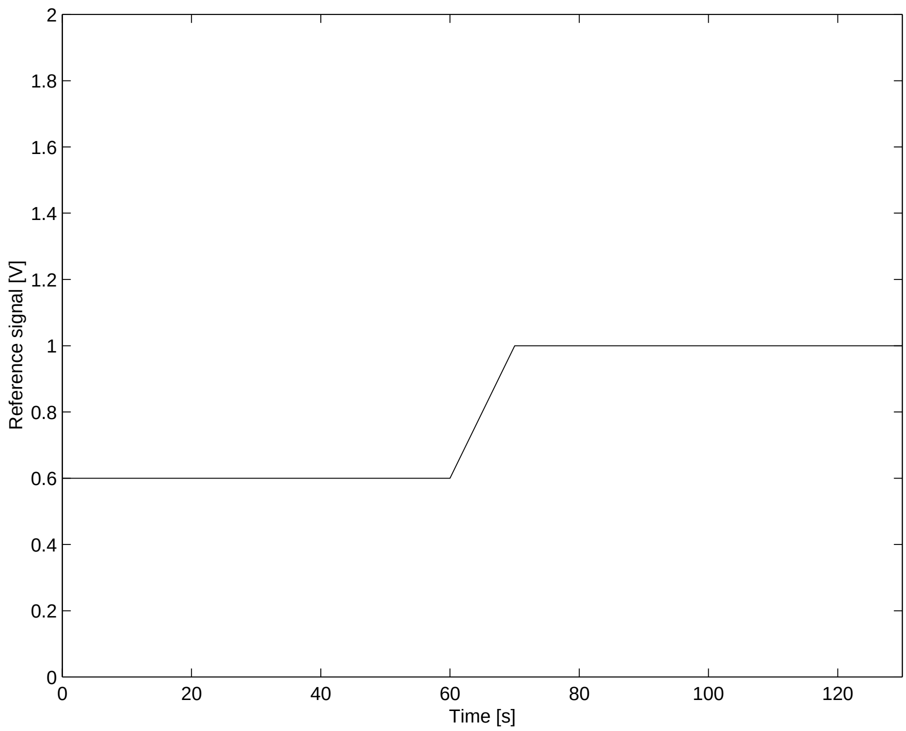

Figure 5.8: The reference signal for a new test cycle with longer steady state periods than in Figure 5.1.

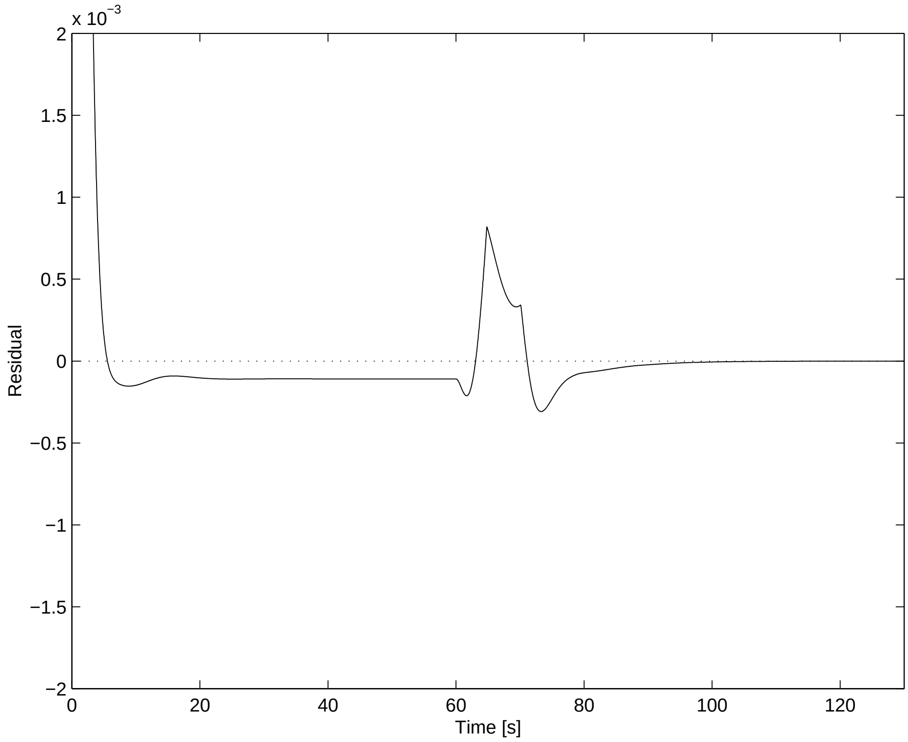

Figure 5.9: The steady state behaviour for residual 1 (linearly interpolated case) in the fault free case, for $h_1 = 0.6$ (between dp 2 and 3) in the time interval 0–60 s and for $h_1 = 1$ (dp 4) in the time interval 70–130 s. Note the small amplitude for the steady-state conditions for $t = 30$ s to $t = 60$ s and for $t = 100$ s to $t = 130$ s. The dotted line is zero.

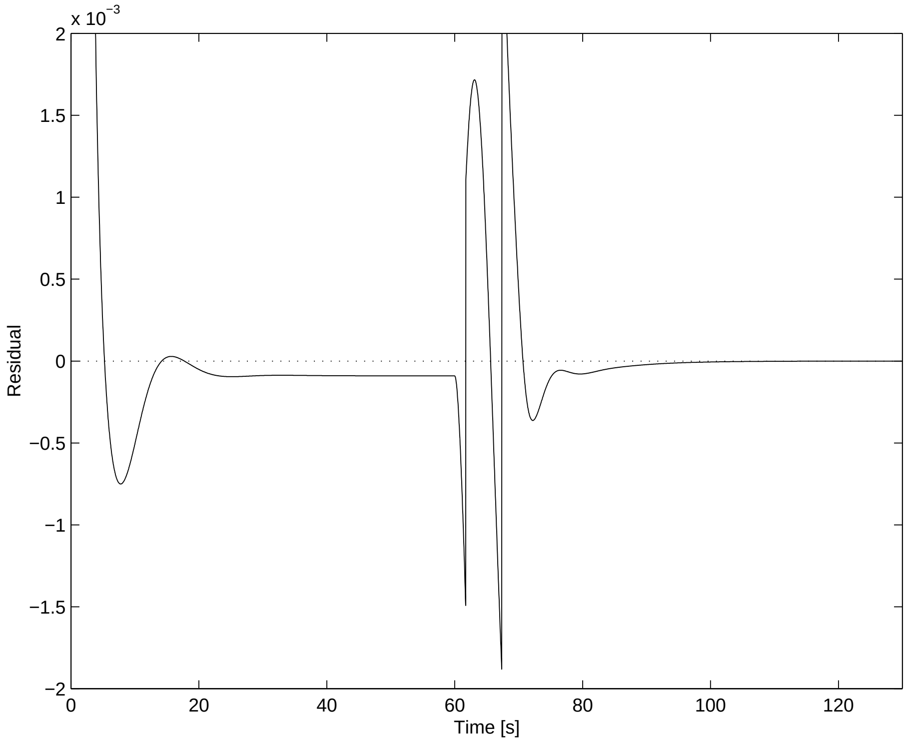

Figure 5.10: The steady state behaviour for residual 1 (gain switched case) in the fault free case, for $h_1 = 0.6$ (between dp 2 and 3) in the time interval 0–60 s and for $h_1 = 1$ (dp 4) in the time interval 70–130 s. Note the small amplitude for the steady-state conditions for $t = 30$ s to $t = 60$ s and for $t = 100$ s to $t = 130$ s. The dotted line is zero.

see that the transient behaviour of the linear interpolation strategy is better than the gain switched strategy.

Chapter 6

Conclusions

Design of residual generators for a coupled water tank system has been done using *gain scheduling*. Simulations of the water tank system have been performed to evaluate the behaviour of the residual generators. The nonlinear system is linearized in different stationary points, and residual generators are then designed using the minimal polynomial basis method and a newly proposed optimization method. Two interpolation methods have then been used as gain scheduling strategies, namely nearest neighbour approximation and linear interpolation for the coefficients in the residual generators. Simulations show that during steady state operation, the residual generators behave well, but during transients the performance of the residual generators decrease significantly. Simulations also propose that linear interpolation as gain scheduling strategy is better than nearest neighbour (gain switching) approximation, especially during transients.

A fault in the scheduling variable causes problems for the diagnosis system, especially for low water tank levels, since the system is not approximated well by the linearizations in that operating range. It is also proposed that different scheduling variables should be used, or rather that they should be calculated in different ways, using different signals from the system. This to make sure that a fault in the scheduling variable can be decoupled.

Future work

To attack the problem with the transient behaviour of the residual generator, one feasible method could be to put in weighting factor on the residual, small during transients on the scheduling variable and big during steady state operation. Another problem is how dense must the distribution of design points be, to give the resulting diagnosis system good performance.

Appendix A

Residuals from the Simulation of the Test Case

In this Appendix the residuals from the simulation of the water tank system with the 480 s long test cycle defined in Chapter 5 (Figure 5.1 and Table 5.1) are shown. The water tank system is simulated both in the fault free case and when a single fault occurs at $t = 15$ s for all of the five faults mentioned in Chapter 3 (Table 3.1). The residuals from both the gain switching strategy (Figure A.1–A.5) and the linear interpolation strategy (Figure A.6–A.10) are shown. Note that the scales in the figures are not always the same.

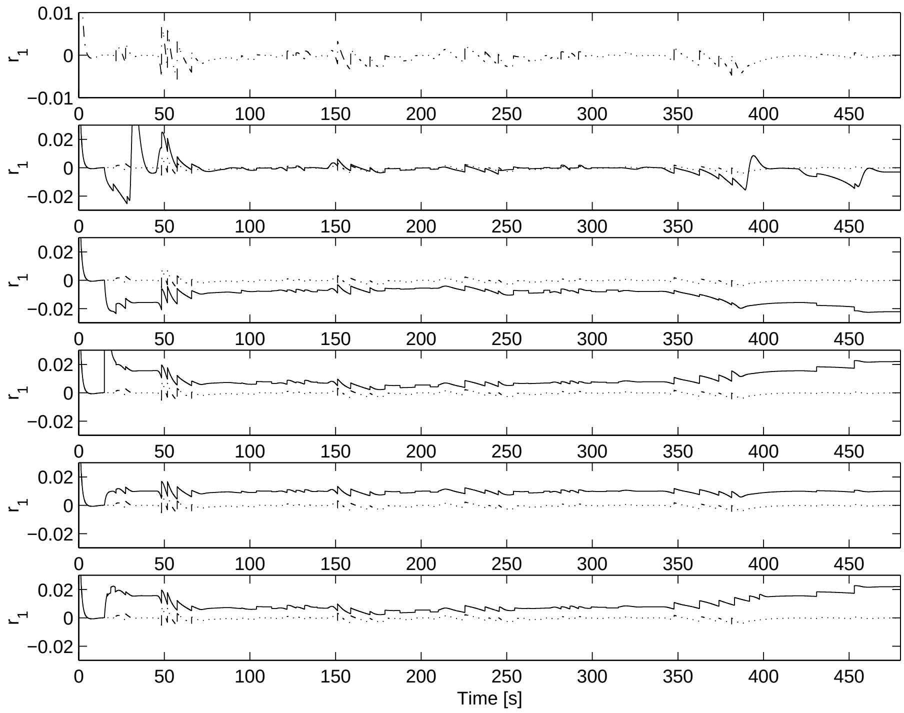

Figure A.1: Residual 1 for the test cycle when using gain switching as gain scheduling strategy. The first figure shows the residual in the fault free case (dotted line). The following figures show residual 1 when the fault f_{h1}, f_{h2}, f_{f1}, f_{f2} and f_a respectively (solid line) have been introduced after 15 s, together with the fault free case (dotted line).

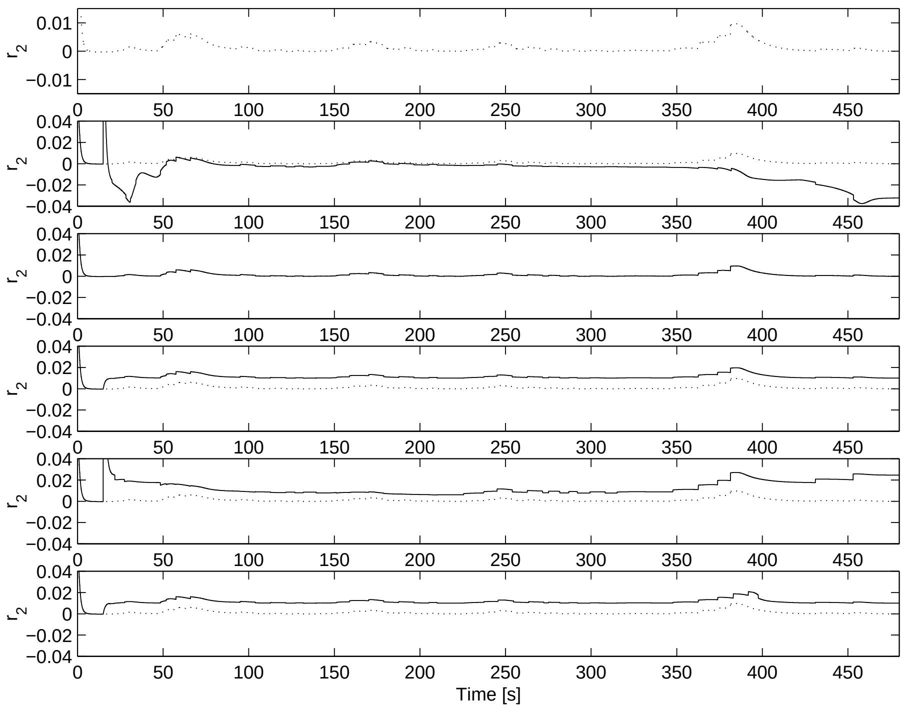

Figure A.2: Residual 2 for the test cycle when using gain switching as gain scheduling strategy. The first figure shows the residual in the fault free case (dotted line). The following figures show residual 2 when the fault f_{h1}, f_{h2}, f_{f1}, f_{f2} and f_a respectively (solid line) have been introduced after 15 s, together with the fault free case (dotted line).

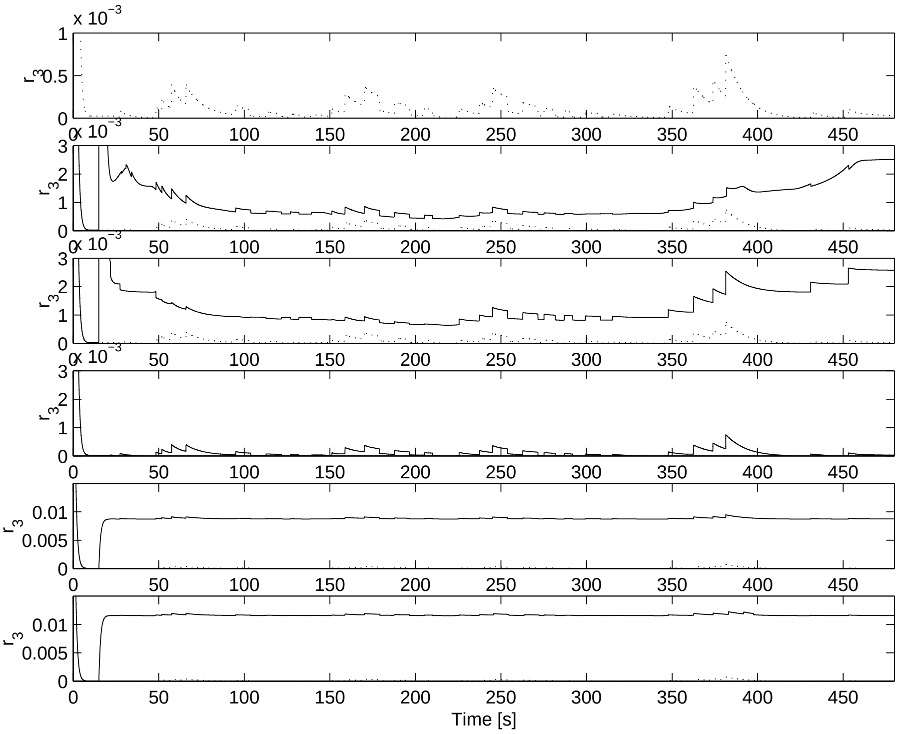

Figure A.3: Residual 3 for the test cycle when using gain switching as gain scheduling strategy. The first figure shows the residual in the fault free case (dotted line). The following figures show residual 3 when the fault f_{h1}, f_{h2}, f_{f1}, f_{f2} and f_a respectively (solid line) have been introduced after 15 s, together with the fault free case (dotted line).

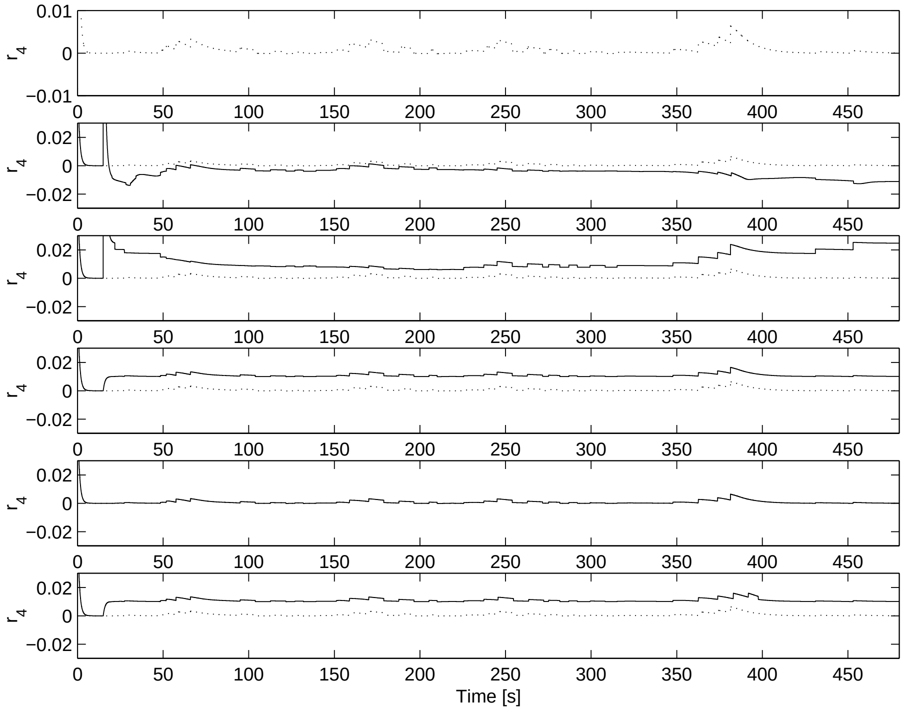

Figure A.4: Residual 4 for the test cycle when using gain switching as gain scheduling strategy. The first figure shows the residual in the fault free case (dotted line). The following figures show residual 4 when the fault f_{h1}, f_{h2}, f_{f1}, f_{f2} and f_a respectively (solid line) have been introduced after 15 s, together with the fault free case(dotted line).

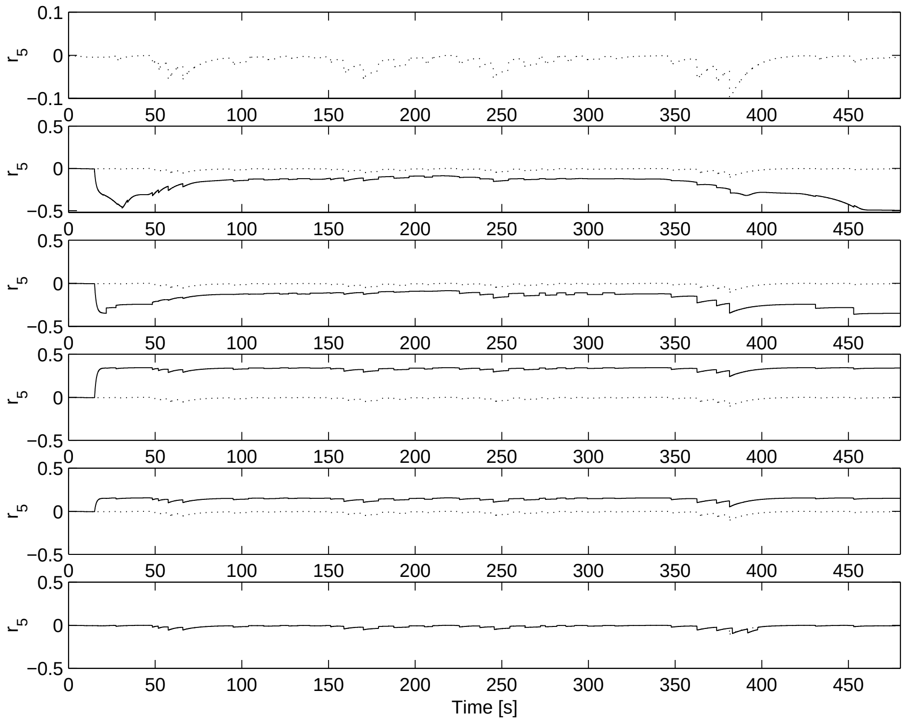

Figure A.5: Residual 5 for the test cycle when using gain switching as gain scheduling strategy. The first figure shows the residual in the fault free case (dotted line). The following figures show residual 5 when the fault f_{h1}, f_{h2}, f_{f1}, f_{f2} and f_a respectively (solid line) have been introduced after 15 s, together with the fault free case (dotted line).

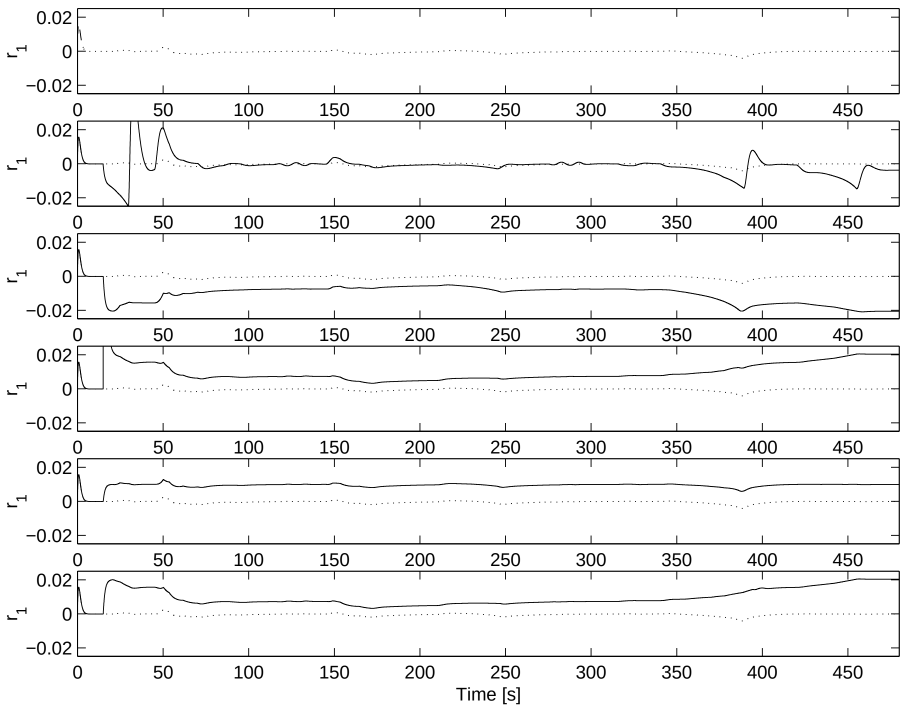

Figure A.6: Residual 1 for the test cycle when using linear interpolation as gain scheduling strategy. The first figure shows the residual in the fault free case (dotted line). The following figures show residual 1 when the fault f_{h1}, f_{h2}, f_{f1}, f_{f2} and f_a respectively (solid line) have been introduced after 15 s, together with the fault free case (dotted line).

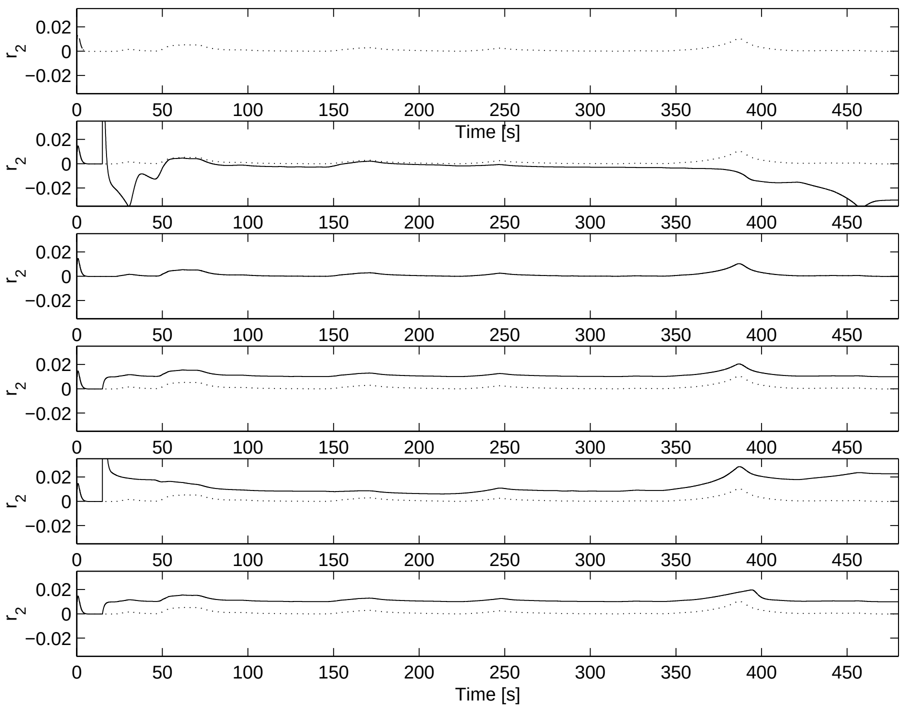

Figure A.7: Residual 2 for the test cycle when using linear interpolation as gain scheduling strategy. The first figure shows the residual in the fault free case (dotted line). The following figures show residual 2 when the fault f_{h1}, f_{h2}, f_{f1}, f_{f2} and f_a respectively (solid line) have been introduced after 15 s, together with the fault free case (dotted line).

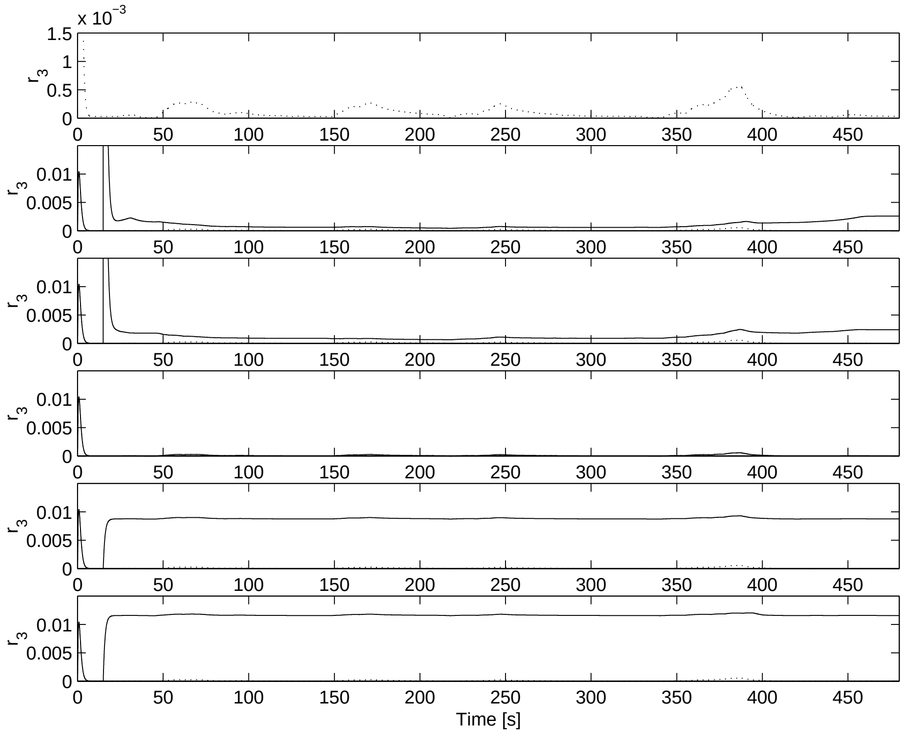

Figure A.8: Residual 3 for the test cycle when using linear interpolation as gain scheduling strategy. The first figure shows the residual in the fault free case (dotted line). The following figures show residual 3 when the fault f_{h1}, f_{h2}, f_{f1}, f_{f2} and f_a respectively (solid line) have been introduced after 15 s, together with the fault free case (dotted line).

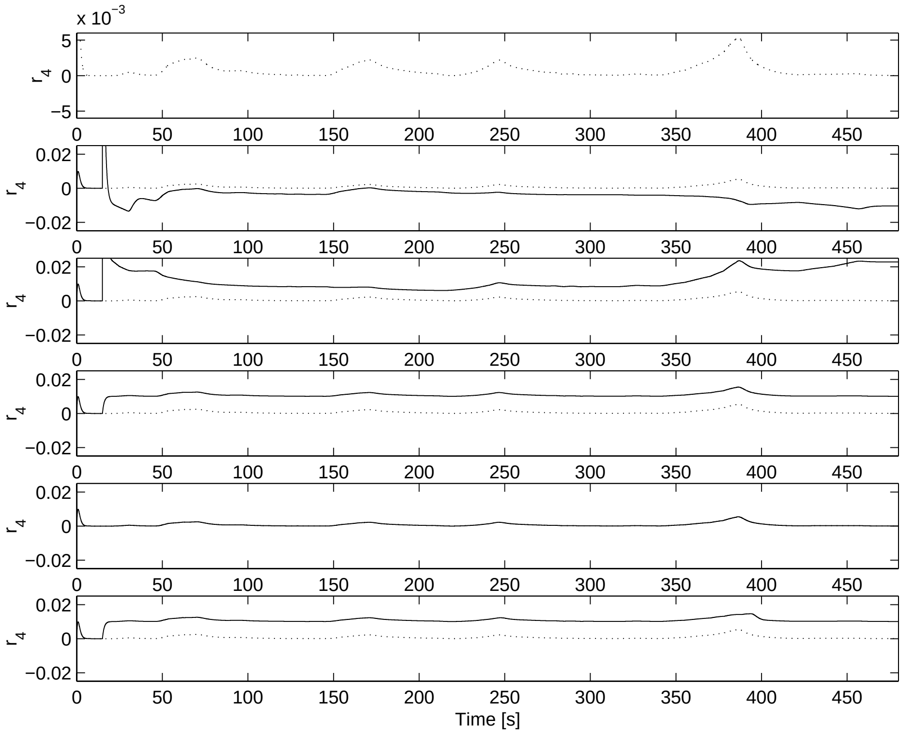

Figure A.9: Residual 4 for the test cycle when using linear interpolation as gain scheduling strategy. The first figure shows the residual in the fault free case (dotted line). The following figures show residual 4 when the fault f_{h1}, f_{h2}, f_{f1}, f_{f2} and f_a respectively (solid line) have been introduced after 15 s, together with the fault free case (dotted line).

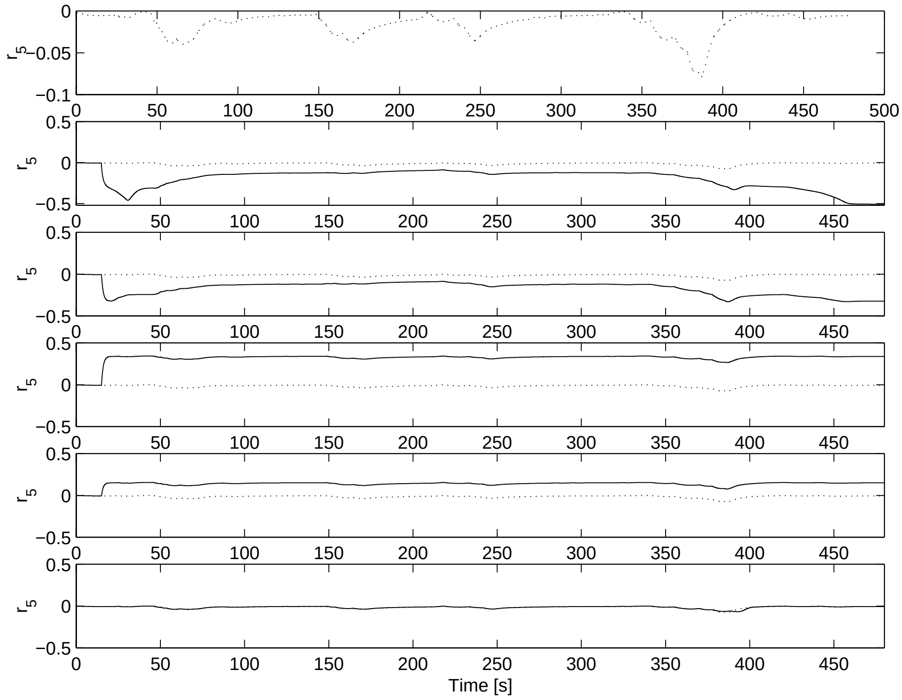

Figure A.10: Residual 5 for the test cycle when using linear interpolation as gain scheduling strategy. The first figure shows the residual in the fault free case (dotted line). The following figures show residual 5 when the fault f_{h1}, f_{h2}, f_{f1}, f_{f2} and f_a respectively (solid line) have been introduced after 15 s, together with the fault free case (dotted line).

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